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New Algorithms on E-Super (a, d)-edge-antimagic Graceful labeling

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Abstract:
An E-super (a, d)-edge-antimagic graceful labeling (EEAGL) is aone-one and onto function λ from the union of the vertex set and edgeset of G into the integers from 1 to $p + q$ where P is the total number of vertices. The absolute value of $\lambda(u) + \lambda(v) - \lambda(uv)$, uv in G consists of integers from a to $a + (q-1)$ d which are consecutive with a, the initial term and d, the common difference. If the edge-weights of the graph G are labeled by the integers from 1 to q then the labeling is named as *EEAGL*. In this paper, we prove the above labeling for the disjoint union of multiple copies (*DUMC*) of cycle graphs, complete graphs and path graphs. Finally, we construct algorithms to find some classes of graphs are EEAGL.

Keywords: Super (a,d)-edge-antimagic graceful labeling, E-Super (a,d)-edge-antimagic graceful labeling.

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1 Introduction

Undirected and simple graphs only used in this paper. The order and size of the graphs are p and q respectively. The notions of graph theory are taken from [7].

The vertices and edges of the graph G assigns some positive value is called graph labeling. Refer [1,2] for *EML* and *EAL* of graphs.

Marimuthu and Balakrishnan introduced the concept of *EMGL* and *ESVML* in [3,6]. In [4] and [5] we get more results about *SEAGL*.

An *EEAGL* is a one-one and onto mapping λ from the union of the vertices and edges of G into the integers 1 to $p + q$. The absolute value of $\lambda(u)+\lambda(v)-\lambda(uv)$, for uv in G, consists of integers from a to $a+(q-1)d$ whichare consecutive with a, the initial term and d, the common difference.

In this paper, we prove the above labeling for the *DUMC* of cycle graphs, complete graphs and path graphs. Finally, we construct algorithms to find some classes of graphs are *EEAGL*.

2 Disconnected Graphs

2.1 Disconnected Cycle graphs

Theorem 2.1. If m ≥ 2 and n ≥3 then the DUMC of the cycle graphs mCn admits an EEAGL.

Proof:

Step 1: (Input)

mC_n: m copies of the cycle graph Cn;

$V(mC_n)$: Vertices of mC_n ;

$V(mC_n)$: { u_i^j : i in 1 to n, j in 1 to m};

$E(mC_n)$: Edges of mC_n ;

$E(mC_n)$: {$u_i^j\ u_{i+1}^j$: i in 1 to n-1, j in 1 to m} U {$u_n^j\ u_1^j$: j in 1 to m};

: $V(mC_n)$ U $E(mC_n)$ → { 1,2,---, 2mn};

W : W_λ^1 U W_λ^2 : the edge-weights of mC_n ;

Step 2:

{
for i in 1 to n , j in 1 to m do
{
(u_i^j) = mn + i+ (j – 1) n
}
for i in 1 to n-1 , j in 1 to m do
{
($u_i^j\ u_{i+1}^j$) = i + n(j-1) + 1
}
for j in 1 to m do
{
($u_n^j\ u_1^j$) = n(j-1) +1
}
for i in 1 to n-1, j in 1 to m do
{
W_λ^1 = { W_λ^1($u_i^j\ u_{i+1}^j$)= 2mn + I + n(j-1)
}
for j in 1 to m do
{
W_λ^2 = { W_λ^2($u_n^j\ u_1^j$)= 2mn + n(j-1) + n
}
{
W = { 2mn+1, 2mn+2,…,3mn} : consecutive integes
}
}
Step 3 : Output (EEAGL of mC_n)

2.2 Disconnected Complete graphs

Theorem 2.2. *If $m, n \geq 2$, then the DUMC of the complete graphs mK_n has an EEAGL.*

Proof:

Step 1: (Input)
mK_n: m copies of the complete graph K_n;
$V(mK_n)$: Vertices of mK_n;
$V(mK_n)$: { u_i^j i in 1 to n, j in 1 to m } ;
{ u_1^j, u_2^j, ---, u_n^j } : The vertex set of the j[th] copies of Kn , j in 1 to m;
E(mK_n): Edges of mK_n;
E (mK_n): $\bigcup_{j=1}^{m} \bigcup_{i=1}^{n-1} \{u_i^j\ u_{i+k}^j$: k in 1 to $n-i$ } ;
$\lambda : V(mK_n) \cup E(mK_n) \to \{1, 2, \ldots, \frac{mn(n+1)}{2}\}$;
Step 2:
{
for i in 1 to *n* , *j* in 1 to *m do*
{
$\lambda(u_i^j) = m (n - i + 1) - j + 1 + \frac{mn(n-1)}{2}$
}
for i in 1 to n-1 , k in 1 to n-i and j in 1 to m do
{
$\lambda(u_i^j\ u_{i+k}^j) = m (n (k + 1) - \frac{k(k+1)}{2} + 1 - i) + 1 - j - m n$

if i+k < n

The edge-weight of $u_i^j\, u_{i+k}^j$ = W($u_i^j\, u_{i+k}^j$) = mn^2 - m(n(k-1) - $\frac{k(k+1)}{2}$ + (k+1) + i) + 1 - j

}

{
W($u_i^j\, u_{i+k}^j$) = {1 + mn(n+1)/2, 2 + mn(n+1)/2, ..., mn(n+1)/2 + mn(n-1)/2}
}
}
Step 3: Output (EEAGL of mK$_n$)

2.3 Disconnected Path graphs

Theorem 2.3. *If* $m, n \geq 2$ *then the DUMC of path graphs* mP_n *permits an EEAGL.*

Proof:

Step 1: (Input)
mP_n: m copies of the path graph P_n;
$V(mP_n)$: Vertices of mP_n;
$V(mP_n)$: {u_i^j i in 1 to n, j in 1 to m};
$E(mP_n)$: Edges of mP_n;
$E(mP_n)$: { $u_i^j\, u_{i+1}^j$: i in 1 to n , j in 1 to m };
W : set of edge-weights ;
Define $\lambda : V(mP_n) \cup E(mP_n) \to \{1, 2, ..., m(2n-1)\}$;
Step 2:
{
for i in 1 to $n-1$, j in 1 to m *do*
{
$\lambda(u_i^j) = m(n-1) + j + (i-1)m$
$\lambda(u_i^j\, u_{i+1}^j) = j + (i-1)m$
}
{
$W = \{m(2n-1)+1, m(2n-1)+2, ..., 3mn-m-3\}$
}
}
Step 3: Output (*EEAGL* of mP_n)

3. Friendship Graphs

Theorem 3.1. *For* $n \geq 1$, $\mathbf{F_n}$, *the friendship graph has an EEAGL.*

Proof:

Step 1: (Input)
$\mathbf{F_n}$, $n \geq 1$ be the friendship graph.
$V(G)$: Vertices of $\mathbf{F_n}$;
$V(G)$: 1 to $2n+1$;
$E(G)$: Edges of $\mathbf{F_n}$;
$E(G)$: $2n+2$ to $5n+1$
λ : The bijective function on $V(G) \cup E(G)$;
c : the center vertex of $\mathbf{F_n}$;
u_i and v_i : the other two vertices of the i^{th} triangle;

W : Edge-weights
Step 2:
{
for i in 1 to n *do*
{
$\lambda(c) = 4n + 1$
$\lambda(u_i) = 3n + i$
$\lambda(v_i) = 5n + 2 - i$
}
for i in 1 to n *do*
{
$\lambda(u_i c) = n + 2i - 1$
$\lambda(v_i c) = 3n + 2 - 2i$
$\lambda(u_i v_i) = i$
$W = \{5n + 2, 5n + 3, \ldots, 8n + 1\}$ is consecutive integers.
}
}
Step 3: Output (*EEAGL* friendship graphs) ■

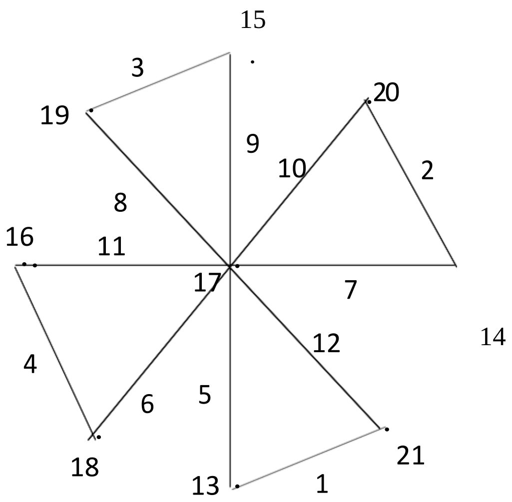

Figure 3.1: An *EEAGL* of $\mathbf{F_4}$

4. Cycles

Theorem 4.1. *If* $n \geq 3$*, then the cycle* C_n *has an EEAGL.*

Proof:

Step 1: (Input)
C_n, $n \geq 3$ be the cycle graphs.
$V(G)$: Vertices of C_n;
$V(G)$: 1 to n;
$E(G)$: Edges of C_n;
$E(G)$: $n + 1$ to $2n$
λ : The bijective function on $V(G) \cup E(G)$;
Step 2:
{
for i in 1 to n *do*

{
$\lambda(u_i) = n + i$
}
for i in 1 to $n - 1$ *do*
{
$\lambda(u_i u_{i+1}) = i$
}
for i in 1 to n *do*
{
$\lambda(u_n u_1) = n$
}
{
set of edge weights = $\{2n + 1, 2n + 2, \text{---}\ 3n\}$ = consecutive integers
}
}
Step 3: Output ($EEAGL$ cycle graphs) ■

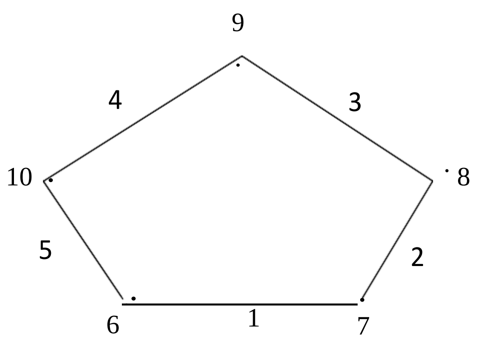

Figure 4.1: An $EEAGL$ of C_5

5. Fan Graphs

Theorem 5.1. *If* $2 \le n \le 6$ *and* $d = 1$ *then the fan graph* F_n *admits an* $EEAGL$.

Proof:

Step 1: (Input)
$F_n, n \ge 2$ be the fan graphs.
$V(G)$: Vertices of F_n;
$V(G)$: 1 to $n + 1$;
$E(G)$: Edges of F_n;
$E(G)$: $n + 2$ to $3n$
c : the center vertex of F_n;
$\lambda_1 : V(F_n) \to \{1, 2, \ldots, n + 1\}$
$\lambda_2 : E(F_n) \to \{n + 2, n + 3, \ldots, 3n\}$
Step 2:
{
for $n = 2$ *do*
{
$\lambda_1(u_1) = 4$
$\lambda_1(u_2) = 5$
$\lambda_1(c) = 6$

}
for $n=3$ *do*
{
$\lambda_1(u_1)=6$
$\lambda_1(u_2)=7$
$\lambda_1(u_3)=8$
$\lambda_1(c)=9$
}
for $n=4$ *do*
{
$\lambda_1(u_1)=8$
$\lambda_1(u_2)=9$
$\lambda_1(u_3)=11$
$\lambda_1(u_4)=12$
$\lambda_1(c)=10$
}
for $n=5$ *do*
{
$\lambda_1(u_1)=11$
$\lambda_1(u_2)=10$
$\lambda_1(u_3)=12$
$\lambda_1(u_4)=14$
$\lambda_1(u_5)=15$
$\lambda_1(c)=13$
}
for $n=6$ *do*
{
$\lambda_1(u_1)=13$
$\lambda_1(u_2)=12$
$\lambda_1(u_3)=14$
$\lambda_1(u_4)=16$
$\lambda_1(u_5)=18$
$\lambda_1(u_6)=17$
$\lambda_1(c)=15$
}
for i in 1 to $2n-1$ *do*
{
The set of edge-weights $w_\lambda = w_\lambda(q_i) = 4 + i \quad \forall q_i \in \mathcal{F}_n$
}
for i is odd *do*
{
$\lambda 2(qi) = \frac{i+1}{2}$
}
for i is even *do*
{
$\lambda 2(qi) = \mathrm{n} + \frac{i}{2}$
}
for i in 1 to $2n-1$ *do*
{
$W = |w_{\lambda_1}(q_i) - w_{\lambda_2}(q_i)|$

}
}
Step 3: Output (*EEAGL* of fan graphs) ■

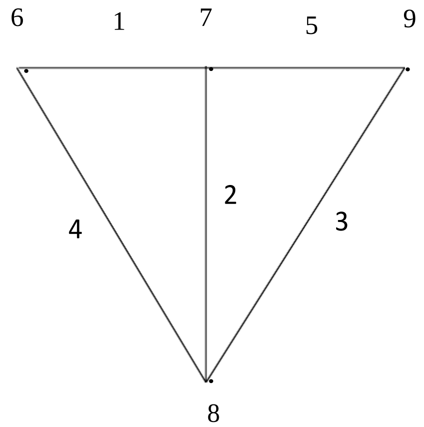

Figure 5.1: An *EEAGL* of F_3

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