Tuijin Jishu/Journal of Propulsion Technology
ISSN: 1001-4055
Vol. 44 No. 6 (2023)

Fixed Points of Erdal-G-α-ψ-Geraghty Contractions Type Mappings and Application to Integral Equations

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ABSTRACT. We present the concept of Erdal-G-α-ψ-Geraghty contraction and develop fixed point theorems in the arrangement of g-metric spaces using Erdal-G-α-ψ-Geraghty contraction with relevant examples and applications to integral equations in this work.

Keywords: *relevant, integral, contraction, g-metric*

Introduction

The Banach contraction concept has proven a useful tool in the study of a fixed point. It is frequently utilized in fields such as nonlinear analysis, applied mathematics, economics, and physics. Because of its significance, the conclusion has been generalized in several ways.

Samet et al. [20] pioneered the notion of α- admissibility. Later, Karapinar et al. [14] extended it to triangular α- admissibility. Abodayeh et al. [1] recently developed the concept of triangular α- admissibility concerning another function β. Chary et al. [23] developed the novel idea if rectangular α-G-admissible mapping in 2021, as well as rectangular α-G-admissible concerning another function β in G-metric space. Karapinar [16] proposed a new form of contraction, the α-ψ-Geraghty contraction, and found fixed point findings for it.

Preliminaries

We are reminded of Geraghty's theorem. To do this, we must notify the Γ class of all functions $\beta:[0,\infty)\to[0,1)$ that satisfy the criteria: $\lim_{n\to\infty}\beta(t_n)=1\Leftrightarrow t_n=0.$

Lemma 2.1. [14] Let f represent a triangular α- admissible mapping. Assume there is a $\varsigma_0\in\Omega$ such that $\alpha(\varsigma_0, f\varsigma_0)\geq 1$. Define the sequence ς_n as $\varsigma_{n+1}=T\varsigma_n$. Then $\alpha(\varsigma_n,\varsigma_m)\geq 1$ for any $m,n\in N$.

First, we write the class of functions that will be utilized extensively in the sequel: Let Ψ indicate the class of $\psi:[0,\infty)\to[0,\infty)$ functions that meet the following conditions: a) ψ is non- decreasing; b) ψ is sub additive, which means that $\psi(s+t)=\psi(s)+\psi(t)$; c) ψ is continuous function; d) $\psi(t)=0\Leftrightarrow t=0$.

1. **Main Results**

Definition 3.1. Assume (Ω,G) is a G -metric space. Let $\alpha:\Omega\times\Omega\times\Omega\to R$ be a function. A mapping $\Lambda:\Omega\to\Omega$ is said to be an Erdal- G - α - ψ -Geraghty contraction if there exists $\beta\in\Gamma$ such that

$$\alpha(\varpi,\rho,\varsigma)\psi\left(G(\Lambda\varpi,\Lambda\rho,\Lambda\varsigma)\right)\leq\beta\Big(\psi\big(\Delta(\varpi,\rho,\varsigma)\big)\Big)\psi\big(\Delta(\varpi,\rho,\varsigma)\big)$$

(3.1)

Where

$\Delta(\varpi,\rho,\varsigma)=max\{G(\varpi,\Lambda\varpi,\Lambda\varpi),G(\rho,\Lambda\rho,\Lambda\rho),G(\varsigma,\Lambda\varsigma,\Lambda\varsigma)\}\ \forall\varpi,\rho,\varsigma\in\Omega$ and $\psi\in\Psi$.

Our first new result is as follows

Theorem 3.1. Let (Ω,G) be a complete G-metric space, $\alpha:\Omega\times\Omega\times\Omega\to R$ be a function, and let $\Lambda:\Omega\to\Omega$ be a map. Assume that the aforementioned circumstances are met.

1) Λ is Erdal- G - α - ψ -Geraghty contraction
2) Λ is rectangular α -admissible
3) there exists $\varpi_1\in\Omega$ so that $\alpha(\varpi_1,\Lambda\varpi_1,\Lambda\varpi_1)\geq 1$
4) Λ is continuous.

Then Λ has a fixed point $\varpi^*\in\Omega$ and $\{\Lambda^n\varpi_1\}$ convergent to ϖ^*.

Proof: Assume $\varpi_1\in\Omega$ is satisfies $\alpha(\varpi_1,\Lambda\varpi_1,\Lambda\varpi_1)\geq 1$. Define $\{\varpi_n\}\subset\Omega$ by $\varpi_{n+1}=\Lambda\varpi_n$ for $n\in N$. Suppose that $\varpi_{n_0}=\varpi_{n_0}+1$ in some cases $n_0\in N$. So, obviously ϖ_{n_0} is a fixed point of Λ and hence the evidence to support is completed. Presume that $\varpi_n\neq\varpi_{n+1}\ \forall n\in N$. According to lemma 2.1, we have

$$\alpha(\varpi_n,\varpi_{n+1},\varpi_{n+1})\geq 1. \tag{3.2}$$

$\forall n\in N$. From (3.1), we aquire

$$\begin{aligned}\psi\left(G\left(\varpi_{n+1},\varpi_{n+2},\varpi_{n+2}\right)\right)&=\psi\left(G\left(\Lambda\varpi_{n},\Lambda\varpi_{n+1},\Lambda\varpi_{n+1}\right)\right)\\&\leq\alpha\left(\varpi_{n},\varpi_{n+1},\varpi_{n+1}\right)\psi\left(G\left(\Lambda\varpi_{n},\Lambda\varpi_{n+1},\Lambda\varpi_{n+1}\right)\right)\\&\leq\beta\left(\psi\left(\Delta\left(\varpi_{n},\varpi_{n+1},\varpi_{n+1}\right)\right)\right)\psi\left(\Delta\left(\varpi_{n},\varpi_{n+1},\varpi_{n+1}\right)\right)\end{aligned}$$

(3.3)

