A Finite Batch Size Priority Queuing Model with Single Server Using Alpha -Cut

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Abstract: In this paper, we extend the bulk arrival size 'b' with priority classes queuing model to a fuzzy environment where fuzzy set theory is used to resolve uncertainty. The triangular fuzzy numbers are used to represent the arrival rates and service times of the fuzzy queue in a parametric programming problem. With the aid of Zadeh's approach, we obtain crisp values from the fuzzy queues using the α -cuts. A numerical example is provided to illustrate the model's performance measure.

Keywords: queuing model, trapezoidal fuzzy number, membership functions, priority queues, bulk arrival queues and parametric programming problem.

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1. INTRODUCTION

In general, when a queuing system is in operation, a single customer arrival is the norm. In most real-world situations, groups of customers arrive to seek services. In certain classified scenarios, customers enter and exit batches. These are services that are served in identical batches, followed by batches that depart. It is possible to model that scenario as queues with bulk arrivals.

Most of the queuing models have the discipline to serve people in the wait according to priority. However, there are other additional queue discipline options, including priority, random order, and last come first served.

Regardless of the time of day they enter the system, customers in a batch priority queue have the highest priority based on the requirements or limits for service before those with other customers. These priority queue models can be divided into two main categories: preemption and non-preemption.

Under the preemptive model, customers with high priority are allowed to enter service immediately, even if there is already a customer with a lower priority. In a single server model, customers can simply wait for their turn, given their high priority in a non-preemptive model. In real time, the arrival rate, service rate, and batch size of the priority queuing system are all uncertain. These ambiguities are resolved by fuzzy set theory. Fuzzy models can therefore be used to expand the classic queuing model with priority queue, thereby increasing its applicability.

Fuzzy queuing models have been examined by a number of researchers, including Kaufmann[13], Negi and Lee [17], Kao and Li [12], and Li and Lee [15]. A large number of researchers, including Bailey [2], Bhat [3], Borthakur [4], Chaudhary, and Templeton [9], have conducted extensive research on single server bulk queues.For the purpose of analyzing the fuzzy queues denoted M/F/1/ and FM/FM/1/, where F stands for fuzzy time and FM for fuzzified exponential distributions, a general technique based on Zadeh's[22] extension principle was employed. Next, the analytical outcomes were acquired. Additionally, Prade [18] and Yager [21] talked about these concepts and the prospective notion. Kao and Li [12] created the membership functions of the fuzzy queue performance measure; the same methodology is used to fuzzy bulk arrival queues.

The basic idea is to transform the fuzzy bulk arrival queues into a family of crisp bulk arrival queues by applying α-cuts and Zadeh's extension principle.

In this paper, fuzzy set theory is used to create the membership function of a fuzzy priority batch queue with two types: one with high priority and the other with normal priority. The types 1 categories are given priority in our model, and the remaining customers are divided into common types 2 categories. The α-cut method is used to derive system characteristics. We quantify key aspects of the model's performance and describe the calculation process.

The remaining portion of the paper is structured as follows. In Section 2, we review the fundamental terms, explanations, presumptions, and notations of the fuzzy batch queuing system. In Section 3, we present the formulation for the fuzzy priority queue mathematical model. In Section 4, we explored a numerical application. Section 5 concludes.

2. PRELIMINARIES

In this section we recall some basic definition.

Definition 2.1. A fuzzy set is a set where the members are allowed to have partial membership and hence the degree of membership varies from 0 to 1. It is expressed as $\tilde{A}$={ (x,$\mu_{\tilde{A}}(X)$)/X∈Z} where Z is the universe of discourse and $\mu_{\tilde{A}}(X)$ is a real number, $\mu_{\tilde{A}}(X)$=0 or 1,ie,x is a non-member in $\tilde{A}$ if $\mu_{\tilde{A}}(X)$=0 and X is a member in $\tilde{A}$ if $\mu_{\tilde{A}}(X)$=1.

Definition 2.2 If a fuzzy set $\tilde{A}$ is defined on X , for any α ∈ [0.1]], the α-cuts of the fuzzy set $\tilde{A}$ is represented by $\tilde{A}_\alpha = \{X/\mu_{\tilde{A}}(X) \geq \alpha, X \in Z\} = \{l_{\tilde{A}}(\alpha), u_{\tilde{A}}(\alpha)\}$,where $l_{\tilde{A}}(\alpha)$ and $u_{\tilde{A}}(\alpha)$ represent the lower bound and upper bound of the α-cut of $\tilde{A}$ respectively.

Definition2.3. The crisp set of elements that belong to the fuzzy set $\tilde{A}$ at least to the degree α is called the α level set $\tilde{A}_\alpha = \{x \in X: \mu_{\tilde{A}}(x) \geq \alpha\}$ where α ∈ [0,1].

Definition2.4. The support of a fuzzy set A is the crisp set such that it is represented as sup $\tilde{A}(X) = \{x \in X/\mu_{\tilde{A}}(x) > 0\}$. Thus, support of a fuzzy set is the set of all members with a strong α-cut , where α=0.

