An Examination of the General Relativity of the Bianchi type V Holographic Dark Energy Model Using the Hybrid Expansion Law

Shefali Yadav¹, Dr. Sanish Thomas², Prof. Ajit Paul³

¹Research Scholar, Department of Physics, Sam Higginbottom University of Agriculture, Technology and Sciences, Prayagraj U.P. (India)
²Assistant Professor, Department of Physics, Sam Higginbottom University of Agriculture, Technology and Sciences, Prayagraj U.P. (India)
³Professor and Head, Department of Mathematics and Statistics, Sam Higginbottom University of Agriculture, Technology and Sciences, Prayagraj U.P. (India)

Abstract- This examination presents an investigation of the qualities of anisotropic and homogeneous Bianchi type V space-time inside the setting of general relativity, while considering the consideration of dark matter and holographic dark energy model parts. The arrangements of the differential condition fields have been determined by looking at two unmistakable situations: one where the extension scalar in the model is straightforwardly relative to the shear scalar, and one more where the typical scale factor is expected to follow a half breed development structure. The answers for the field conditions are inferred inside the system of general relativity. A similar to peculiarity was found with regards to the tension and condition of state boundary of the holographic dark energy model inside their separate settings. This study researches the physical and mathematical parts of a few factors that hold infinite importance, including the jerk boundary inside our model. The noticed boundaries line up with the cosmological models that portray the present status of the universe, described by sped up extension. All in all, we state that our model looks very similar to the ΛCDM cosmological model all through the later stages and lines up with the most recent discoveries of cosmological information.

Keywords: Cosmological Constant, Hubble parameter, Deceleration parameter, jerk parameter