$\forall n\in N$, where

$$\begin{aligned}\Delta\left(\varpi_{n},\varpi_{n+1},\varpi_{n+1}\right)&=max\left\{\begin{matrix}G\left(\varpi_{n},\varpi_{n+1},\varpi_{n+1}\right),G\left(\varpi_{n},\Lambda\varpi_{n},\Lambda\varpi_{n}\right),G\left(\varpi_{n+1},\Lambda\varpi_{n+1},\Lambda\varpi_{n+1}\right),\\G\left(\varpi_{n+1},\Lambda\varpi_{n+1},\Lambda\varpi_{n+1}\right)\end{matrix}\right\}\\&=max\left\{\begin{matrix}G\left(\varpi_{n},\varpi_{n+1},\varpi_{n+1}\right),G\left(\varpi_{n},\varpi_{n+1},\varpi_{n+1}\right),G\left(\varpi_{n+1},\varpi_{n+2},\varpi_{n+2}\right),\\G\left(\varpi_{n+1},\varpi_{n+2},\varpi_{n+2}\right)\end{matrix}\right\}\end{aligned}$$

Here, we observed, $\Delta\left(\varpi_{n},\varpi_{n+1},\varpi_{n+1}\right)=G\left(\varpi_{n+1},\varpi_{n+2},\varpi_{n+2}\right)$ is inconceivable because of the precise meaning of β.

$$\begin{aligned}\psi\left(G\left(\varpi_{n+1},\varpi_{n+2},\varpi_{n+2},\right)\right)&\leq\beta\left(\psi\left(\Delta\left(\varpi_{n},\varpi_{n+1},\varpi_{n+1}\right)\right)\right)\psi\left(\Delta\left(\varpi_{n},\varpi_{n+1},\varpi_{n+1}\right)\right)\\&\leq\beta\left(\psi\left(G\left(\varpi_{n+1},\varpi_{n+2},\varpi_{n+2}\right)\right)\right)\psi\left(G\left(\varpi_{n+1},\varpi_{n+2},\varpi_{n+2}\right)\right)\\&<\psi\left(G\left(\varpi_{n+1},\varpi_{n+2},\varpi_{n+2}\right)\right)\end{aligned}$$

Therefore, we declared $\Delta\left(\varpi_{n},\varpi_{n+1},\varpi_{n+1}\right)=G\left(\varpi_{n},\varpi_{n+1},\varpi_{n+1}\right)$. From (3.3), we get

$$\psi\left(G\left(\varpi_{n+1},\varpi_{n+2},\varpi_{n+2}\right)\right)<\psi\left(G\left(\varpi_{n},\varpi_{n+1},\varpi_{n+1}\right)\right)$$

$\forall n\in N$. According to the circumstance of ψ, declared that

$$G\left(\varpi_{n+1},\varpi_{n+2},\varpi_{n+2}\right)<G\left(\varpi_{n},\varpi_{n+1},\varpi_{n+1}\right)$$

$\forall n\in N$. Hence, we conclude that $G\left(\varpi_{n},\varpi_{n+1},\varpi_{n+1}\right)$ is neither negative nor growing. Given a consequence, $s\geq 0$ occurs in a way that

$$\lim_{n\to\infty}G\left(\varpi_{n},\varpi_{n+1},\varpi_{n+1}\right)=s.$$

We assert that $s=0$. Assume, on the other hand, that $s>0$. Then, as a result of (3.3), our situation is

$$\frac{\psi\left(G\left(\varpi_{n+1},\varpi_{n+2},\varpi_{n+2}\right)\right)}{\psi\left(\Delta\left(\varpi_{n},\varpi_{n+1},\varpi_{n+1}\right)\right)}\leq\beta\left(\psi\left(\Delta\left(\varpi_{n},\varpi_{n+1},\varpi_{n+1}\right)\right)\right)<1$$

In the following,

$$\lim_{n\to\infty}\beta\left(\psi\left(\Delta\left(\varpi_{n},\varpi_{n+1},\varpi_{n+1}\right)\right)\right)=1$$

Because of $\beta\in\Gamma$, there is

$$\lim_{n\to\infty}\psi\left(\Delta\left(\varpi_{n},\varpi_{n+1},\varpi_{n+1}\right)\right)=0, \tag{3.4}$$

as a result of which

$$s=\lim_{n\to\infty}G\left(\varpi_{n},\varpi_{n+1},\varpi_{n+1}\right)=0 \tag{3.5}$$

We notice that

$$\Delta(\varpi_m,\varpi_n,\varpi_n)=max\{G(\varpi_m,\varpi_n,\varpi_n),G(\varpi_m,\Lambda\varpi_m,\Lambda\varpi_m)G(\varpi_n,\Lambda\varpi_n,\Lambda\varpi_n),G(\varpi_n,\Lambda\varpi_n,\Lambda\varpi_n)\}$$

utilising the conclusion

$$\lim_{n\to\infty} G(\varpi_n,\varpi_{n+1},\varpi_{n+1})=0,$$

We obtain that

$$\lim_{m,n\to\infty}\Delta(\varpi_m,\varpi_n,\varpi_n)=\lim_{m,n\to\infty}G(\varpi_m,\varpi_n,\varpi_n) \tag{3.6}$$

We conclude that $\{\varpi_n\}$ is a G-Cauchy. On the other hand, pretend there is

$$\varepsilon=\lim_{m,n\to\infty} sup\{G(\varpi_n,\varpi_m,\varpi_m)\}>0 \tag{3.7}$$

We infer from rectangular inequality

$$G(\varpi_n,\varpi_m,\varpi_m)\le G(\varpi_n,\varpi_{n+1},\varpi_{n+1})+G(\varpi_{n+1},\varpi_{m+1},\varpi_{m+1})+G(\varpi_{m+1},\varpi_m,\varpi_m) \tag{3.8}$$