Definition2.5. Triangular fuzzy number

A fuzzy number $\tilde{A} = (a, b, c)$ is called triangular fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a \\ \dfrac{x-a}{b-a}, & a \leq x \leq b \\ \dfrac{c-x}{c-b}, & b \leq x \leq c \\ 0, & x > c. \end{cases}$$

Definition2.6. Conversion of triangular fuzzy number into Interval using $\alpha - $cut

Let $\tilde{A} = \{a, b, c\}$ be the triangular fuzzy number then to fine α −cut of $\tilde{A}$. we first set α equal to the left and right membership function of $\tilde{A}$.

That is $\alpha = \frac{x-a}{b-a}$ and $\alpha = \frac{c-x}{c-b}$. Expressing x in terms of α we have,

$x = \alpha(b-a) + a$ and $x = -\alpha(c-b) + c$.

Therefore we can write the fuzzy interval in terms of α − cut interval:

$\tilde{A}_\alpha = [\alpha(b-a) + a \ , \ -\alpha(c-b) + c]$.

3. Development of model for Fuzzy Bulk Queue Arrival with Priority Model FM^b/FM/I solving by parametric programming problem

We take into consideration the fuzzy queueing model FM^b/FM/1 of bulk arrival rate size ‘b’ with priority discipline to illustrate our model.

3.1 CONSTRUCTION OF MODEL

We consider the FM^b/FM/1 fuzzy batch priority queuing model. Every arrival of customer is classified as belonging to one of two priority types. Then, we assumed that, in batches of size "b," arrivals of type 1 or higher priority have a mean arrival rate of $\widetilde{\lambda_1}$ and those of type 2 or lower priority or no priority have a mean arrival rate of $\widetilde{\lambda_2}$, so that $\tilde{\lambda} = \widetilde{\lambda_1} + \widetilde{\lambda_2}$. is true. Customers of type 1 are allowed to get service prior to all other customers without pre-emption. We presumptively used a single server system and an infinite number of calling sources.

The average system lengths of type 1 and type 2 priority queues are represented in this model by the letters $\bar{L}_1$ and $\bar{L}_2$, respectively. The letters $\bar{L}_{q1}$ and $\bar{L}_{q2}$ stand for the length of the queues of type 1 and type 2 priority queues, respectively. The average waiting times for customers in the type 1 and type 2 priority queues, respectively, are denoted by the symbols $\bar{W}_{q1}$ and $\bar{W}_{q2}$, respectively. Devaraj & Jayalakshmi explored priority queues using a fuzzy approach; both the system capacity and the population of calling sources are infinite.

In this model, we considered that each batch's size is 'b' and that the batch arrival rate is finite. All service times are distributed according to an exponential distribution with fuzzy service rate $\tilde{\mu}$ at a single-server facility in batches as a Poisson process with group arrival mean rate $\tilde{\lambda}$, where $\tilde{\lambda}$, is a fuzzy number. The batch size 'b' of arrival is represented by triangular fuzzy number. The size of the calling population is infinite, and the system capacity is infinite. Customers are served using a priority model. The arrival rate triangular fuzzy number is represented by various intervals of confidence using α- cuts.

3.2 REPRESENTATION OF FUZZY SETS

The inter-arrival times $\tilde{A}_i$, i = 1,2 of units in the type 1 and type 2 priority and service times $\tilde{S}$are represented by the fuzzy sets in the following form in the single server queuing system $FM^b/FM/1$

$$\tilde{A}_i = \{(a, \mu_{\widetilde{A_i}}(a))/a \in X\}, i = 1, \quad \text{..(1)}$$

$$\tilde{S} = \{(s, \mu_{\tilde{S}}(s))/s \in Y\} \quad \text{..(2)}$$

Where X is universal crisp set of the inter arrival rate with its membership function $\mu_{\widetilde{A_i}}(a), i = 1,2$ and Y is also crisp universal set of service time with its membership function $\mu_{\tilde{S}}(s)$.

3.3 REPRESENTATION OF Α- CUTS

The $\alpha -$ cuts of arrival rate $\tilde{A}_i$, i = 1,2 and service rate $\tilde{S}$ are denoted by

$$A_i(\alpha) = \{a \in X/\mu_{\widetilde{A_i}}(a) \geq \alpha\}, i = 1,2 \quad \text{..(3)}$$

$$S(\alpha) = \{sYX/\mu_s(s) \geq \alpha\} \quad \text{..(4)}$$

Where $0 < \alpha \leq 1$. Both $A_i(\alpha)$, i = 1,2 and S(α) are the crisp sets and they can be represented through different levels of confidence intervals [0,1] using $\alpha -$ cut. So We can easily reduce the fuzzy queue to a family of crisp queue with different $\alpha -$ cuts $\{A_i(\alpha)/0 < \alpha \leq 1\}, i = 1,2$ and $\{S(\alpha)/0 < \alpha \leq 1\}$. These two sets represent sets of movable boundaries, and they form nested structure [22] for expressing the relationship between the crisp sets and fuzzy sets.