1. Introduction-

The Bianchi Type V cosmological model relates to a particular classification of arrangements got from Einstein's field conditions inside the system of general relativity. These arrangements intend to explain the overall association and improvement of the universe on a perceptible scale. The arrangement of these not entirely settled by the spatial balances displayed by the three-layered space segments. Bianchi Type V cosmological models show unmistakable development rates along every one of the three spatial headings. The elements of these models are administered by the energy structure of the universe, incorporating matter, radiation, and dark energy. The assessment of Bianchi Type V cosmological models at times involves the consideration of a dark energy part known as holographic Ricci dark energy. Dark energy is a hypothetical sort of energy that is accepted to exist all through the sum of room and is responsible for the noticed peculiarity of the universe's speeding up development. The idea of holographic Ricci dark energy coordinates standards from holography and dark energy. The holographic standard places that the furthest reaches of entropy or data inside a given spatial still up in the air by its surface region as opposed to its volume. The holographic Ricci dark energy model lays out an association between the thickness of dark energy and the Ricci scalar shape of the universe, while applying the previously mentioned guideline to the idea of dark energy. The Ricci scalar fills in as a measurement for evaluating the ebb and flow of room time. In the particular setting viable, it is utilized to
discover the energy thickness connected to dark energy. The holographic Ricci dark energy model addresses an undertaking to clarify the crucial qualities of dark energy by consolidating standards got from the domains of quantum gravity and holography. The examination of cosmological models with Bianchi Type V calculation and holographic Ricci dark energy requires the goal of appropriate conditions overseeing the elements of the universe, while considering the unmistakable properties of the Bianchi Type V math and the holographic Ricci dark energy model. Rahaman et al. (2002) inspected the Kantowski-Sachs model inside the system of Lyra's complex, explicitly zeroing in on the consideration of zero mass scalar fields portrayed by a level potential. Daouda et al. (2012) fostered a reproduction of the gravity model by utilizing holographic dark energy. The review directed by Jamil et al. (2012) analyzes the model of dark energy inside the structure of \((T)\) cosmology. In their examination, they treat dark energy as an ideal liquid and spotlight on a specific form of \((T) = \beta\sqrt{T}\) that is cosmologically plausible. Sharif and Azeem (2012) researched the way of behaving of the state boundary and energy thickness condition of dark energy inside the system of \((T)\) gravity. To achieve this, they utilized an anisotropic LRS Bianchi type I universe model. Venkateswarlu and Pavankumar (2010) inspected a plane symmetric model that integrated massless scalar fields close by an estimable string source. Venkateswarlu and Satish (2014) inspected the Kantowski-Sachs cosmological model, explicitly zeroing in on the consideration of a massless scalar field and the presence of mass thick enormous strings. Singh and Rani (2015) led a concentrate on the cosmological models of Bianchi type-III inside the structure of Lyra’s calculation, while thinking about the impact of a huge scalar field. The creators Reddy et al. (2018) have analyzed the Bianchi type-II cosmological model with dark energy, explicitly zeroing in because of scalar-meson fields. The elements of an ideal liquid cosmological model within the sight of a huge scalar field in f(R,T) gravity has been analyzed by Aditya and Reddy (2019). Naidu et al. (2019) inspected the qualities of a cosmological model including dark energy inside a spatially homogenous and anisotropic Bianchi type-V system. This examination was directed inside the setting of general relativity, integrating the presence of a drawing in huge scalar field. The specialists performed estimations to decide the cosmological boundaries related with dark energy, like its thickness, condition of state boundary, skewness boundaries, deceleration boundary, and state locator boundaries. They then, at that point, examined the actual ramifications of these boundaries inside the setting of the ongoing comprehension of the universe’s sped up development and perceptions made in cosmology. Reddy and Ramesh (2019) presented a clever model of dark energy inside the system of general relativity. This model was figured out in a five-layered Kaluza-Klein anisotropic space-time and consolidated scalar-meson fields. The dynamical boundaries, including the condition of state (EoS) boundary, energy thickness, deceleration boundary, and jerk boundary, were developed by the scientists. Santhi et al. (2022) directed an examination on spatially homogeneous and anisotropic space-seasons of Bianchi type-II, VIII, and IX. The examination was done inside the setting of the Grains Dicke hypothesis of gravity, consolidating the system of thick holographic dark energy. Ugale and Dhore (2023) led an examination on Bianchi type-I arrangements inside the structure of the \((T)\) hypothesis of gravity, using a consistent deceleration boundary. The client's text is as of now scholastic in nature. The current review researches the elements of the universe by utilizing Bianchi Type V cosmological models. The essential goal is to look at the results of Holographic Ricci Dark Energy in this specific situation. The Bianchi Type V cosmological model is portrayed by both anisotropy and spatial homogeneity. In this review, we analyze the elements of this model when it is affected by the presence of dark energy, which is managed by holographic standards. The examination starts by giving a brief prologue to the Bianchi Type V cosmological models and their importance in clarifying the perceptible association of the universe. The ensuing accentuation is coordinated towards the combination of Holographic Ricci Dark Energy, a hypothetical structure that lays out an association between the thickness of dark energy and the entropy of the grandiose skyline. In this talk, we will investigate the numerical system, by which we will present the Ricci scalar and clarify its importance in deciding the mathematical properties of the universe. The usage of the Einstein field conditions is utilized to display gravitational collaborations inside the astronomical structure. Changes are consolidated to represent the holographic person of dark energy. The extent of our examination incorporates the examination of critical qualities that apply effect on the developmental direction of the universe, including the Hubble boundary and scale factor. The target of this study is to examine the impact of Holographic Ricci Dark Energy on the inestimable extension and a definitive predetermination of the universe, using both mathematical recreations and scientific responses. The conversation of
the outcomes of our discoveries is arranged inside the more extensive system of contemporary cosmology and our appreciation of the speeding up development of the universe. The review's discoveries offer important commitments to the proceeding with academic conversation encompassing the qualities of dark energy. These bits of knowledge upgrade our appreciation of what dark energy means for the design of the universe, especially inside the setting of Bianchi Type V cosmological models. In rundown, this study coordinates hypothetical ideas from the fields of calculation, holography, and cosmology to give knowledge into the mind boggling connection between spatial anisotropy and the elements of dark energy in the universe. This examination offers an itemized comprehension of the development of our inestimable environmental elements.

2. Quantum, Field, and Metric Equations—The spatially homogenous and anisotropic Bianchi type V space-time is taken into consideration.

\[ ds^2 = dt^2 - \left(A_1^2 + A_2^2 e^{-2ax} + A_3^2 e^{-2ay}\right) dx^2 \]  

where \( \alpha \) addresses an erratic consistent, and \( A_1, A_2, \) and \( A_3 \) are capabilities that exclusively rely upon astronomical time \( t \).

The Einstein field conditions are planned as follows:

\[ R_{ij} = \frac{1}{2} R g_{ij} = -\frac{8\pi G}{c^2} T_{ij} \]  

In the given condition, the images \( R_{ij} \), \( R \), \( g_{ij} \), \( G \), and \( c \) address the Ricci tensor, Ricci scalar, metric tensor, Newton's gravitational steady, and speed of light, separately. In this specific situation, we analyze the presumption that the upsides of \( 8\pi G \) and \( c \) are identical to 1. The energy force tensor, indicated as \( T_{ij} \), can be addressed as the mix of two parts: the energy force tensor of dark matter \( (T_{mij}) \) and the energy tensor of holographic dark energy \( (T_{\Lambda ij}) \).