We obtain through the use of (3.3), (3.8) and the characteristics of ψ

$$\begin{aligned}\psi(G(\varpi_n,\varpi_m,\varpi_m))&\le\psi(G(\varpi_n,\varpi_{n+1},\varpi_{n+1})+G(\Lambda\varpi_n,\Lambda\varpi_m,\Lambda\varpi_m)+G(\varpi_{m+1},\varpi_{m'},\varpi_m))\\&\le\psi(G(\varpi_n,\varpi_{n+1},\varpi_{n+1}))+\psi(G(\Lambda\varpi_n,\Lambda\varpi_m,\Lambda\varpi_m))+\psi(G(\varpi_{m+1},\varpi_{m'},\varpi_m))\\&\le\psi(G(\varpi_n,\varpi_{n+1},\varpi_{n+1}))+\beta(\psi(\Delta(\varpi_n,\varpi_m,\varpi_m)))\psi(\Delta(\varpi_n,\varpi_m,\varpi_m))\\&\quad+\psi(G(\varpi_{m+1},\varpi_m,\varpi_m))\end{aligned}$$

(3.9)

We may conclude from (3.6), (3.9) and (3.5)

$$\begin{aligned}\operatorname{Lim}_{m,n\to\infty}\psi(G(\varpi_n,\varpi_m,\varpi_m))&\le\lim_{m,n\to\infty}\beta(\psi(\Delta(\varpi_n,\varpi_m,\varpi_m)))\lim_{m,n\to\infty}\psi(\Delta(\varpi_m,\varpi_n,\varpi_n))\\&\le\lim_{m,n\to\infty}\beta(\psi(\Delta(\varpi_n,\varpi_m,\varpi_m)))\lim_{m,n\to\infty}\psi(G(\varpi_m,\varpi_n,\varpi_n))\end{aligned}$$

From (3.7), obtain

$$1\le\lim_{m,n\to\infty}\beta(\psi(\Delta(\varpi_n,\varpi_m,\varpi_m)))$$

Which deals

$$\lim_{m,n\to\infty}\beta(\psi(\Delta(\varpi_n,\varpi_m,\varpi_m)))=1$$

As a result, we obtain

$$\lim_{m,n\to\infty}\Delta(\varpi_n,\varpi_m,\varpi_m)=0$$

and hence $G(\varpi_n,\varpi_m,\varpi_m)=0$, which is incongruous. Finally, $\{\varpi_n\}$ is a G-Cauchy. Given the completeness of Ω, we are able to infer that there is one

$$\varpi^*=\lim_{n\to\infty}\varpi_n\in\Omega.$$

Because Λ is continuous, we possess $\lim_{n\to\infty}\varpi_n=\Lambda\varpi^*$ and so $\varpi^*=\Lambda\varpi^*$.

Definition 3.2. Let (Ω,G) be complete G-metric space $\alpha:\Omega\times\Omega\times\Omega\to R$ be a map. Let $\Lambda:\Omega\to\Omega$ be a map. Assume $\{\varpi_n\}$ is a α-G-regular if the subsequent criteria is fulfilled: If $\{\varpi_n\}$ is a sequence in Ω such that $\alpha(\varpi_n,\varpi_{n+1},\varpi_{n+1})\geq 1\ \forall n$ and $\varpi_n\to\varpi\in\Omega$ as $n\to+\infty$, then there exists a sub-sequence $\{\varpi_{n(k)}\}$ of $\{\varpi_n\}$ such that $\alpha(\varpi_{n(k)},\varpi_{n(k)},\varpi)\geq 1\ \forall k.$

The continuity constraint of the mapping Λ in the preceding claim is removed in the next statement.

Theorem 3.2. Assume (Ω,G) is a complete G-metric space and α is a mapping from $\Omega\times\Omega\times\Omega$ to R and assume $\Lambda:\Omega\to\Omega$ is a mapping. Assume that the theorem 3.1 circumstances are fulfilled with $\{\varpi_n\}$ is an α-G-regular. Then Λ has a fixed point $\varpi^*\in\Omega$, and $\{\Lambda^n\varpi_1\}$ convergent to ϖ^*.

Proof. From the above theorem, we recognize that $\{\varpi_n\}$ is given by $\varpi_{n+1}=\Lambda\varpi_n$ for $n\geq 0$, and con-verges to a certain $\varpi^*\in\Omega$. Based on (3.2) and the theorem's condition (4), there is a sub-sequence $\{\varpi_{n(k)}\}$ of $\{\varpi_n\}$ in a way that

$$\lim_{k\to\infty}\alpha\left(\varpi_{n_k},\varpi_{n_k},\varpi^*\right)\geq 1$$

Using (3.1) for every k, we get that

$$\begin{aligned}\alpha\left(\varpi_{n_k},\varpi_{n_k},\varpi^*\right)\psi\left(G\left(\varpi_{n(k)+1},\varpi_{n(k)+1},\Lambda\varpi^*\right)\right)&=\alpha\left(\varpi_{n_k},\varpi_{n_k},\varpi^*\right)\psi\left(G\left(\Lambda\varpi_{n(k)},\Lambda\varpi_{n(k)},\Lambda\varpi^*\right)\right)\\&\leq\beta\left(\psi\left(\Delta\left(\varpi_{n(k)},\varpi_{n(k)},\varpi^*\right)\right)\right)\psi\left(\Delta\left(\varpi_{n(k)},\varpi_{n(k)},\varpi^*\right)\right)\end{aligned}$$

(3.10)