3.4 REPRESENTATION OF CONFIDENCE INTERVALS

To estimate the value of parameter, $\widetilde{A}_i$, and $\tilde{S}$ is represented by different levels confidence intervals [0,1] using α-cuts. The α-cuts are given by.

$$A_i(\alpha) = \left\{a \in \frac{X}{\mu_{\widetilde{A_i}}(a)} \geq \alpha\right\}, i = 1,2 \text{ and } S(\alpha) = \{sYX/\mu_s(s) \geq \alpha\}.$$

Using these α-cuts, fuzzy queue is reduced to family of crisp queues.

The confidence interval to be estimated is given the fuzzy sets $\widetilde{A}_i$, i = 1,2 and $\tilde{S}$ be.

$[l_{A_i(\alpha)}, u_{A_i(\alpha)}], i = 1,2$ and $[l_{s(\alpha)}, u_{s(\alpha)}]$ respectively.

3.5 USE OF ZADEH'S EXTENSION PRINCIPLE

Since the $\tilde{A}_i$ and $\tilde{S}$ are fuzzy numbers, by Zadeh's extension principle [23], the membership function of the performance measure $p(\tilde{A},\tilde{S}), i = 1,2$ cab be defined as

$$\mu_{p(\tilde{A},\tilde{S})}(z) = \sup_{a \in X, s \in Y} \min\{\mu_{\widetilde{Ai}}(a), \mu_s(s)/z = p(a,s)\}, i = 1,2 \quad \text{.............(5)}$$

Construction of the membership function $\mu_{p(\tilde{A},\tilde{S})}(z), i = 1,2$ is equivalent to derivation of $\alpha -$ cuts of $\mu_{p(\tilde{A},\tilde{S})}$.
From the equation (5), the equation
$\mu_{p(\tilde{A},\tilde{S})}(z) = \alpha, i = 1,2$ is true only when either,
$\mu_{\tilde{A}_i}(a) = \alpha, \mu_{\tilde{s}}(s) \geq \alpha$ or $\mu_{\tilde{A}_i}(a) \geq \alpha, \mu_{\tilde{s}}(s) = \alpha$ is true.

3.6 PARAMETRIC PROGRAMMING PROBLEM

Different levels of confidence intervals are given to the inter-arrival times and service time after considering the fuzzy sets of inter-arrival times and their membership functions. The membership function of a performance measure is defined using Zadeh's extension principle. We obtained the following parametric programming problem.
In the parametric programming problem $l_{p(\alpha)}$ and $u_{p(\alpha)}$are represent as,

$$l_{p(\alpha)} = min\, p(a,s) \quad \text{.....................................(6)}$$

Such that $l_{A_i(\alpha)} \leq a \leq u_{A_i(\alpha)}, i = 1,2$
$l_{s(\alpha)} \leq s \leq u_{s(\alpha)}$

and

$$u_{p(\alpha)} = max\, p(a,s) \quad \text{.....................................(7)}$$

Such that $l_{A_i(\alpha)} \leq a \leq u_{A_i(\alpha)}, i = 1,2$
$l_{s(\alpha)} \leq s \leq u_{s(\alpha)}$

3.7 REPRESENTATION OF SHAPE FUNCTION

If both $l_{p(\alpha)}$ and $u_{p(\alpha)}$ are invertible with respect to α, then
the left shape function $L(z) = \left(l_{p(\alpha)}\right)^{-1}$ and
the right shape function $R(z) = \left(u_{p(\alpha)}\right)^{-1}$ can be obtained,
from which the membership function $\mu_{p(\tilde{A},\tilde{S})}(z), i = 1,2$ is constructed as

$$\mu_{p(\tilde{A},\tilde{S})}(z) = \begin{cases} L(z), & for\ z_1 \leq z \leq z_2 \\ R(z), & for\ z_2 \leq z \leq z_3 \\ 0, & otherwise \end{cases} \quad \text{............................(8)}$$

Where $z_1 \leq z_2 \leq z_3$ and $L(z_1) = R(z_3) = 0$ for the triangular fuzzy number.

3.8 The fuzzy batch arrival FM^b/FM/I reduced to crisp M/M/1 priority service

Using the concept of $\alpha -$ cut the $FM^b/FM/1$ queue with priority can be in the following form by [21]

$$L_{q1} = \frac{\rho(b-1+2\frac{\lambda_1}{\mu})}{2\left(1-\frac{\lambda_1}{\mu}\right)}$$

$$L_{q2} = \frac{\rho(b-1+2\frac{\lambda_2}{\mu})}{2(1-\rho)\left(1-\frac{\lambda_2}{\mu}\right)}$$

$$W_{q1} = \frac{b+2\rho-1}{2\mu\left(1-\frac{\lambda_1}{\mu}\right)}$$

$$W_{q2} = \frac{b+2\rho-1}{2(\mu-\lambda)\left(1-\frac{\lambda_2}{\mu}\right)}$$

Where $\widetilde{\lambda_1}$ and $\widetilde{\lambda_2}$ are the arrival rates of type 1 priority and type 2 priority units respectively and $\tilde{\mu}$ is the service rate. Further $\tilde{\lambda} = \widetilde{\lambda_1} + \widetilde{\lambda_2}$ and
$\rho = \frac{\lambda b}{\mu}$ from [21].