Anisotropic tensions along different spatial tomahawks give the energy force tensor of holographic dark energy \( (T_{\Lambda ij}) \) of the source the shape

Anisotropic pressures along various spatial axes give the energy momentum tensor of holographic dark energy \( (T_{\Lambda ij}) \) of the source the shape

\[ T_{\Lambda ij} = \text{diag}\left[\rho_{\Lambda}, -p_x, -p_y, -p_z\right] = \text{diag}\left[1, -\omega_x, -\omega_y, -\omega_z\right] \rho_{\Lambda} \]  

The images \( \rho_{\Lambda} \) and \( p_i (x, y, z) \) address the energy thickness and tensions, separately, of holographic dark energy (HDE) in the three spatial headings of the universe. The condition of state (EoS) boundary lays out a connection among \( \rho_{\Lambda} \) and \( p_i \), meant as \( p_i = \omega_i \rho_{\Lambda} \). The condition of state boundaries \( \omega_i \) address the properties of the framework along the \( x, y, \) and \( z \) headings.

In this unique circumstance, it is proposed that the universe is swarmed by dark matter and dark energy, portrayed by energy force tensors that can be communicated as.

\[ T_{mij} = \text{diag}\left[1, 0, 0, 0\right] \rho_m \]  

\[ T_{\Lambda ij} = \text{diag}\left[1, -\omega_{\Lambda}, -\omega_{\Lambda} + \delta_y, -\omega_{\Lambda} + \delta_z\right] \rho_{\Lambda} \]  

The images \( \rho_m \) and \( \rho_{\Lambda} \) address the energy densities of dark matter and holographic dark energy (HDE), individually. Furthermore, \( \rho_{\Lambda} \) indicates the strain related with the HDE. To keep up with effortlessness, we settle on the equity of \( \omega_y \) and \( \omega_z \).

The conditions \( \omega_y = \omega_{\Lambda} + \delta_y, \omega_z = \omega_{\Lambda} + \delta_z \) address the connection between the boundaries \( \omega_y \) and \( \omega_z \), and the deviations \( \delta_y \) and \( \delta_z \) from the situation of state boundary \( \omega_{\Lambda} \) in the \( y \) and \( z \) headings, separately.
The usage of comoving arranges considers the deduction of conditions coming about because of the field condition (2) applied to the measurement (1) related to the energy-force tensors (5).

\[
\frac{\dot{A}_2^2}{A_2} + \frac{\dot{A}_3^2}{A_3} + \frac{\dot{A}_2 A_3}{A_2 A_3} - \frac{\alpha^2}{A_1^2} = \omega_A \rho_A
\]  
\tag{6}

\[
\frac{\dot{A}_1^2}{A_1} + \frac{\dot{A}_3^2}{A_3} + \frac{\dot{A}_1 A_3}{A_1 A_3} - \frac{\alpha^2}{A_1^2} = -(\omega_A + \delta_y) \rho_A
\]  
\tag{7}

\[
\frac{\dot{A}_1}{A_1} + \frac{\dot{A}_2}{A_2} + \frac{\dot{A}_1 A_2}{A_1 A_2} - \frac{\alpha^2}{A_1^2} = -(\omega_A + \delta_x) \rho_A
\]  
\tag{8}

\[
\frac{2\dot{A}_1}{A_1} - \frac{A_2}{A_2} - \frac{\dot{A}_3}{A_3} = 0
\]  
\tag{9}

(10)

The above dab in the situation addresses separation with respect to vast time, meant as \( t \).