On the different one, there is

$$\begin{aligned}\Delta\left(\varpi_{n_k},\varpi_{n_k},\varpi^*\right)&=\max\left\{G\left(\varpi_{n_k},\varpi_{n_k},\varpi^*\right),G\left(\varpi_{n_k},\Lambda\varpi_{n_k},\Lambda\varpi_{n_k}\right),G\left(\varpi_{n_k},\Lambda\varpi_{n_k},\Lambda\varpi_{n_k}\right),G\left(\varpi^*,\Lambda\varpi^*,\Lambda\varpi^*\right)\right\}\\&=\max\left\{\begin{matrix}G\left(\varpi_{n_k},\varpi_{n_k},\varpi^*\right),G\left(\varpi_{n_k},\Lambda\varpi_{n_k+1},\Lambda\varpi_{n_k+1}\right),G\left(\varpi_{n_k},\Lambda\varpi_{n_k+1},\Lambda\varpi_{n_k+1}\right),\\G\left(\varpi^*,\Lambda\varpi^*,\Lambda\varpi^*\right)\end{matrix}\right\}\end{aligned}$$

and hence,

$$\lim_{n\to\infty}\psi\left(\Delta\left(\varpi_{n_k},\varpi_{n_k},\varpi^*\right)\right)=\psi\left(G\left(\varpi^*,\varpi^*,\Lambda\varpi^*\right)\right)\qquad(3.11)$$

From (3.10), we have

$$\alpha\left(\varpi_{n_k},\varpi_{n_k},\varpi^*\right)\frac{\psi\left(G\left(\varpi_{n(k)+1},\varpi_{n(k)+1},\Lambda\varpi^*\right)\right)}{\psi\left(\Delta\left(\varpi_{n_k},\varpi_{n_k},\varpi^*\right)\right)}\leq\beta\left(\psi\left(\Delta\left(\varpi_{n_k},\varpi_{n_k},\varpi^*\right)\right)\right)<1$$

By allowing $k \to \infty$ in the preceding disparity, we obtain

$$\lim_{k\to\infty} \beta\left(\psi\left(\Delta\left(\varpi_{n_k}, \varpi_{n_k}, \varpi^*\right)\right)\right) = 1,$$

And so

$$\psi\left(G\left(\varpi^*, \varpi^*, \Lambda\varpi^*\right)\right) = \lim_{k\to\infty} \psi\left(\Delta\left(\varpi_{n_k}, \varpi_{n_k}, \varpi^*\right)\right) = 0.$$

Hence $\varpi^* = \Lambda\varpi^*$.

For the uniqueness of a fixed point of Λ, assume that the subsequent circumstance.

(H_1) For all $\varpi, \rho \in Fix(\Lambda)$, there exists $\varsigma \in \Omega$ such that $\alpha(\varpi, \varsigma, \varsigma) \geq 1$ $\alpha(\rho, \varsigma, \varsigma) \geq 1.$

Theorem 3.3. Putting criterion (H_1) to argument 3.1 results in ϖ^* being a distinct fixed point of Λ.

Proof. According to argument 3.1, we begin with a fixed point, namely $\varpi^* \in \Omega$ and take $\rho^* \in \Omega$ to be another fixed point of Λ. Then, by presumption, $\varsigma \in \Omega$ occurs in a way that

$$\alpha\left(\varpi^*, \varsigma, \varsigma\right) \geq 1, \quad \alpha\left(\rho^*, \varsigma, \varsigma\right) \geq 1. \tag{3.12}$$

Because Λ is $\alpha - G -$admissible, one obtains from (3.12),

$$\alpha\left(\varpi^*, \Lambda^n\varsigma, \Lambda^n\varsigma\right) \geq 1 \text{ and } \alpha\left(\rho^*, \Lambda^n\varsigma, \Lambda^n\varsigma\right) \geq 1$$

for all n. Hence we have

$$\begin{aligned} G\left(\varpi^*, \Lambda^n\varsigma, \Lambda^n\varsigma\right) &\leq \alpha\left(\varpi^*, \Lambda^{n-1}\varsigma, \Lambda^{n-1}\varsigma\right) G\left(\Lambda\varpi^*, \Lambda\Lambda^{n-1}\varsigma, \Lambda\Lambda^{n-1}\varsigma\right) \\ &\leq \beta\left(G\left(\varpi^*, \Lambda^{n-1}\varsigma, \Lambda^{n-1}\varsigma\right)\right) G\left(\varpi^*, \Lambda^{n-1}\varsigma, \Lambda^{n-1}\varsigma\right) \\ &< G\left(\varpi^*, \Lambda^{n-1}\varsigma, \Lambda^{n-1}\varsigma\right) \end{aligned}$$

(3.13)

$\forall n \in N$. Thus $G\left(\varpi^*, \Lambda^n\varsigma, \Lambda^n\varsigma\right)$ is non-increasing, and $u \geq 0$ occurs in a way that

$$\lim_{n\to\infty} G\left(\varpi^*, \Lambda^n\varsigma, \Lambda^n\varsigma\right) = u$$

From (3.13), we have

$$\frac{G\left(\varpi^*, \Lambda^n\varsigma, \Lambda^n\varsigma\right)}{G\left(\varpi^*, \Lambda^{n-1}\varsigma, \Lambda^{n-1}\varsigma\right)} \leq \beta\left(G\left(\varpi^*, \Lambda^{n-1}\varsigma, \Lambda^{n-1}\varsigma\right)\right).$$

And thus

$$\lim_{n\to\infty} \beta\left(G\left(\varpi^*, \Lambda^n\varsigma, \Lambda^n\varsigma\right)\right) = 1.$$

Hence

$$\lim_{n\to\infty} G\left(\varpi^*, \Lambda^n\varsigma, \Lambda^n\varsigma\right) = 0$$

Which implies $\lim_{n\to\infty} \Lambda^n\varsigma = \rho^*$, therefore, we get $\varpi^* = \rho^*$.

2. Consequences

If we take $\Lambda(\varpi,\rho,\varsigma)=G(\varpi,\rho,\varsigma)$ in theorem 3.1. Then we get the bellow contraction. We say that this contraction is called Erdal $-G-\alpha-\psi-$Geraghty contraction.

$$\alpha(\varpi,\rho,\varsigma)\psi\left(G(\Lambda\varpi,\Lambda\rho,\Lambda\varsigma)\right)\le\beta\left(\psi\left(G(\varpi,\rho,\varsigma)\right)\right)\psi\left(G(\varpi,\rho,\varsigma)\right) \tag{4.1}$$

for all $\varpi,\rho,\varsigma\in\Omega$ and $\psi\in\Psi$.