If the functions $l_{p(\alpha)}$ and $u_{p(\alpha)}$ are not invertible with respect to α, the membership functions $\mu_{p\,(\tilde{A},\tilde{S})}(z)$ in not derived. But we can trace the graph of $\mu_{p(\tilde{A},\tilde{S})}(z)$ from the $\alpha -$ cuts of $[l_{p(\alpha)}, u_{p(\alpha)}]$.

This procedure can be applied to find the membership functions $[l_{s(\alpha)}, u_{s(\alpha)}]$ for the batch priority queuing model with priority can be obtained.

4. NUMERICAL EXAMPLE

Expected number of customer and expected waiting time for $FM^b/FM/1$ batch priority queue with two priority types

Suppose the arrival rate of the two types of priority queues with the same service rate are fuzzy numbers represented by $\tilde{A}_1 = [3,4,5], \tilde{A}_2 = [6,7,8]$ and $\tilde{S} = [14,15,16]$ per hour respectively with batch size b=12. The $\alpha -$ cut of the membership functions $u_{\tilde{A}_1}(\alpha), u_{\tilde{A}_2}(\alpha)$ and $u_{\tilde{S}}(\alpha)$ are $[\alpha + 3, 5 - \alpha], [\alpha + 6, 8 - \alpha]$ and $[\alpha + 14, 16 - \alpha]$ respectively using definition 2.6. From equation (6) and (7) the parametric programming problem are formulated to derive the membership function $\bar{L}_{q1}$, $\bar{L}_{q2}$, $\bar{W}_{q1}$ and $\bar{W}_{q2}$.

The performance functions of
(i) $\bar{L}_{q1}$ - average queue length of highest priority type 1
(ii) $\bar{L}_{q2}$ - average queue length of less priority type 2
(iii) $\bar{W}_{q1}$- average waiting time of units highest priority in the queue type 1
(iv) $\overline{W_{q2}}$- average waiting time of units less priority in the queue type 2
are derived from the respective parametric programming problems from equations (6) and (7).

4.1 PERFORMANCE MEASURES

The performance measures $\bar{L}_{q1}$, $\bar{L}_{q2}$, $\bar{W}_{q1}$ and $\overline{W_{q2}}$ are differ only in their objective functions.

4.2 OBJECTIVE FUNCTIONS AND OPTIMAL SOLUTIONS

The objective functions and optimal solutions are listed below.

4.2.1 For type 1 or higher priority - Length of the queue

The objective function: The performance measure of $\bar{L}_{q1}$

$$\left.\begin{aligned} l_{L_{q1}(\alpha)} &= min \left\{ \frac{\left(\frac{r1+r2}{k}\right)\left(b-1+2\left(\frac{r1}{k}\right)\right)}{2\left(1-\frac{r1}{k}\right)} \right\} \\ such\ that \quad & 3+\alpha \le r1 \le 5-\alpha \\ & 6+\alpha \le r2 \le 8-\alpha \\ & 14+\alpha \le k \le 16-\alpha \end{aligned}\right\} \quad \text{..................(9)}$$

and

$$u_{L_{q1}(\alpha)} = max \left\{ \frac{\left(\frac{r1+r2}{k}\right)\left(b-1+2\left(\frac{r1}{k}\right)\right)}{2\left(1-\frac{r1}{k}\right)} \right\} \quad \text{such that} \quad \begin{array}{c} 3+\alpha \le r1 \le 5-\alpha \\ 6+\alpha \le r2 \le 8-\alpha \\ 14+\alpha \le k \le 16-\alpha \end{array} \quad \text{.................(10)}$$

Where $0 < \alpha \le 1$.
$l_{L_{q1}(\alpha)}$ is found when r1 and r2 approaches their lower bounds and k approaches its upper bound.

Consequently the optimal solutions for (9) is

$$l_{L_{q1}(\alpha)} = \frac{1638+283\alpha-18\alpha^2}{416-90\alpha+4\alpha^2} \quad \text{................(11)}$$

Also, $u_{L_{q1}(\alpha)}$ is found when r1 and r2 approaches their upper bounds and k approaches its lower bound.