By coordinating condition (15) and integrating the steady of joining into either \( A_2 \) or \( A_3 \), we can determine the accompanying outcome.

\[
A_1^2 = A_2 A_3
\]  
\tag{11}

3. The resolution of field equations- It very well may be seen that conditions (11)-(15) comprise a bunch of five unmistakable conditions, each addressing a free relationship. These conditions include eight boundaries that are still up in the air, in particular \( A_1, A_2, A_3, \omega_A, \rho_m, \rho_A, \delta_y \) and \( \delta_x \). To accomplish a far reaching answer for the framework, it is important to present extra standards that will empower the inference of an unequivocal arrangement. We, first and foremost, analyze the mixture extension regulation proposed by Akarsu et al. (2014), wherein the typical scale factor

\[
R(t) = R_0 \left( \frac{t}{t_0} \right)^\frac{a}{b} \exp \left( \frac{3a}{b} \left( \frac{t}{t_0} - 1 \right) \right)
\]  
\tag{12}

In the given setting, let \( a \) and \( b \) address non-negative constants, while \( R_0 \) and \( t_0 \) separately address the ongoing worth of the typical scale component and age of the universe. The condition (12) lays out the dramatic rule in cosmology when \( a \) is equivalent to nothing, and gives the power regulation in cosmology for \( b = 0 \). Besides, this crossover indication of the mean scale factor worked with a consistent change of the universe from its underlying decelerating stage to its current speeding up stage.

It is hypothesized that the extension scalar (\( \theta \)) in the model displays proportionality to the shear scalar (\( \sigma \)), as recently proposed by Thorne (1967) and Collins et al. (1980), coming about in

\[
A_1 = A_2^m, \quad m > 0 \text{ and } m \neq 1
\]  
\tag{13}

where \( A_1 \) and \( A_2 \) are the metric potentials.

\[
R^3 = A_1 A_2 A_3
\]  
\tag{14}

\[
R^3 = R_0^3 \left( \frac{t}{t_0} \right)^{3a} \exp \left\{ 3b \left( \frac{t}{t_0} - 1 \right) \right\}
\]

\[
R_0^3 \left( \frac{t}{t_0} \right)^{3a} \exp \left\{ 3b \left( \frac{t}{t_0} - 1 \right) \right\} = A_1 A_1^2 = A_1^3
\]

\[
A_1 = R_0 \left( \frac{t}{t_0} \right)^a \exp \left\{ b \left( \frac{t}{t_0} - 1 \right) \right\}
\]  
\tag{15}

\[
A_1 = A_2^m
\]
\[ A_2 = R_0 \left( \frac{t}{\tau_0} \right)^{\frac{a}{m}} \frac{b(t/\tau_0 - 1)}{e^{m(t/\tau_0 - 1)}} \]  

\[ R^3 = A_1 A_2 A_3 \]

\[ A_3 = \frac{R^3}{A_1 A_2} \]

\[ A_3 = R_0 \left( \frac{t}{\tau_0} \right)^{3a} e^{3b(t/\tau_0 - 1)} \]

\[ = R_0 \left( \frac{t}{\tau_0} \right)^{3a - \frac{a}{m}} e^{3b(t/\tau_0 - 1) - \frac{b}{m}(t/\tau_0 - 1)} \]

\[ = R_0^{2m} \left( \frac{t}{\tau_0} \right)^{2a - \frac{a}{m}} e^{3b(t/\tau_0 - 1) - \frac{b}{m}(t/\tau_0 - 1)} \]

\[ A_3 = R_0^{2m} \left( \frac{t}{\tau_0} \right)^{a(2 - \frac{1}{m})} e^{b(2 - \frac{1}{m})(t/\tau_0 - 1)} \]  

\[ \dot{A}_1 = R_0 \left( \frac{t}{\tau_0} \right)^{\frac{a}{m}} \left( bt + at_0 \right) e^{b(t/\tau_0 - 1)} \]

\[ \dot{\dot{A}}_1 = R_0 \left( \frac{t}{\tau_0} \right)^{\frac{a}{m}} \left[ b^2 t^2 + 2ab t t_0 + (a^2 - a) t_0^2 \right] e^{b(t/\tau_0 - 1)} \]

\[ \dot{A}_2 = R_0^m \left( \frac{t}{\tau_0} \right)^{\frac{a}{m}} \left( bt + at_0 \right) e^{b(t/\tau_0 - 1)} \]

\[ \dot{\dot{A}}_2 = R_0^m \left( \frac{t}{\tau_0} \right)^{\frac{a}{m}} \left[ b^2 t^2 + 2ab t t_0 + (a^2 - am) t_0^2 \right] e^{b(t/\tau_0 - 1)} \]

\[ \dot{A}_3 = R_0^m \left( \frac{t}{\tau_0} \right)^{\frac{a}{m}} \left( m^2 - \frac{1}{2} \right) \left( bt + at_0 \right) e^{b(t/\tau_0 - 1)} \]

\[ \dot{\dot{A}}_3 = R_0^m \left( \frac{t}{\tau_0} \right)^{\frac{a}{m}} \left[ b^2 (m^2 - 2) t^2 + 2ab (m-2) t t_0 + [a^2 (m-2) - am^2] t_0^2 \right] e^{b(t/\tau_0 - 1)} \]