Theorem 4.1. Let (Ω,G) be a complete G-metric space, $\alpha:\Omega\times\Omega\times\Omega\to R$ be a function, and let $\Lambda:\Omega\to\Omega$ be a map. Assume that the theorem 3.1 circumstances are fulfilled with (4.1). Then Λ has a fixed point $\varpi^*\in\Omega$ and $\{\Lambda^n\varpi_1\}$ convergent to ϖ^*.

Proof. Assume $\varpi_1\in\Omega$ is a sequence such that $\alpha(\varpi_1,\Lambda\varpi_1,\Lambda\varpi_1)\ge1$, we observe through theorem 3.1 that $\{\varpi_n\}$ determined by $\varpi_{n+1}=\Lambda\varpi_n$ for all n converges to some $\varpi^*\in\Omega$ and $\alpha(\varpi_1,\Lambda\varpi_1,\Lambda\varpi_1)\ge1$, for all n. Because Λ is continuous, ϖ^* is a fixed point of Λ.

The continuity constraint of the mapping Λ in the preceding claim is removed in the next statement.

Theorem 4.2. Let (Ω,G) be a complete G-metric space, $\alpha:\Omega\times\Omega\times\Omega\to R$ be a function and let $\Lambda:\Omega\to\Omega$ be a map. Assume that the theorem 3.2 circumstances are fulfilled with (4.1)

Proof. Let $\varpi_1\in\Omega$ be such that $\alpha(\varpi_1,\Lambda\varpi_1,\Lambda\varpi_1)\ge1$, From theorem 3.1, we are aware that the sequence $\{\varpi_n\}$ determined by $\varpi_{n+1}=\Lambda\varpi_n$ for all n, converges to some $\varpi^*\in\Omega$, and $\alpha(\varpi_1,\Lambda\varpi_1,\Lambda\varpi_1)\ge1$, for all n. Assume that the circumstance α -G-regular holds. As a result, there is

$$\lim_{n\to\infty} sup\alpha\left(\varpi_n,\varpi^*,\varpi^*\right)\ge0.$$

Thus, there exists a sub sequence ϖ_{n_k} of ϖ_n such that

$$\lim_{n\to\infty}\alpha\left(\varpi_n,\varpi^*,\varpi^*\right)=p>0.$$

Therefore it occurs

$$\begin{aligned}\psi\left(G\left(\varpi_{n(k)+1},\Lambda\varpi^*,\Lambda\varpi^*\right)\right)&=\psi\left(G\left(\Lambda\varpi_{n(k)},\Lambda\varpi^*,\Lambda\varpi^*\right)\right)\\&\le\frac{1}{\alpha\left(\varpi_{n_k},\varpi^*,\varpi^*\right)}\beta\left(\psi\left(G\left(\varpi_{n(k)},\varpi^*,\varpi^*\right)\right)\right)\psi\left(G\left(\varpi_{n(k)},\varpi^*,\varpi^*\right)\right)\\&\le\frac{1}{\alpha\left(\varpi_{n_k},\varpi^*,\varpi^*\right)}\psi\left(G\left(\varpi_{n(k)},\varpi^*,\varpi^*\right)\right)\end{aligned}$$

for all sufficiently large k. Hence, we obtain

$$\psi\left(G\left(\varpi^*,\Lambda\varpi^*,\Lambda\varpi^*\right)\right)=\lim_{k\to\infty}\psi\left(G\left(\varpi_{n(k)+1},\Lambda\varpi^*,\Lambda\varpi^*\right)\right)$$
$$\le\frac{1}{p}\lim_{k\to\infty}\psi\left(G\left(\varpi_{n(k)+1},\Lambda\varpi^*,\Lambda\varpi^*\right)\right)=0.$$

Therefore ϖ^* is a fixed point of Λ.

Theorem 4.3. By including premise (H_1) into theorem 4.1, we find that ϖ^* is the unique fixed point of Λ.

Proof. From theorem 3.2, we've got a fixed point, namely $\varpi^*\in\Omega$. Now let $\rho^*\in\Omega$ be a different fixed point of Λ. Then, by the presumption $\varsigma\in\Omega$ occurs in a way that

$$\alpha\left(\varpi^*,\varsigma,\varsigma\right)\ge 1,\ \ \alpha\left(\rho^*,\varsigma,\varsigma\right)\ge 1. \tag{4.2}$$

From (4.2) and noted that Λ is $\alpha-G-$admissible, get $\alpha\left(\varpi^*,\Lambda^n\varsigma,\Lambda^n\varsigma\right)\ge 1,$ and $\alpha\left(\rho^*,\Lambda^n\varsigma,\Lambda^n\varsigma\right)\ge 1,$ For all n. Hence we have

$$\psi\left(G\left(\varpi^*,\Lambda^n\varsigma,\Lambda^n\varsigma\right)\right)\le\alpha\left(\varpi^*,\Lambda^{n-1}\varsigma,\Lambda^{n-1}\varsigma\right)\psi\left(G\left(\Lambda\varpi^*,\Lambda\Lambda^{n-1}\varsigma,\Lambda\Lambda^{n-1}\varsigma\right)\right)$$
$$\le\beta\left(G\left(\varpi^*,\Lambda^{n-1}\varsigma,\Lambda^{n-1}\varsigma\right)\right)\psi\left(G\left(\varpi^*,\Lambda^{n-1}\varsigma,\Lambda^{n-1}\varsigma\right)\right)$$
$$<\psi\left(G\left(\varpi^*,\Lambda^{n-1}\varsigma,\Lambda^{n-1}\varsigma\right)\right)$$