Consequently the optimal solutions for (10) is

$$u_{L_{q1}(\alpha)} = \frac{2132-211\alpha-18\alpha^2}{252+74\alpha+4\alpha^2} \quad \text{................(12)}$$

The membership function

$$u_{\bar{L}_{q1}}(z) = \begin{cases} L(z), & [l_{L_{q1}(\alpha)}]_{\alpha=0} \le z \le [l_{L_{q1}(\alpha)}]_{\alpha=1} \\ R(z), & [u_{L_{q1}(\alpha)}]_{\alpha=1} \le z \le [u_{L_{q1}(\alpha)}]_{\alpha=0} \\ 0, & otherwise \end{cases}$$

which is estimated as

$$u_{\bar{L}_{q1}}(z) = \begin{cases} \frac{(90z+283)-\left(1444z^2+47196z+198025\right)^{1/2}}{4(2z+9)}, & 3.93 \le z \le 5.766 \\ \frac{-(74z+211)+\left(1444z^2+47196z+198025\right)^{1/2}}{4(2z+9)}, & 5.766 \le z \le 8.46 \\ 0, & otherwise \end{cases} \quad \text{...........(13)}$$

4.2.2 For type 2 or less priority - Length of the queue

The objective function : The performance measure of $\bar{L}_{q2}$

$$l_{L_{q2}(\alpha)} = min \left\{ \frac{\left(\frac{r1+r2}{k}\right)\left(b-1+2\left(\frac{r1}{k}\right)\right)}{2\left(1-\left(\frac{r1+r2}{k}\right)\right)\left(1-\frac{r1}{k}\right)} \right\} \quad \text{such that} \quad \begin{array}{c} 3+\alpha \le r1 \le 5-\alpha \\ 6+\alpha \le r2 \le 8-\alpha \\ 14+\alpha \le k \le 16-\alpha \end{array} \quad \text{..................(14)}$$

and

$$u_{L_{q2}(\alpha)} = max \left\{ \frac{\left(\frac{r1+r2}{k}\right)\left(b-1+2\left(\frac{r1}{k}\right)\right)}{2\left(1-\left(\frac{r1+r2}{k}\right)\right)\left(1-\frac{r1}{k}\right)} \right\} \quad \text{such that} \quad \begin{array}{c} 3+\alpha \le r1 \le 5-\alpha \\ 6+\alpha \le r2 \le 8-\alpha \\ 14+\alpha \le k \le 16-\alpha \end{array} \quad \text{.................(15)}$$

$l_{L_{q2}(\alpha)}$ is found when r1 and r2 approaches their lower bounds and k approaches its upper bound.

Then, the optimal solutions for (14) is

$$l_{L_{q2}(\alpha)} = \frac{1692+295\alpha-18\alpha^2}{140-88\alpha+12\alpha^2} \quad \text{................(16)}$$

Also, $u_{L_{q2}(\alpha)}$ is found when r1 and r2 approaches their upper bounds and k approaches its lower bound.

Consequently the optimal solutions for (15) is

$$u_{L_{q2}(\alpha)} = \frac{2210-223\alpha-18\alpha^2}{12+40\alpha+12\alpha^2} \quad \text{................(17)}$$

The membership function

$$u_{\tilde{L}_{q2}}(z) = \begin{cases} L(z), & [l_{L_{q1}(\alpha)}]_{\alpha=0} \le z \le [l_{L_{q1}(\alpha)}]_{\alpha=1} \\ R(z), & [u_{L_{q1}(\alpha)}]_{\alpha=1} \le z \le [u_{L_{q1}(\alpha)}]_{\alpha=0} \\ 0, & otherwise \end{cases}$$

which is estimated as

$$u_{\tilde{L}_{q2}}(z) = \begin{cases} \frac{(88z+225)-(1024z^2+143216z+208849)^{1/2}}{12(2z+3)}, & 12.09 \le z \le 30.77 \\ \frac{-(40z+223)+(1024z^2+143216z+208849)^{1/2}}{12(2z+3)}, & 30.77 \le z \le 184.17 \\ 0, & otherwise \end{cases} \text{...........(18)}$$

4.2.3 For type 1 and type 2 priority - waiting time in the queue

The performance functions of $\widetilde{W_{qP1}}$ and $\widetilde{W_{qP2}}$ are derived from the respective parametric programs.

$$\left.\begin{array}{ll} l_{W_{q1}(\alpha)} & = min\left\{\frac{\left(b+2\left(\frac{r1+r2}{k}\right)-1\right)}{2k\left(1-\frac{r1}{k}\right)}\right\} \\ such\ that & 3+\alpha \le r1 \le 5-\alpha \\ & 6+\alpha \le r2 \le 8-\alpha \\ & 14+\alpha \le k \le 16-\alpha \end{array}\right\} \quad \text{...................(19)}$$

and

$$\left.\begin{array}{ll} u_{W_{q1}(\alpha)} & = max\left\{\frac{\left(b+2\left(\frac{r1+r2}{k}\right)-1\right)}{2k\left(1-\frac{r1}{k}\right)}\right\} \\ such\ that & 3+\alpha \le r1 \le 5-\alpha \\ & 6+\alpha \le r2 \le 8-\alpha \\ & 14+\alpha \le k \le 16-\alpha \end{array}\right\} \quad \text{..................(20)}$$