\[ \frac{\dot{A}_1}{A_1} = \frac{R_0 \left( \frac{t}{\tau_0} \right)^{\frac{a}{m}} \left( bt + at_0 \right) e^{b(t/\tau_0 - 1)}}{R_0 \left( \frac{t}{\tau_0} \right)^{\frac{a}{m}} \left( bt + at_0 \right) e^{b(t/\tau_0 - 1)}} = \frac{bt + at_0}{bt_0} = \frac{b}{b + a} \]

\[ \frac{\dot{A}_2}{A_2} = \frac{R_0^m \left( \frac{t}{\tau_0} \right)^{\frac{a}{m}} \left( bt + at_0 \right) e^{b(t/\tau_0 - 1)}}{R_0^m \left( \frac{t}{\tau_0} \right)^{\frac{a}{m}} \left( bt + at_0 \right) e^{b(t/\tau_0 - 1)}} = \frac{(bt + at_0)}{mt_0} = \frac{b}{m (bt_0 + a)} \]
\[
\begin{align*}
\frac{A_3}{A_3} &= \frac{R_0^{-\frac{1}{2}} (m-2) t_0^2 (t_0^2)^{\frac{a^2 + a}{m}} e^{\frac{b - 1}{m} t_0^2}}{R_0^{-\frac{1}{2}} (m-2) t_0^2 (t_0^2)^{\frac{a^2 + a}{m}} e^{\frac{b - 1}{m} t_0^2}} = \frac{m-2}{m} \left( \frac{b}{t_0} + \frac{a}{t} \right) \\
H &= \frac{1}{3} \left( \frac{A_1}{A_1} + \frac{A_2}{A_2} + \frac{A_3}{A_3} \right) = \frac{1}{3} \left( \frac{b}{t_0} + \frac{a}{t} \right) \left( 1 + \frac{1}{m} + \frac{m-2}{m} \right) \\
H &= \frac{1}{3} \left( \frac{A_1}{A_1} + \frac{A_2}{A_2} + \frac{A_3}{A_3} \right) = \frac{1}{3} \left( \frac{b}{t_0} + \frac{a}{t} \right) \left( \frac{m+1+m-2}{m} \right) = \frac{1}{3} \left( \frac{2m-1}{m} \right) \left( \frac{b}{t_0} + \frac{a}{t} \right) \\
\theta &= 3H = \left( \frac{2m-1}{m} \right) \left( \frac{b}{t_0} + \frac{a}{t} \right) \\
q &= -1 + \frac{d}{dt} \left( \frac{1}{H} \right) \\
q &= -1 + \frac{d}{dt} \left( \frac{3t_0m}{2m-1 (bt+at)} \right) \\
q &= -1 + \frac{3t_0m}{2m-1 (bt+at)} \frac{at_0}{2m-1 (bt+at)^2} = -1 + \frac{3m \cdot at_0}{2m-1 (bt+at)^2} \\
V &= R^3 = R_0^3 \left( \frac{t}{t_0} \right)^{3a} e^{3b (\frac{t}{t_0} - 1)} \\
\frac{\dot{A}_1}{A_1} &= \frac{b^2}{t_0^2} + \frac{2ab}{t_0} + \frac{a^2-a}{t^2} \\
\frac{\dot{A}_2}{A_2} &= \frac{b^2 t^2 + 2ab t t_0 + (a^2-a) t_0}{m^2 t^2 t_0^2} = \frac{b^2}{m^2 t_0^2} + \frac{2ab}{m^2 t_0} + \frac{a^2-a}{m^2 t^2} \\
\frac{\dot{A}_3}{A_3} &= \frac{R_0^{-2} (m-2) t_0^2 (t_0^2)^{\frac{a^2 + a}{m}} e^{\frac{b - 1}{m} t_0^2}}{R_0^{-2} (m-2) t_0^2 (t_0^2)^{\frac{a^2 + a}{m}} e^{\frac{b - 1}{m} t_0^2}} = \frac{m-2}{m^2 t_0^2} \frac{2ab}{m^2 t_0} + \frac{a^2-a}{m^2 t^2} \\
\frac{\dot{A}_3}{A_3} &= (m-2) \left[ \frac{b^2 (m-2)}{m^2 t_0^2} + \frac{2ab (m-2)}{m^2 t_0} + \frac{a^2 (m-2) - am}{m^2 t^2} \right] \\
\end{align*}
\]
3. Details of the Model's Physics and Geometry:

3.1 Hubble Parameter: \( H = \frac{1}{3} \left( \frac{A_1}{A_1} + \frac{A_2}{A_2} + \frac{A_3}{A_3} \right) = \frac{1}{3} \left( \frac{2m-1}{m} \right) \left( \frac{b}{t_0} + \frac{a}{t} \right) \) (18)

3.2 Expansion Scalar: \( \theta = 3H = \frac{2m-1}{m} \left( \frac{b}{t_0} + \frac{a}{t} \right) \) (19)

3.3 Deceleration Parameter: \( q = -1 + \frac{\frac{d}{dt} \left( \frac{1}{H} \right)}{\frac{1}{H}} = -1 + \frac{3m}{2m-1} \left( \frac{at_0}{(br+at_0)^2} \right) \) (20)

3.4 Spatial Volume: \( V = R^3 = R_0^3 \left( \frac{t}{t_0} \right)^{3a} e^{3b \left( \frac{t}{t_0} - 1 \right)} \) (21)

3.5 Energy Density of HDE: According to the research conducted by Granda and Oliveros (2009), the energy density of holographic dark energy (HDE) with an infrared (IR) cut-off can be expressed as follows:

\[
\rho_\Lambda = 3 \left( \beta H^2 + \gamma H \right)
\]

\[
\rho_\Lambda = 3 \left[ \beta \left( \frac{1}{3} \left( \frac{2m-1}{m} \right) \left( \frac{b}{t_0} + \frac{a}{t} \right) \right)^2 + \gamma \left( \frac{1}{3} \left( \frac{2m-1}{m} \right) \left( - \frac{a}{t} \right) \right) \right]
\]

\[
\rho_\Lambda = 3 \left[ \beta \left( \frac{1}{3} \left( \frac{2m-1}{m} \right) \left( \frac{b}{t_0} + \frac{a}{t} \right) \right)^2 - \frac{(2m-1) \gamma a y}{3m} \right]
\]

\[
\rho_\Lambda = 3 \left[ \frac{\beta (2m-1)^2}{3} \left( \frac{b}{t_0} + \frac{a}{t} \right)^2 - \frac{(2m-1) \gamma a y}{m} \right]
\]

\[
\rho_\Lambda = \frac{3}{3} \frac{(2m-1)^2}{m^2} \left( \frac{b}{t_0} + \frac{a}{t} \right)^2 - \frac{(2m-1) \gamma a y}{m} \quad (23)
\]

3.6 Energy Density of Matter: Using Equations (9) and (23), we get

\[
\frac{A_1 A_2 + A_2 A_3 + A_3}{A_1 A_2 A_3} - \frac{3a^2}{A_1} = \left( \rho_\Lambda + \rho_m \right)
\]

\[
\rho_m = 3 \left[ \beta \left( \frac{1}{3} \left( \frac{2m-1}{m} \right) \left( \frac{b}{t_0} + \frac{a}{t} \right) \right)^2 + \gamma \left( \frac{1}{3} \left( \frac{2m-1}{m} \right) \left( - \frac{a}{t} \right) \right) \right]
\]

\[
\rho_m = 3 \left[ \beta \left( \frac{1}{3} \left( \frac{2m-1}{m} \right) \left( \frac{b}{t_0} + \frac{a}{t} \right) \right)^2 - \frac{(2m-1) \gamma a y}{3m} \right]
\]

\[
\rho_m = \frac{3}{3} \frac{(2m-1)^2}{m^2} \left( \frac{b}{t_0} + \frac{a}{t} \right)^2 - \frac{(2m-1) \gamma a y}{m} \quad (24)
\]

3.7 EoS Parameter of HDE: Using Equations (6) and (24), we get

\[
\omega_\Lambda \rho_\Lambda = \frac{A_2 A_3 + A_3 A_1 + A_1 A_2}{A_2 A_3 A_1} - \frac{a^2}{A_1}
\]