(4.3)

$\forall n\in N$. Then $\psi\left(G\left(\varpi^*,\Lambda^n\varsigma,\Lambda^n\varsigma\right)\right)$ is non increasing and there is a value $u\ge 0$ that is so

$$\lim_{n\to\infty}\psi\left(G\left(\varpi^*,\Lambda^n\varsigma,\Lambda^n\varsigma\right)\right)=u.$$

From (4.3), we have

$$\frac{\psi\left(G\left(\varpi^*,\Lambda^n\varsigma,\Lambda^n\varsigma\right)\right)}{\psi\left(G\left(\varpi^*,\Lambda^{n-1}\varsigma,\Lambda^{n-1}\varsigma\right)\right)}\le\beta\left(\psi\left(G\left(\varpi^*,\Lambda^{n-1}\varsigma,\Lambda^{n-1}\varsigma\right)\right)\right)$$

And thus

$$\lim_{n\to\infty}\beta\left(\psi\left(G\left(\varpi^*,\Lambda^n\varsigma,\Lambda^n\varsigma\right)\right)\right)=1.$$

Hence

$$\lim_{n\to\infty}\psi\left(G\left(\varpi^*,\Lambda^n\varsigma,\Lambda^n\varsigma\right)\right)=0$$

Which implies

$$\lim_{n\to\infty}\Lambda^n\varsigma=\varpi^*$$

Similarly, we have

$$\lim_{n\to\infty}\Lambda^n\varsigma=\rho^*$$

Therefore, we get $\rho^*=\varpi^*$.

The below one is suitable for the theorem 4.1.

Example 4.1. Assume $\Omega=[0,\infty)$ and $G(\varpi,\rho,\varsigma)=|\varpi-\rho|+|\rho-\varsigma|+|\varsigma-\varpi|\quad \forall \varpi,\rho,\varsigma\in\Omega$

Assume $\beta(t)=\frac{1}{1+t}\quad \forall t\geq 0$ then $\beta\in\Gamma$. Assume $\psi(t)=\frac{t}{2}$ a mapping $\Lambda:\Omega\to\Omega$ be given by

$$\Lambda\varpi=\begin{cases}\frac{\varpi}{6}, & if\ 0\leq\varpi\leq 1,\\ 6\varpi, & if\ \varpi\geq 1.\end{cases}$$

And $\alpha:\Omega\times\Omega\times\Omega\to[0,\infty)$ is given by

$$\alpha(\varpi,\rho,\varsigma)=\begin{cases}1, & if\ \ 0\leq\varpi,\rho,\varsigma\leq 1\\ 0, & otherwise.\end{cases}$$

Criterion (3) of theorem 4.1 is met by $\varpi=1$. Criterion (4) of theorem 4.1 can be fulfilled by $\varpi_n=\Lambda^n\varpi_1=\frac{1}{6^n}$, obviously, condition (2) is satisfied. Let $\varpi,\rho,\varsigma\in\Omega$ be such that $\alpha(\varpi,\rho,\varsigma)\geq 1$. Then $\varpi,\rho,\varsigma\in[0,1]$ and so $\Lambda\varpi\in[0,1],\Lambda\rho\in[0,1],\Lambda\varsigma\in[0,1]$ and $\alpha(\Lambda\varpi,\Lambda\rho,\Lambda\varsigma)=1$. Hence, Λ is α-G-admissible and hence (2) is fulfilled. At last, we are going to show that (1) is satisfied. If $0\leq\varpi,\rho,\varsigma\leq 1$, Then $\alpha(\varpi,\rho,\varsigma)=1$ and we get

$$\begin{aligned}
&\beta\big(\psi\big(G(\varpi,\rho,\varsigma)\big)\big)\psi\big(G(\varpi,\rho,\varsigma)\big)-\alpha(\varpi,\rho,\varsigma)\psi\big(G(\Lambda\varpi,\Lambda\rho,\Lambda\varsigma)\big)\\
&=\beta\big(\psi\big(G(\varpi,\rho,\varsigma)\big)\big)\psi\big(G(\varpi,\rho,\varsigma)\big)-\psi\big(G(\Lambda\varpi,\Lambda\rho,\Lambda\varsigma)\big)\\
&=\frac{\frac{|\varpi-\rho|+|\rho-\varsigma|+|\varsigma-\varpi|}{2}}{1+\frac{|\varpi-\rho|+|\rho-\varsigma|+|\varsigma-\varpi|}{2}}-\frac{1}{12}[|\varpi-\rho|+|\rho-\varsigma|+|\varsigma-\varpi|]\\
&=\frac{|\varpi-\rho|+|\rho-\varsigma|+|\varsigma-\varpi|}{2+|\varpi-\rho|+|\rho-\varsigma|+|\varsigma-\varpi|}-\frac{1}{12}[|\varpi-\rho|+|\rho-\varsigma|+|\varsigma-\varpi|]\\
&=\frac{[|\varpi-\rho|+|\rho-\varsigma|+|\varsigma-\varpi|][12-(2+|\varpi-\rho|+|\rho-\varsigma|+|\varsigma-\varpi|)]}{12(2+|\varpi-\rho|+|\rho-\varsigma|+|\varsigma-\varpi|)}\\
&=\frac{[|\varpi-\rho|+|\rho-\varsigma|+|\varsigma-\varpi|][10-(|\varpi-\rho|+|\rho-\varsigma|+|\varsigma-\varpi|)]}{12(2+|\varpi-\rho|+|\rho-\varsigma|+|\varsigma-\varpi|)}\\
&\geq 0
\end{aligned}$$

For any $\varpi,\rho,\varsigma\in\Omega$. Therefore

$$\alpha(\varpi,\rho,\varsigma)\psi\big(G(\Lambda\varpi,\Lambda\rho,\Lambda\varsigma)\big)=\beta\big(\psi\big(G(\varpi,\rho,\varsigma)\big)\big)\psi\big(G(\varpi,\rho,\varsigma)\big)$$

Since $\alpha(\varpi,\rho,\varsigma)=0$. As a result, all of the conditions of theorem 4.1 are met, and Λ has a fixed point $\varpi^*=0$.