$$\left.\begin{array}{ll} l_{W_{q2}(\alpha)} & = min\left\{\frac{\left(b+2\left(\frac{r1+r2}{k}\right)-1\right)}{2(k-(r1+r2))\left(1-\frac{r2}{k}\right)}\right\} \\ such\ that & 3+\alpha \le r1 \le 5-\alpha \\ & 6+\alpha \le r2 \le 8-\alpha \\ & 14+\alpha \le k \le 16-\alpha \end{array}\right\} \quad \text{...................(21)}$$

and

$$\left.\begin{array}{ll} u_{W_{q2}(\alpha)} & = max \left\{ \frac{\left(b+2\left(\frac{r1+r2}{k}\right)-1\right)}{2(k-(r1+r2))\left(1-\frac{r2}{k}\right)} \right\} \\ such\ that & 3+\alpha \le r1 \le 5-\alpha \\ & 6+\alpha \le r2 \le 8-\alpha \\ & 14+\alpha \le k \le 16-\alpha \end{array}\right\} \quad \ldots\ldots\ldots\ldots\ldots(22)$$

For the above equations $l_{W_{q1}(\alpha)}$,$l_{W_{q2}(\alpha)}$ is found when r1 and r2 approaches their lower bounds and k approaches its upper bound. Also, $u_{W_{q1}(\alpha)}$,$u_{W_{q2}(\alpha)}$ is found when r1 and r2 approaches their upper bounds and k approaches its lower bound.

The optimal solutions for (18), (19) are

$$l_{W_{q1}(\alpha)} = \frac{194-7\alpha}{416-90\alpha+4\alpha^2} \quad \ldots\ldots\ldots\ldots\ldots(23)$$

$$u_{W_{q1}(\alpha)} = \frac{180+7\alpha}{252+74\alpha+4\alpha^2} \quad \ldots\ldots\ldots\ldots\ldots(24)$$

The optimal solutions for (20), (21) are

$$l_{W_{q2}(\alpha)} = \frac{194-7\alpha}{140-88\alpha+12\alpha^2} \quad \ldots\ldots\ldots\ldots\ldots(25)$$

$$u_{W_{q2}(\alpha)} = \frac{180+7\alpha}{12+40\alpha+12\alpha^2} \quad \ldots\ldots\ldots\ldots\ldots(26)$$

The membership function

$$\mu_{\overline{W}_{q1}}(z) = \begin{cases} L(z), & [l_{L_{q1}(\alpha)}]_{\alpha=0} \le z \le [l_{L_{q1}(\alpha)}]_{\alpha=1} \\ R(z), & [u_{L_{q1}(\alpha)}]_{\alpha=1} \le z \le [u_{L_{q1}(\alpha)}]_{\alpha=0} \\ 0, & otherwise \end{cases}$$

which is estimated as

$$\mu_{\overline{W}_{q1}}(z) = \begin{cases} \frac{(90z-7)-\left(1444z^2+1844z+49\right)^{1/2}}{8z}, & 0.46635 \le z \le 0.56667 \\ \frac{(-74z+7)+\left(1444z^2+1844z+49\right)^{1/2}}{4(2z+9)}, & 0.56667 \le z \le 0.72222 \\ 0, & otherwise \end{cases} \quad \ldots\ldots\ldots(27)$$

The membership function

$$\mu_{\overline{W}_{q2}}(z) = \begin{cases} L(z), & [l_{L_{q1}(\alpha)}]_{\alpha=0} \le z \le [l_{L_{q1}(\alpha)}]_{\alpha=1} \\ R(z), & [u_{L_{q1}(\alpha)}]_{\alpha=1} \le z \le [u_{L_{q1}(\alpha)}]_{\alpha=0} \\ 0, & otherwise \end{cases}$$

which is estimated as

$$\mu_{\overline{W}_{q2}}(z) = \begin{cases} \frac{(88z-7)-(1024z^2+8080z+49)^{1/2}}{24z}, & 1.3857 \leq z \leq 2.92181 \\ \frac{(-40z+7)+(1024z^2+8080z+49)^{1/2}}{24z}, & 2.92181 \leq z \leq 15 \\ 0, & otherwise \end{cases} \quad \text{............(28)}$$

4.3 TABLES AND GRAPHS

TABLE -1 The α-cuts of the performance measure length of the queue for priority 1 and priority 2 at different α values

LENGTH OF THE QUEUE				
For **Priority 1** Lq_ L P1 is LOWER VALUE of P1, Lq_UP1 is UPPER VALUE of P1 , & **Priority 2** Lq_ L P2 is LOWER VALUE of P2, Lq_UP2 is UPPER VALUE of P2 .				
α	Lq_L P1	Lq_U P1	Lq_L P2	Lq_UP2
0	3.937500	8.460317	12.08571	184.16667
0.1	4.093259	8.135677	13.10783	135.70223
0.2	4.254270	7.825442	14.24382	105.69727
0.3	4.420793	7.528700	15.51168	85.38596
0.4	4.593106	7.244615	16.93328	51.33234
0.5	4.771505	6.972414	18.53535	70.78610
0.6	4.956306	6.711389	20.35096	59.82857
0.7	5.147848	6.460884	22.42145	44.57454
0.8	5.346491	6.220296	24.79917	39.08824
0.9	5.552625	5.989063	27.55133	34.55856
1	5.766667	5.766667	30.76562	30.76562