\[
\omega_\Lambda \rho_\Lambda = \frac{b^2}{m^2 t_0} + \frac{2ab}{m^2 t_0} + \frac{a^2 - \rho_m}{m^2 t_0} + (m - 2) \left[ \frac{b^2 (m-2)}{m^2 t_0} + \frac{2ab(m-2)}{m^2 t_0} + \frac{a^2 (m-2) - a m}{m^2 t^2} \right] + \frac{1}{m} \left( \frac{b}{t_0} + \frac{a}{t} \right) \frac{m-2}{m} \left( \frac{b}{t_0} + \frac{a}{t} \right) - \frac{3a^2}{R_0^2 \left( \frac{t}{t_0} \right)^{2a} e^{2b \left( \frac{t}{t_0} - 1 \right)}}
\]
In the context of the flat ΛCDM model, the cosmic jerk parameter is a dark expansionless quantity that is defined by

$$j = \frac{R^2}{R^3} = \frac{R_0^2 \left( \frac{t}{t_0} \right)^2 b^2 \rho_0}{\left( \frac{t}{t_0} \right)^3 b^2 \rho_0 e^{2b(t/t_0 - 1)} - \left( \frac{t}{t_0} \right)^2 2b(t/t_0 - 1)}$$

In the context of the flat ΛCDM model, it is necessary for the value of the jerk parameter, denoted as $j$, to be equal to 1.

3.10 The state finder pair:

$$r = \frac{1}{H^2 R} = \frac{1}{R_0^2 \left( \frac{t}{t_0} \right)^3 b^2 \rho_0 e^{2b(t/t_0 - 1)}}$$

$$r = \frac{1}{H^2 R} = \frac{1}{R_0^2 \left( \frac{t}{t_0} \right)^3 b^2 \rho_0 e^{2b(t/t_0 - 1)}}$$

$$r = \frac{1}{H^2 R} = \frac{27m^2 \left( b^3 t^3 + 3ab^2 t^2 + (3a^2 - 2a)bt_0 t + (a^3 - 3a^2 + 2a)t_0^3 \right)}{2m^2 - 1 \frac{t}{t_0} e^{2b(t/t_0 - 1)}}$$

$$s = \frac{r - 1}{3(q - 2)} = \frac{r - 1}{3(q - 2)}$$

3.8 The coincidence parameter: The average coincidence parameter, denoted as $\bar{r}$, exhibits variability during the initial stages of evolution. However, after a finite duration, it reaches a stable value and maintains this constancy during the subsequent evolution. Consequently, it effectively circumvents the coincidence problem, which is not the case for the ΛCDM model.

$$\bar{r} = \frac{\rho_0}{\rho_m} = \frac{b^2 \rho_0}{\left( \frac{t}{t_0} \right)^3 b^2 \rho_0 e^{2b(t/t_0 - 1)}} \frac{(2m - 1)\gamma y}{m \gamma x}$$

3.9 Cosmic jerk parameter: The cosmic jerk parameter is a dark expansionless quantity that is defined by

$$j = \frac{R^2}{R^3} = \frac{R_0^2 \left( \frac{t}{t_0} \right)^2 b^2 \rho_0}{\left( \frac{t}{t_0} \right)^3 b^2 \rho_0 e^{2b(t/t_0 - 1)} - \left( \frac{t}{t_0} \right)^2 2b(t/t_0 - 1)}$$

$$j = \frac{R^2}{R^3} = \frac{b^2 t^3 + 3ab^2 t^2 + (3a^2 - 2a)bt_0 t + (a^3 - 3a^2 + 2a)t_0^3}{(bt + at_0)^3}$$

$$j = \frac{R^2}{R^3} = \frac{b^2 t^3 + 3ab^2 t^2 + (3a^2 - 2a)bt_0 t + (a^3 - 3a^2 + 2a)t_0^3}{(bt + at_0)^3}$$

(26)
4. Results and Discussion -

Graph 1: Variation in average scale factor with time for different values of $t_0$

Graph 2: Variation in Hubble parameter with time

Graph 3: Variation in deceleration parameter with time for different values of $t_0$
The scale factor in the field of cosmology is a parameter that quantifies the relative dimensions of the cosmos at various points in time. The aforementioned metric plays a crucial role in characterizing the phenomenon of the universe's expansion or contraction. In the cosmological setting, a hybrid model denotes a theoretical framework that integrates diverse constituents or characteristics. The phenomenon under consideration encompasses a combination of many forms of matter, including as dark matter, radiation, and normal matter, as well as several physical mechanisms that exert an influence on the development of the universe. The empirical evidence suggests that the average scale factor exhibits exponential growth as cosmic time increases, but it diminishes as the beginning time \( t_0 \) becomes larger.

The Hubble parameter, commonly symbolized as \( H(t) \), signifies the temporal rate of expansion of the cosmos. Within the realm of cosmology, the Hubble parameter assumes a pivotal role as a parameter of utmost significance, elucidating the velocity at which celestial objects diverge from one another as a consequence of the universe's ongoing expansion.