5. Application

Assume the second order differential equation's boundary value problem

$$-\frac{d^2\varpi}{d\sigma^2}=\begin{cases} f\left(\sigma,\varpi\left(t\right)\right),\sigma\in\left[0,1\right]. \\ \varpi\left(0\right)=\varsigma\left(1\right)=0. \end{cases} \tag{5.1}$$

Where $f:[0,1]\times R\rightarrow R$ is continuous mapping. Green's function determined by

$$G'\left(\sigma,\upsilon\right)=\begin{cases} \sigma\left(1-\upsilon\right); if\ 0\le\sigma\le\upsilon\le 1; \\ \upsilon\left(1-\sigma\right); if\ 0\le\upsilon\le\sigma\le 1. \end{cases}$$

Assume $\Omega=C([0,1])$ is continuous function defined on I = [0, 1]. Now that we've determined the generalized metric G on Ω.

$$G\left(\varpi,\rho,\varsigma\right)=\|\varpi-\rho\|+\|\rho-\varsigma\|+\|\varsigma-\varpi\|$$
$$=\sup_{\sigma\in I}|\varpi(\sigma)-\rho(\sigma)|+\sup_{\sigma\in I}|\rho(\sigma)-\varsigma(\sigma)|+\sup_{\sigma\in I}|\varsigma(\sigma)-\varpi(\sigma)|$$

$\forall\varpi,\rho,\varsigma\in\Omega$. Then (Ω,G) is a complete G-metric space. Assume the subsequent circumstances

(a) Occurs $\zeta:R^3\rightarrow R$ so that $\forall\rho\in I, a,b\in R$ using $\zeta(q,w,e)\ge 0$, we have
$|f(\sigma,q)-f(\sigma,w)|\le\ln(|q-w|+1)$;

(b) Occurs $\varpi_1\in C(I)$ so that $\forall\rho\in I$,
$\zeta(\varpi_1(\sigma),\int_0^1 G'(\sigma,\upsilon)f(\upsilon,\varpi_1(\upsilon))d\upsilon,\int_0^1 G'(\sigma,\upsilon)f(\upsilon,\varpi_1(\upsilon))d\upsilon)\ge 0$;

(c) For all $\rho\in I$ and for all $\varpi,\rho,\varsigma\in\Omega,\zeta(\varpi(\sigma),\rho(\sigma),\varsigma(\sigma))\ge 0$ implies
$\zeta(\int_0^1 G'(\sigma,\upsilon)f(\upsilon,\varpi(\upsilon))d\upsilon,\int_0^1 G'(\sigma,\upsilon)f(\upsilon,\rho(\upsilon))d\upsilon,\int_0^1 G'(\sigma,\upsilon)f(\upsilon,\varsigma(\upsilon))d\upsilon)\ge 0$;

(d) for each ϖ of ϖ_n of points in C(I) with $\zeta(\varpi(\sigma),\rho(\sigma),\varsigma(\sigma))\ge 0$, $\liminf_{n\rightarrow\infty}\zeta(\varpi_n,\varpi,\varpi)=0$.

Theorem 5.1. Assume that requirements (a)-(d) are met, then (5.1) has at least one solution $\varpi^*\in C^2(I)$.

Proof. $\varpi^*\in C^2(I)$ is assumed to be a solution of (5.1) if and only if $\varpi\in C(I)$ is a solution of the integral equation.

$$\varpi(\sigma)=\int_0^1 G'(\sigma,\upsilon)f(\upsilon,\varpi(\upsilon))d\upsilon, \tag{5.2}$$

For all $\sigma\in I$. We derive $\Lambda:C(I)\rightarrow C(I)$ by

$$\Lambda\varpi(\sigma)=\int_0^1 G'(\sigma,\upsilon)f(\upsilon,\varpi(\upsilon))d\upsilon, \tag{5.3}$$

$\forall\rho\in I$. Then, the difficulty (5.1) is equal to $\varpi^*\in C(I)$ fixed point of Λ. Assume $\varpi,\rho,\varsigma\in\Omega$ such that $\zeta(\varpi(\sigma),\rho(\sigma),\varsigma(\sigma))\ge 0$, for all $\sigma\in I$. From (a), we have

$$
\begin{aligned}
G(\Lambda\varpi,\Lambda\rho,\Lambda\varsigma) &= |\Lambda\varpi(\sigma)-\Lambda\rho(\sigma)|+|\Lambda\rho(\sigma)-\Lambda\varsigma(\sigma)|+|\Lambda\varsigma(\sigma)-\Lambda\varpi(\sigma)| \\
&= |\int_0^1 G'(\sigma,\upsilon)f(\upsilon,\varpi(\upsilon))d\upsilon-\int_0^1 G'(\sigma,\upsilon)f(\upsilon,\rho(\upsilon))d\upsilon| \\
&\quad +|\int_0^1 G'(\sigma,\upsilon)f(\upsilon,\rho(\upsilon))d\upsilon-\int_0^1 G'(\sigma,\upsilon)f(\upsilon,\varsigma(\upsilon))d\upsilon| \\
&\quad +\int_0^1 G'(\sigma,\upsilon)f(\upsilon,\varsigma(\upsilon))d\upsilon-\int_0^1 G'(\sigma,\upsilon)f(\upsilon,\varpi(\upsilon))d\upsilon| \\
&= |\int_0^1 G'(\sigma,\upsilon)[f(\upsilon,\varpi(\upsilon))-f(\upsilon,\rho(\upsilon))]d\upsilon| \\
&\quad +|\int_0^1 G'(\sigma,\upsilon)[f(\upsilon,\rho(\upsilon))-f(\upsilon,\varsigma(\upsilon))]d\upsilon| \\
&\quad +|\int_0^1 G'(\sigma,\upsilon)[f(\upsilon,\varsigma(\upsilon))-f(\upsilon,\varpi(\upsilon))]d\upsilon| \\
&\le \int_0^1 G'(\sigma,\upsilon)\{|f(\upsilon,\varpi(\upsilon))-f(\upsilon,\rho(\upsilon))|+|f(\upsilon,\rho(\upsilon))-f(\upsilon,\varsigma(\upsilon))| \\
&\qquad +|f(\upsilon,\varsigma(\upsilon))-f(\upsilon,\varpi(\upsilon))|\}d\upsilon \\
&\le \int_0^1 G'(\sigma,\upsilon)\{\ln(|\varpi(\upsilon)-\rho(\upsilon)|+1)+\ln(|\rho(\upsilon)-\varsigma(\upsilon)|+1)+\ln(|\varsigma(\upsilon)-\varpi(\upsilon)|+1)\}d\upsilon \\
&\le \sup_{\sigma\in I}\int_0^1 G'(\sigma,\upsilon)d\upsilon\{\ln(|\varpi(\upsilon)-\rho(\upsilon)|+1)+\ln(|\rho(\upsilon)-\varsigma(\upsilon)|+1)+\ln(|\varsigma(\upsilon)-\varpi(\upsilon)|+1)\} \\
&= \frac{1}{2}[\ln(|\varpi(\upsilon)-\rho(\upsilon)|+1)+\ln(|\rho(\upsilon)-\varsigma(\upsilon)|+1)+\ln(|\varsigma(\upsilon)-\varpi(\upsilon)|+1)] \\
&\le \ln(|\varpi(\upsilon)-\rho(\upsilon)|+1)+\ln(|\rho(\upsilon)-\varsigma(\upsilon)|+1)+\ln(|\varsigma(\upsilon)-\varpi(\upsilon)|+1) \\
&\le \ln(|\varpi(\upsilon)-\rho(\upsilon)|)+\ln(|\rho(\upsilon)-\varsigma(\upsilon)|)+\ln(|\varsigma(\upsilon)-\varpi(\upsilon)|) \\
&\le \ln(|\varpi(\upsilon)-\rho(\upsilon)|+|\rho(\upsilon)-\varsigma(\upsilon)|+|\varsigma(\upsilon)-\varpi(\upsilon)|) \\
&= \ln(G(\varpi,\rho,\varsigma))=\ln(G(\varpi,\rho,\varsigma))+\ln 1=\ln(G(\varpi,\rho,\varsigma)+1)
\end{aligned}
$$

Which yields that

$$\ln(G(\varpi,\rho,\varsigma)+1)\le \ln(\ln(G(\varpi,\rho,\varsigma)+1))+1=\frac{\ln(\ln(G(\varpi,\rho,\varsigma)+1))+1}{\ln(G(\varpi,\rho,\varsigma)+1)}\ln(G(\varpi,\rho,\varsigma)+1)$$

Place $\psi(\varpi)=\ln(\varpi+1)$ and $\beta(\varpi)=\frac{\psi(\varpi)}{\varpi}$. Undoubtedly $\psi:[0,\infty)\to[0,\infty)$ is continuous, sub additive, and non-decreasing, and ψ is positive in $(0,\infty)$ with $\psi(0)=0$, as well as $\psi(\varpi)<\varpi$ for any $\beta\in\Gamma$. Thus we have $\psi(G(\Lambda\varpi,\Lambda\rho,\Lambda\varsigma))\le\beta(\psi(G(\varpi,\rho,\varsigma)))\psi(G(\varpi,\rho,\varsigma))$, for all $\varpi,\rho,\varsigma\in C(I)$ such that $\zeta(\varpi(\sigma),\rho(\sigma),\varsigma(\sigma))\ge 0$, for all $\sigma\in I$. We derive $\alpha:C(I)\times C(I)\times C(I)\to[0,\infty)$ by

$$\alpha(\varpi,\rho,\varsigma)=\begin{cases}1, if\ \zeta(\varpi(\sigma),\rho(\sigma),\varsigma(\sigma))\ge 0;\\ 0, otherwise.\end{cases}$$

Then, for all $\varpi,\rho,\varsigma\in C(I)$, we have $\alpha(\varpi,\rho,\varsigma)G(\Lambda\varpi,\Lambda\rho,\Lambda\varsigma)<\beta(G(\varpi,\rho,\varsigma))G(\varpi,\rho,\varsigma)$ obviously, $\alpha(\varpi,\rho,p)=1$ and $\alpha(p,\rho,\varsigma)=1$ implies $\alpha(\varpi,\rho,\varsigma)=1$, for all $\varpi,\rho,\varsigma\in C(I)$. If $\alpha(\varpi,\rho,\varsigma)=1$ for all $\varpi,\rho,\varsigma\in C(I)$, then $\zeta(\varpi(\sigma),\rho(\sigma),\varsigma(\sigma))\ge 0$. From (c) we have

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$\zeta(\Lambda\varpi(\sigma),\Lambda\rho(\sigma),\Lambda\varsigma(\sigma))\geq 1$. Therefore Λ is rectangular α-G-admissible. According to (b) there exists $\varpi_1 \in C(I)$ so that $\alpha(\varpi_1,\Lambda\varpi_1,\Lambda\varpi_1)=1$. According to (d), for any point ϖ of ϖ_n of points in C (I) using $\alpha(\varpi_n,\varpi_{n+1},\varpi_{n+1})=1$, $\liminf_{n\to\infty}\alpha(\varpi_n,\varpi,\varpi)=1$. Apply theorem (4.1), Λ has a fixed point in C (I) that is there occurs $\varpi^* \in C(I)$ so that $\Lambda\varpi^*=\varpi^*$ and ϖ^* is a solution of (5.1).

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