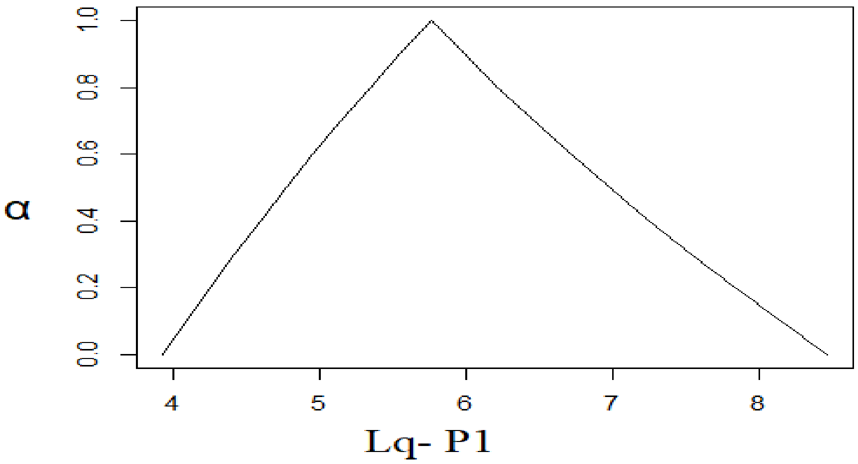

Fig. 1 The membership function of the Expected queue length Lq of priority 1

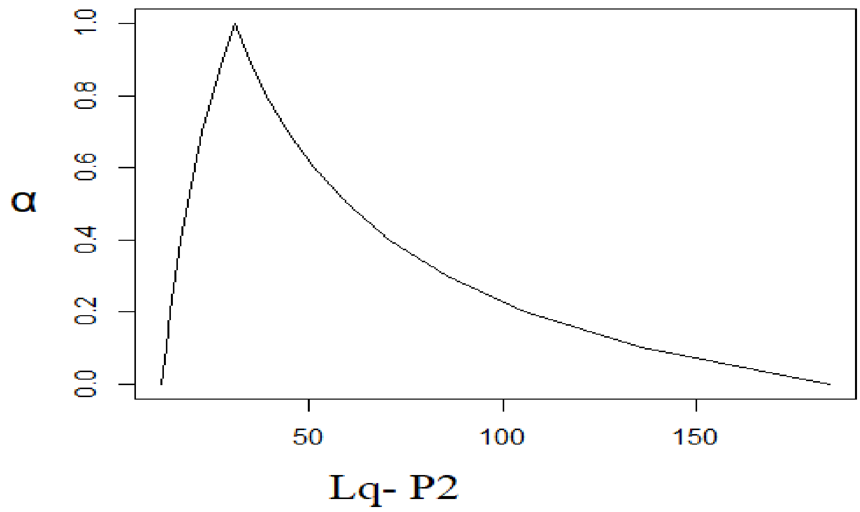

Fig. 2 The membership function of expected queue length Lq of priority 2

TABLE -2 The α-cuts of the performance measure waiting time in the queue for priority 1 and priority 2 at different α values

WAITING TIME IN THE QUEUE for **Priority 1** Wq_ L P1 is LOWER VALUE of P1, Wq_UP1 is UPPER VALUE of P1 , & **Priority 2** Wq_ L P2 is LOWER VALUE of P2, Wq_UP2 is UPPER VALUE of P2 .				
α	Wq_L P1	Wq_UP1	Wq_L P2	Wq_U P2
0	0.4663462	0.7142857	1.385714	15.000000
0.1	0.4748919	0.6965002	1.471977	11.209677
0.2	0.4837251	0.6795025	1.567383	8.857422
0.3	0.4928601	0.6632430	1.673352	7.260766
0.4	0.5023119	0.6476757	1.791604	6.109626
0.5	0.5120968	0.6327586	1.924242	5.242857
0.6	0.5222320	0.6184529	2.073864	4.568452
0.7	0.5327361	0.6047227	2.243711	4.030078
0.8	0.5436288	0.5915349	2.437888	3.591331
0.9	0.5549314	0.5788591	2.661656	3.227651
1	0.5666667	0.5666667	2.921875	2.921875

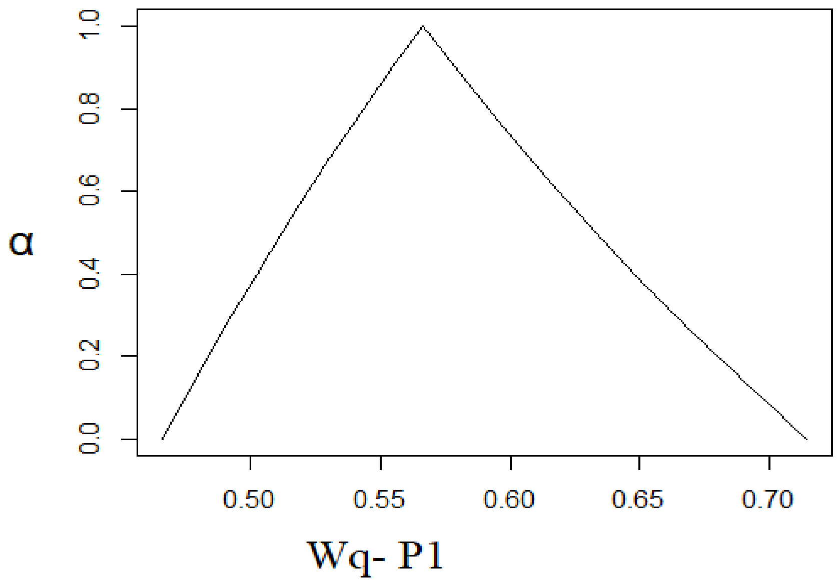

Fig. 3 The membership function of the expected waiting time Wq of priority 1

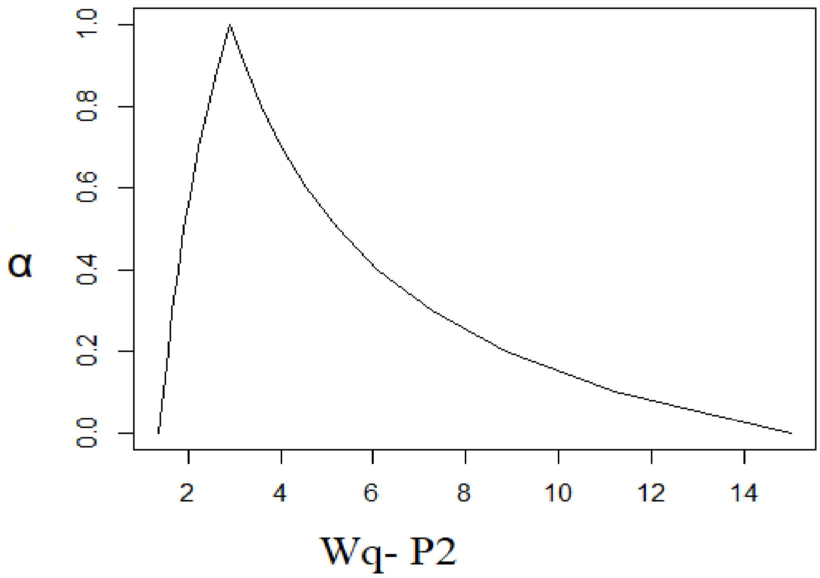

Fig. 4 The membership function of the expected waiting time Wq of priority 2

4.4 DISCUSSION

For the fuzzy priority queue with batch arrivals, the range of a performance measure at different probability levels is computed using the

α -cut approach. Consider the expected queue length Lq-LP1 using the previous example. For possibility levels $\alpha = 0$ and $\alpha = 1$, the range of the estimated queue length for high priority is, at one extreme, [1638/416, 2132/252] (see Eq. (11)). According to this range, there will never be an average of fewer consumers in line than 3.9375 or more than 5.7667.

There are many benefits to using the computational technique for the parametric programming problem. For example, the α -cuts of the Membership functions for Lq-UP1, Lq-LP2, and Lq-UP2 from table 1 can be used to obtain the interval of performance measure at different possibility levels. The waiting time for Wq-LP1 between α = 0 and α = 1 is calculated in Table 2 to be [180/252, 187/330] and will never be approximately less than 0.466342 or more than 0.566667. The knowledge gained from a system evaluation will be very beneficial when building a queuing system, which involves deciding one or more decisions involving things like the capacity of the system, the size of the constant bulk of consumers, and the efficiency of the servers.

This paper's main goal is to show that recent advancements in the field of parametric programming problems may be applied to bulk and priority queuing models while providing an entirely new potential for obtaining the best crisp points inside intervals.

5. CONCLUSION

Bulk arrival queue systems have many real-world applications. In this research, we present an approach to give customers priority when batch size "b," arrival rate, and service rate are fuzzy: the membership function of the system performance measure. It transforms a fuzzy batch-arrival queue into a family of crisp queues that may be represented by a parametric programming problem by applying Zadeh's extension principle and the concept of α-cuts.

Next, one can locate the α-cuts of the performance measures' membership functions. The membership function is obtained proportionally from different possibility levels α. The findings can be utilized to more correctly depict the fuzzy system while maintaining the fuzziness of the input data. An example has successfully solved the efficiency and the exact values of our suggested solution, and they have been graphically contrasted.

The method's applicability and creativity in determining the system performance measure for bulk arrival with fixed batch size and priority service were illustrated by the numerical results. It would be fascinating to find out about further queueing system elements in the future that broaden multi-channel scanning across an ongoing batch.

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