The graph (2) depicts the relationship between the Hubble parameter and time, aiming to elucidate the temporal variations in the rate of expansion along the evolutionary trajectory of the cosmos. The Hubble parameter is commonly defined as the quotient of the pace at which the universe is expanding and the present value of the scale factor.

The deceleration parameter \( q(t) \) is a fundamental cosmological parameter that characterizes the temporal evolution of the deceleration of the universe's expansion. The graphical representation denoted as (3) depicting the deceleration parameter as a function of time offers valuable insights into the underlying mechanics governing the expansion of the cosmos. It has been observed that the expansion of the cosmos is speeding. This observation implies that the acceleration of the expansion could potentially be attributed to the presence of a repulsive force, such as dark energy.

The spatial volume with respect to time graph (4) offers valuable insights into the temporal evolution of the universe's spatial extent. The spatial volume evolution in a Bianchi Type-V universe is subject to the impact of its particular geometry and dynamics. The theoretical notion known as "holographic dark energy" pertains to the field of cosmology and involves establishing a connection between the density of dark energy in the universe and the informational content present on a bounding surface, as opposed to the entirety of the volume. The concept is frequently linked to notions pertaining to the holographic principle. The graphical representation denoted as (5) illustrates the temporal variation of the holographic dark energy density. This graph elucidates the manner in which the density of dark energy undergoes modifications over the evolution of the universe.
The jerk parameter is a term utilized to characterize the temporal variation of the acceleration of the cosmos. The characterization of various stages of cosmic expansion is facilitated by this phenomenon. The analysis of the jerk parameter graph (6) offers valuable insights into the temporal evolution of the universe's acceleration.

Redshift serves as a metric for quantifying the extent to which light originating from remote galaxies or cosmic entities has undergone elongation due to the expansion of the universe. The analysis of the redshift-time graph (7) necessitates comprehending the temporal evolution of the scale factor and the rate of expansion throughout cosmic history. The phenomenon of rising redshift is observed as a consequence of the expansion of the cosmos, whereby things situated at greater distances from our location exhibit a redshift.

5. Concluding Remarks: In the culmination of our investigation into Bianchi Type V cosmological models, with a particular focus on Holographic Ricci Dark Energy, this research has revealed captivating revelations regarding the underlying dynamics of the cosmos. The Bianchi Type V model is known for its anisotropic and spatially homogenous characteristics. It provides a framework for exploring the interactions between geometry, dark energy, and holography. The incorporation of Holographic Ricci Dark Energy inside the cosmological framework has emerged as a crucial factor to be taken into account. Through the establishment of a connection between the density of dark energy and the entropy of the cosmic horizon, we have embarked upon a new and innovative approach to comprehending the mysterious factor responsible for propelling the universe's accelerating expansion. The Ricci scalar plays a crucial role in shaping the history of the universe by influencing important parameters such as the Hubble parameter and scale factor. It acts as a geometric component that impacts the overall dynamics of the cosmos. The comprehensive examination of the cosmic tapestry in the framework of Bianchi Type V models has been conducted by a combination of numerical and analytical approaches, yielding a sophisticated understanding. The findings indicate that the incorporation of Holographic Ricci Dark Energy not only concurs with empirical evidence about the acceleration of the universe but also introduces a more intricate aspect to the dynamics of the cosmos. In the holographic paradigm, the universe undergoes a dynamic interaction between geometry and dark energy, giving rise to a story that presents a challenge to conventional understandings. Upon careful consideration of the wider ramifications of our research outcomes, it becomes apparent that the utilization of Bianchi Type V cosmological models presents a distinctive perspective from which to investigate the expansive arrangement of the cosmos. The utilization of holographic lenses in the study of dark energy offers a novel vantage point for understanding the enigmatic aspects of the universe, so enriching the ongoing scholarly dialogue concerning the fundamental characteristics of the imperceptible forces that dictate the trajectory of our cosmic existence. This inquiry provides as a fundamental starting point, facilitating the invitation for additional exploration and enhancement of cosmological models that integrate holographic concepts. The complex interplay between geometry and dark energy within the context of Bianchi Type V calls for ongoing examination and verification through empirical findings. By engaging in these pursuits, our aim is to enhance our understanding of the intricate orchestration of the cosmos and decipher the enigmatic aspects that obscure the essential essence of our universe.

REFERENCES:


