Tuijin Jishu/Journal of Propulsion Technology
ISSN: 1001-4055
Vol.44 No. 4 (2023)

Analysis of Simply Supported Laminated Beam using Shear Deformation Theory

Renu P Pathak[1], Mahesh V. Ghotkar[2], Satyawan L. Dhondge[3] and Durgesh H. Tupe[4], Gajendra Gandhe[5] , Ramdas Biradar[6]

[1] *Professor, Department of Mathematics, School of Science, Sandip University, Nashik. (Maharashtra), India.*

[2] *Research Scholar, Department of Mathematics, School of Science, Sandip University, Nashik. (Maharashtra), India.*

[3] *Professor, Department of Basic Science and Humanities, Deogiri Institute of Engineering and Management Studies, Aurangabad (Maharashtra), India.*

[4] *Assistant Professor, Department of Civil Engineering, Deogiri Institute of Engineering and Management Studies, Aurangabad (Maharashtra), India.*

[5] *Professor, Department of Civil Engineering, Deogiri Institute of Engineering and Management Studies, Aurangabad (Maharashtra), India.*

[6] *Professor, School of Engineering and Technology, Pimpri Chinchwad University, Pune (Maharashtra), India.*

Abstract:

The shear deformation effects of an isotropic beam were investigated in this study utilising Elementary Beam Theory (ETB). Based on Elementary Beam speculation, systematic approach and blends for the tension investigation of a directly hold up joist put through to linear thermal bale are offered. The fundamental expulsion, horizontal expulsion, fundamental curve tension, and horizontal trim tension of a simply supported beam are all calculated.

Keywords: Displacement, Laminated beam, Elementary Beam Theory, Shear Deformation Theory.

1. Introduction:

The ETB premise states that flat segment across to the impartial bend remain flat and across before crook. As a result, the supposition ignores the consequences of trim strain**.** The principle only applies to thin joist and may perhaps not used on broad joists. We used the Navier equation in this chapter to get the desired solution. This theory's displacement field is as follows

$$u\left(x,z\right)=-z\frac{\partial w}{\partial x}.$$

$$w\left(x,z\right)=w\left(x\right)$$

Ali *et al.* [1] proposes a new higher-order hypothesis based on relocation. The concept is based on actual removal options and has been demonstrated to be highly accurate for even thick overlays and any combination of mechanical and thermal stacking. Ali *et al.* [2] A microstructure subordinate Reddy beam hypothesis is developed in contrast to the conventional theory, which only has one material length scale barrier and is unable to account for the size influence in small size materials. Anderse and Lars [3 They firstly introduce the criteria for harmony, and then determine the classic bar hypotheses in the context of Bernoulli-Euler and Timoshenko beam kinematics. Messina [4] provided; cross-employ layerwise circular barrel-shaped boards are explored; and the impact of various edge limit restrictions is discussed. Arya *et al.* [5] presented a laminated composite beam zigzag model.

2. The Displacement Field for ETB

As per on the earlier than stated expectations, the exile area of the current composite laminated joint thesis can be given bellow:

$$u(x,z) = -z\frac{dw}{dx}$$
$$w(x,z) = w(x) \qquad (2.1)$$

A location on the joist in the center is described by the exiles u in the x direction and w in the horizontal order.

$$\varepsilon_x^0 = \frac{\partial u}{\partial x}, \quad k_x^0 = -\frac{\partial^2 w}{\partial x^2}, \quad k_x^2 = \frac{\partial \phi}{\partial x}$$

$$f\ (z)\ =\ h\sinh\frac{z}{h} - \frac{4}{3}\frac{z^3}{h^2}\cosh\frac{1}{2}$$

$$g\ (z)\ = 1 - f^{'}\ (z)$$

$$g\ (z) = \ 1 - \left[\cosh\frac{z}{h} - 4\frac{z^2}{h^2}\cosh\frac{1}{2}\right] \qquad (2.2)$$

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & B_{66} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_{xy}^0 \\ k_x^0 \\ k_y^0 \\ k_{xy}^0 \end{Bmatrix} \qquad (2.3)$$

Where N are horizontal squads, M are curve and wrap squads, ε are centre shear, k curve and wrap bends. $A_{ij}, B_{ij}, D_{ij}\ (i, j = 1,2,6)$ are the stiffness coefficient. Consider N_y, N_{xy}, M_y, M_{xy} equal to zero. While ε_y^0, $\varepsilon_{xy}^0, k_y^0, k_{xy}^0$ are assume to be non-zero.

$$\begin{Bmatrix} N_x \\ M_x \end{Bmatrix} = \begin{bmatrix} \bar{A}_{11} & \bar{B}_{11} \\ \bar{B}_{11} & \bar{D}_{11} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ k_x^0 \end{Bmatrix}$$

$$\begin{Bmatrix} N_x \\ M_x \end{Bmatrix} = \begin{bmatrix} \bar{A}_{11} & \bar{B}_{11} \\ \bar{B}_{11} & \bar{D}_{11} \end{bmatrix} \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial \theta}{\partial x} \end{Bmatrix}$$

$$\begin{bmatrix} \bar{A}_{11} & \bar{B}_{11} \\ \bar{B}_{11} & \bar{D}_{11} \end{bmatrix} = \begin{bmatrix} A_{11} & B_{11} \\ B_{11} & D_{11} \end{bmatrix} - \begin{bmatrix} A_{12} & A_{16} & B_{12} & B_{16} \\ B_{12} & B_{16} & D_{12} & D_{16} \end{bmatrix}$$
$$\begin{bmatrix} A_{22} & A_{26} & B_{22} & B_{26} \\ A_{26} & A_{66} & B_{26} & B_{66} \\ B_{22} & B_{26} & D_{22} & D_{26} \\ B_{26} & B_{66} & D_{26} & D_{66} \end{bmatrix}^{-1} \begin{bmatrix} A_{12} & B_{12} \\ A_{16} & B_{16} \\ B_{12} & D_{12} \\ B_{16} & D_{16} \end{bmatrix} \qquad (2.4)$$

— — —

The transformed reduced stiffness constant, $\overline{Q_{11}} = Q_{11} \cos^4 \theta$

Where θ is the perspective between the fiber route and along the axis of joist.

$$Q_{11} = \frac{E_1}{1-\mu_{12}\mu_{21}}$$

$$N_x = \int_{-h/2}^{h/2} \sigma_x^k dz,$$

$$M_x = \int_{-h/2}^{h/2} \sigma_x^k z\ dz, \qquad (2.5)$$

The following is the detailed formation of the in effect toil concept:

$$b \int_{x=0}^{x=L} \int_{-h/2}^{h/2} \left(\sigma_x^k \delta\varepsilon_x + \tau_{zx}^k \delta\gamma_{zx}\right) dz\ dx - \int_0^L q\ \delta w\ dx = 0$$

$$b \int_0^L \int_{-h/2}^{h/2} \sigma_x^k \left(\frac{d\delta u_0}{dx} - Z\ \frac{d^2 \delta w}{dx^2} \right) dxdz\ - \int_0^L q\ \delta w\ dx = 0$$

$$b \int_0^L \left(\mathrm{N}_x\ \frac{d^2 \delta u_0}{dx^2} - \mathrm{M}_x\ \frac{d^3 \delta w}{dx^3} \right) dx - \int_0^L q\ \delta w\ dx = 0 \qquad (2.6)$$

where δ is the modification administrator.

$$\delta u_0 : \frac{dN}{dx} = 0\ :\ ,\ \delta w : \frac{d^2 M}{dx^2} + q = 0 \qquad (2.7)$$

Either $\mathrm{N}=0$ or $\mathrm{u}_0 = 0$

Either $\frac{dM}{dx}=0$ or $\mathrm{W} = 0$

Either $\mathrm{M}=0$ or $\frac{dW}{dx}=0 \qquad (2.8)$

Using equation (2.4) in equation (2.6) , then we can in terms of displacement and variables are,

$$\begin{bmatrix} N_x \\ M_x \end{bmatrix} = \begin{bmatrix} \overline{A}_{11} & \overline{B}_{11} \\ \overline{B}_{11} & \overline{D}_{11} \end{bmatrix} \begin{bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{d^2\theta}{dx^2} \end{bmatrix}$$

$$\frac{dN_x}{dx} = 0$$

$$\overline{A}_{11} \frac{d^2 u_0}{dx^2} + \overline{B}_{11} \left(-\frac{d^3 w_b}{dx^3} \right) = 0$$

$$\overline{A}_{11} \frac{d^2 u_0}{dx^2} - \overline{B}_{11} \left(\frac{d^3 w}{dx^3} \right) = 0 \qquad (2.9)$$

If Poisson's effect is informed, the coefficients $\overline{A}_{11}, \overline{B}_{11}, \overline{D}_{11}$ should be replaced by the laminated stiffness coefficient A_{11}, B_{11}, D_{11} respectively,

$$A_{11}\frac{d^2u_0}{dx^2} - B_{11}\left(\frac{d^3w}{dx^3}\right) = 0$$

$$\frac{d^2M_x}{dx^2} + q = 0$$

$$\overline{B}_{11}\frac{d^3u_0}{dx^3} + \overline{D}_{11}\left(-\frac{d^4w}{dx^4}\right) + q = 0$$

$$\overline{B}_{11}\frac{d^3u_0}{dx^3} - \overline{D}_{11}\frac{d^4w}{dx^4} = -q$$

$$-\overline{B}_{11}\frac{d^3u_0}{dx^3} + \overline{D}_{11}\frac{d^4w}{dx^4} = q \qquad (2.10)$$

$\overline{B}_{11}, \overline{D}_{11}$ are replaced by B_{11}, D_{11},

$$-B_{11}\frac{d^3u_0}{dx^3} + D_{11}\frac{d^4w}{dx^4} = q$$

$$\left(A_{ij}, B_{ij}, D_{ij}\right) = \int_{-h/2}^{h/2} \overline{Q}_{ij}\left(1, z, z^2\right)dz$$

Example : A simply supported beam with uniformly distributed load, $q(x) = q_0$

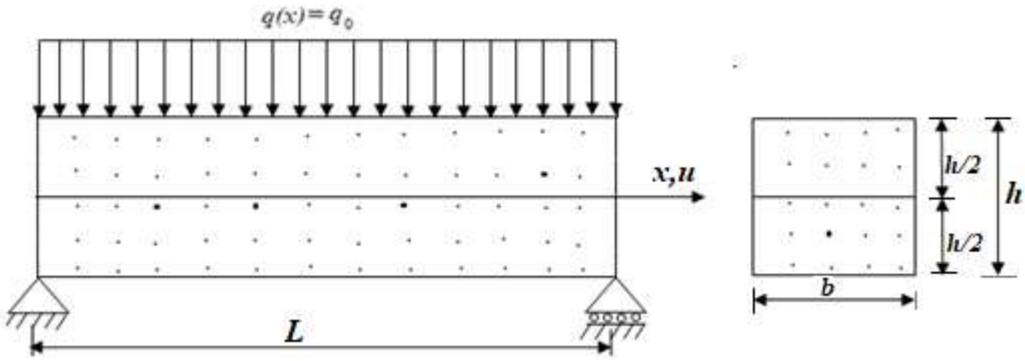

Fig.1. Simply supported beam with uniformly distributed load of single layer beam (90^0).

Non dimensional transverse displacement $\overline{w}$,

For 0^0

$$\overline{w} = \left[\frac{200}{\pi^3}\sin\frac{\pi x}{L}\right],$$

For 90^0

$$\overline{w} = \left[\frac{8.0171}{\pi^3}\sin\frac{\pi x}{L}\right],$$

For $0^0/90^0/0^0$

$$\overline{w} = \left[\frac{409.23}{\pi^3}\sin\frac{\pi x}{L}\right],$$

For $90^0/0^0/90^0$

$$\overline{w} = \left[\frac{216.54}{\pi^3}\sin\frac{\pi x}{L}\right],$$

For $0^0/90^0/90^0/0^0$

$$\overline{w} = \left[\frac{417.043}{\pi^3}\sin\frac{\pi x}{L}\right],$$

For $90^0/0^0/0^0/90^0$

$$\overline{w} = \left[\frac{417.043}{\pi^3}\sin\frac{\pi x}{L}\right].$$

Non dimensional axial displacement $\overline{u}$,

Tuijin Jishu/Journal of Propulsion Technology
ISSN: 1001-4055
Vol.44 No. 4 (2023)

For 0^0 For 90^0 For $0^0/90^0/0^0$

$$\bar{u}=\left[\frac{200}{\pi^2}\frac{z}{h}\cos\frac{\pi x}{L}\right],\quad \bar{u}=\left[\frac{8.0171}{\pi^2}\frac{z}{h}\cos\frac{\pi x}{L}\right],\quad \bar{u}=\left[\frac{409.23}{\pi^2}\frac{z}{h}\cos\frac{\pi x}{L}\right],$$

For $90^0/0^0/90^0$ For $0^0/90^0/90^0/0^0$ For $90^0/0^0/0^0/90^0$

$$\bar{u}=\left[\frac{216.54}{\pi^2}\frac{z}{h}\cos\frac{\pi x}{L}\right],\ \bar{u}=\left[\frac{417.043}{\pi^2}\frac{z}{h}\cos\frac{\pi x}{L}\right],\ \bar{u}=\left[\frac{417.043}{\pi^2}\frac{z}{h}\cos\frac{\pi x}{L}\right].$$

Non dimensional axial stresses $\bar{\sigma}_x$,

For 0^0 For 90^0 For $0^0/90^0/0^0$

$$\bar{\sigma}_x=\left[\frac{200}{\pi E_2}\frac{z}{h}\sin\frac{\pi x}{L}\right],\ \bar{\sigma}_x=\left[\frac{8.0171}{\pi E_2}\frac{z}{h}\sin\frac{\pi x}{L}\right],\ \bar{\sigma}_x=\left[\frac{409.23}{\pi E_2}\frac{z}{h}\sin\frac{\pi x}{L}\right],$$

For $90^0/0^0/90^0$ For $0^0/90^0/90^0/0^0$ For $90^0/0^0/0^0/90^0$

$$\bar{\sigma}_x=\left[\frac{216.54}{\pi E_2}\frac{z}{h}\sin\frac{\pi x}{L}\right],\ \bar{\sigma}_x=\left[\frac{417.043}{\pi E_2}\frac{z}{h}\sin\frac{\pi x}{L}\right],\ \bar{\sigma}_x=\left[\frac{417.043}{\pi E_2}\frac{z}{h}\sin\frac{\pi x}{L}\right].$$

Non dimensional transverse shear stresses $\tau_{zx}^{\overline{EE}}$ using equilibrium equation,

For 0^0 For 90^0

$$\tau_{zx}^{\overline{EE}}=\left[\frac{200}{E_2}\frac{h}{L}\frac{1}{8}\cos\frac{\pi x}{L}\right]\left[1-\frac{4z^2}{h^2}\right],\ \tau_{zx}^{\overline{EE}}=\left[\frac{8.0171}{E_2}\frac{h}{L}\frac{1}{8}\cos\frac{\pi x}{L}\right]\left[1-\frac{4z^2}{h^2}\right],$$

For $0^0/90^0/0^0$ For $90^0/0^0/90^0$

$$\tau_{zx}^{\overline{EE}}=\left[\frac{409.23}{E_2}\frac{h}{L}\frac{1}{8}\cos\frac{\pi x}{L}\right]\left[1-\frac{4z^2}{h^2}\right],\ \tau_{zx}^{\overline{EE}}=\left[\frac{216.54}{E_2}\frac{h}{L}\frac{1}{8}\cos\frac{\pi x}{L}\right]\left[1-\frac{4z^2}{h^2}\right],$$

For $0^0/90^0/90^0/0^0$ For $0^0/90^0/90^0/0^0$

$$\tau_{zx}^{\overline{EE}}=\left[\frac{417.043}{E_2}\frac{h}{L}\frac{1}{8}\cos\frac{\pi x}{L}\right]\left[1-\frac{4z^2}{h^2}\right],\ \tau_{zx}^{\overline{EE}}=\left[\frac{417.043}{E_2}\frac{h}{L}\frac{1}{8}\cos\frac{\pi x}{L}\right]\left[1-\frac{4z^2}{h^2}\right].$$

Table 1.1: Non-Dimensional Deflection $\bar{w}$, $\bar{u}$ and Stress $\bar{\sigma}_x$, $\tau_{zx}^{\overline{EE}}$ for Single Layer, Three Layers and Four Layers of Laminated Simply Supported Beam treat with UDL $[q_0]$ for Aspect Ratio 4.

AS **(Aspect Ratio)**	Ply Angle	ETB			
		$\bar{w}$	$\bar{u}$	$\bar{\sigma}_x$	$\tau_{zx}^{\overline{EE}}$
	0^0	0.258	10.132	31.830	6.250
	90^0	0.258	0.406	1.275	0.250
	$0^0/90^0/0^0$	0.527	20.731	65.130	0.511

4	$90^0/0^0/90^0$	3.010	10.970	34.463	0.270
	$0^0/90^0/90^0/0^0$	0.611	21.127	66.374	0.521
	$90^0/0^0/0^0/90^0$	3.362	21.127	66.374	0.521

Table 1.1: Non-Dimensional Deflection $\overline{w}$, $\overline{u}$ and Stress $\overline{\sigma}_x$, $\overline{\tau_{zx}^{EE}}$ for Single Layer, Three Layers and Four Layers of Laminated Simply Supported Beam treat with UDL $[q_0]$ for Aspect Ratio 10.

AS (**Aspect Ratio)**	Ply Angle	ETB			
		$\overline{w}$	$\overline{u}$	$\overline{\sigma}_x$	$\overline{\tau_{zx}^{EE}}$
10	0^0	0.258	10.132	31.83	2.500
	90^0	0.258	0.406	1.275	0.100
	$0^0/90^0/0^0$	0.527	20.731	65.130	0.204
	$90^0/0^0/90^0$	3.010	10.970	34.643	0.108
	$0^0/90^0/90^0/0^0$	0.611	21.127	66.374	0.002
	$90^0/0^0/0^0/90^0$	3.362	21.127	66.374	0.013

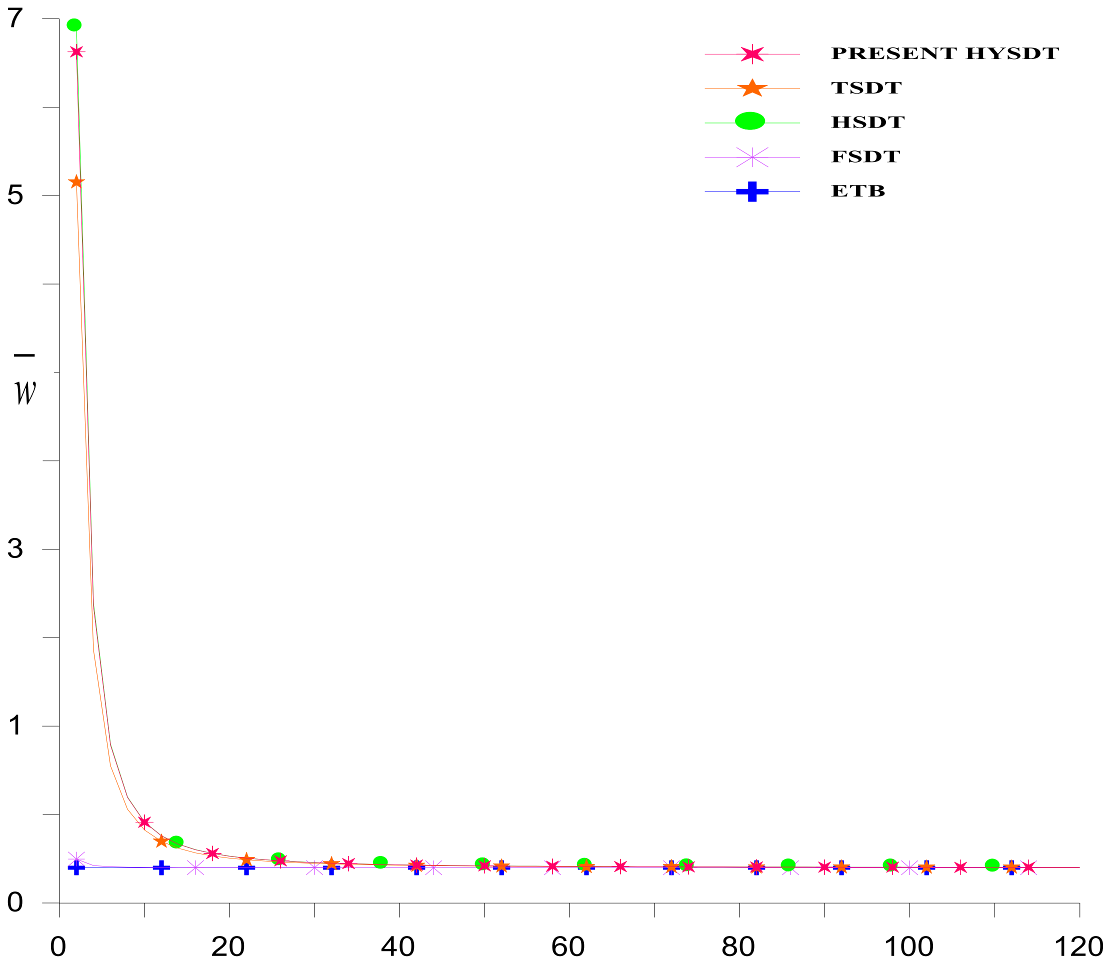

Figure 2: Variation of maximum transverse displacement for ply angle (0^0)

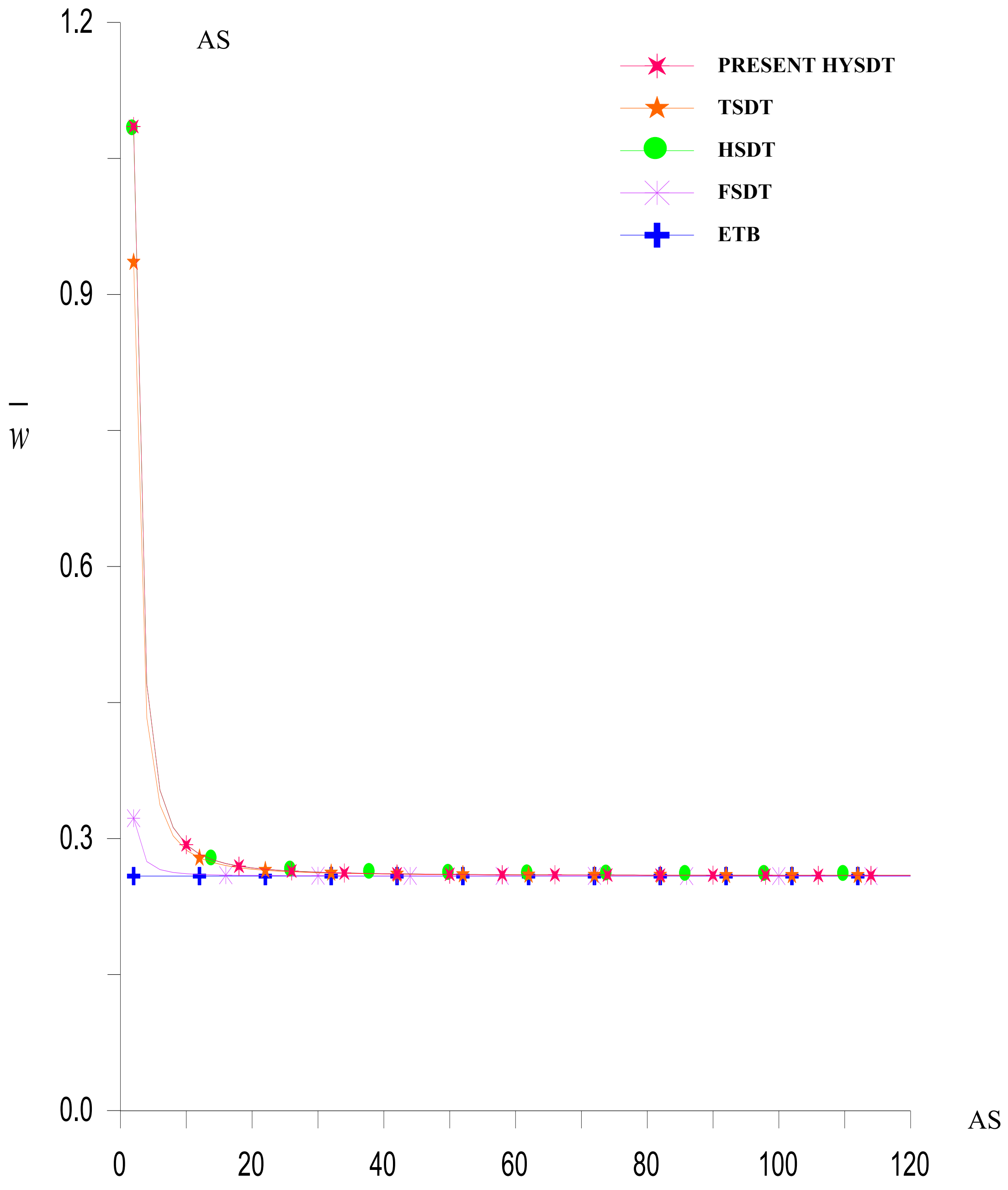

Figure 3: Variation of maximum transverse displacement for ply angle (90^0)

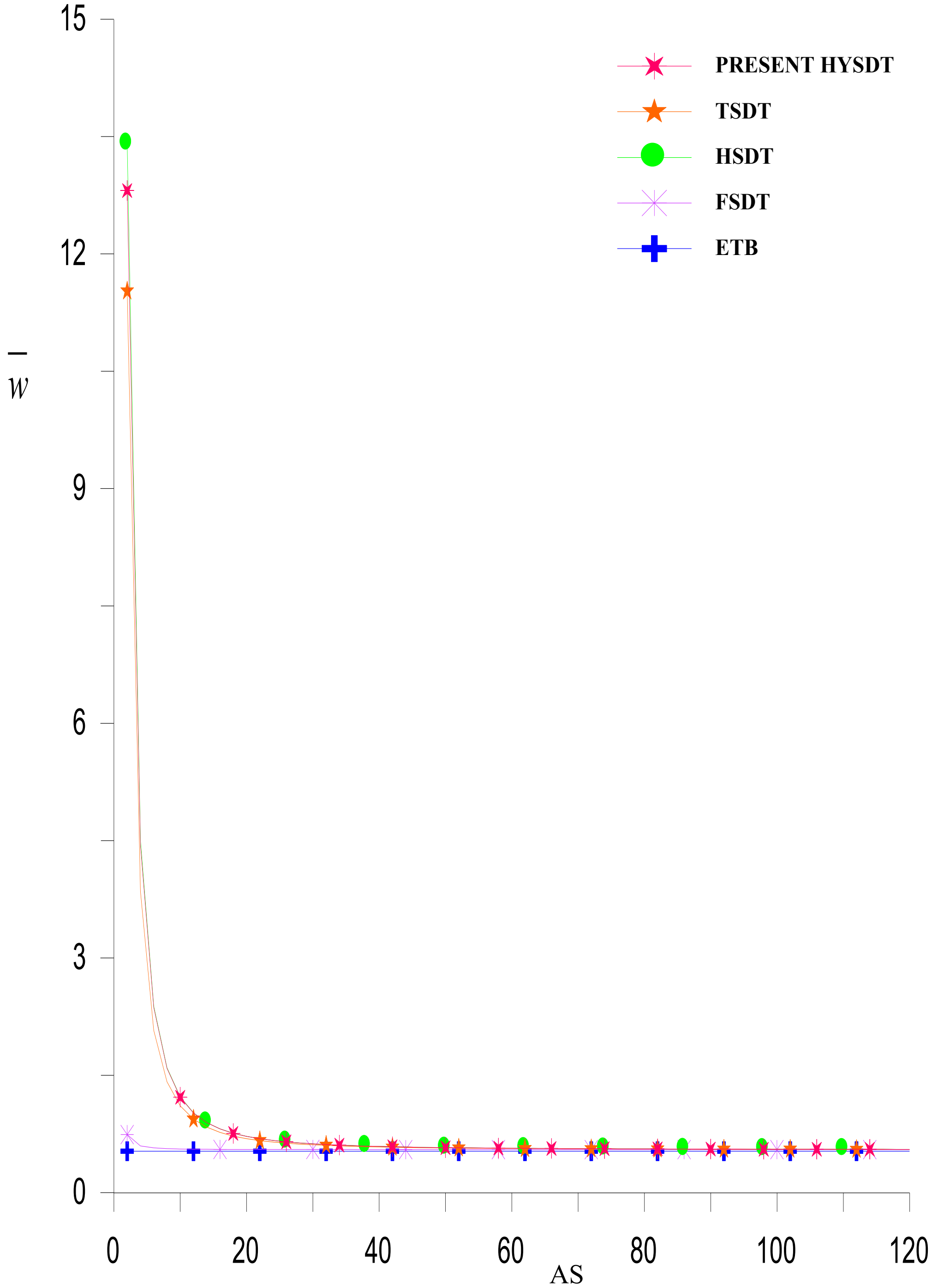

Figure 4: Variation of maximum transverse displacement for ply angle ($0^0/90^0/0^0$)

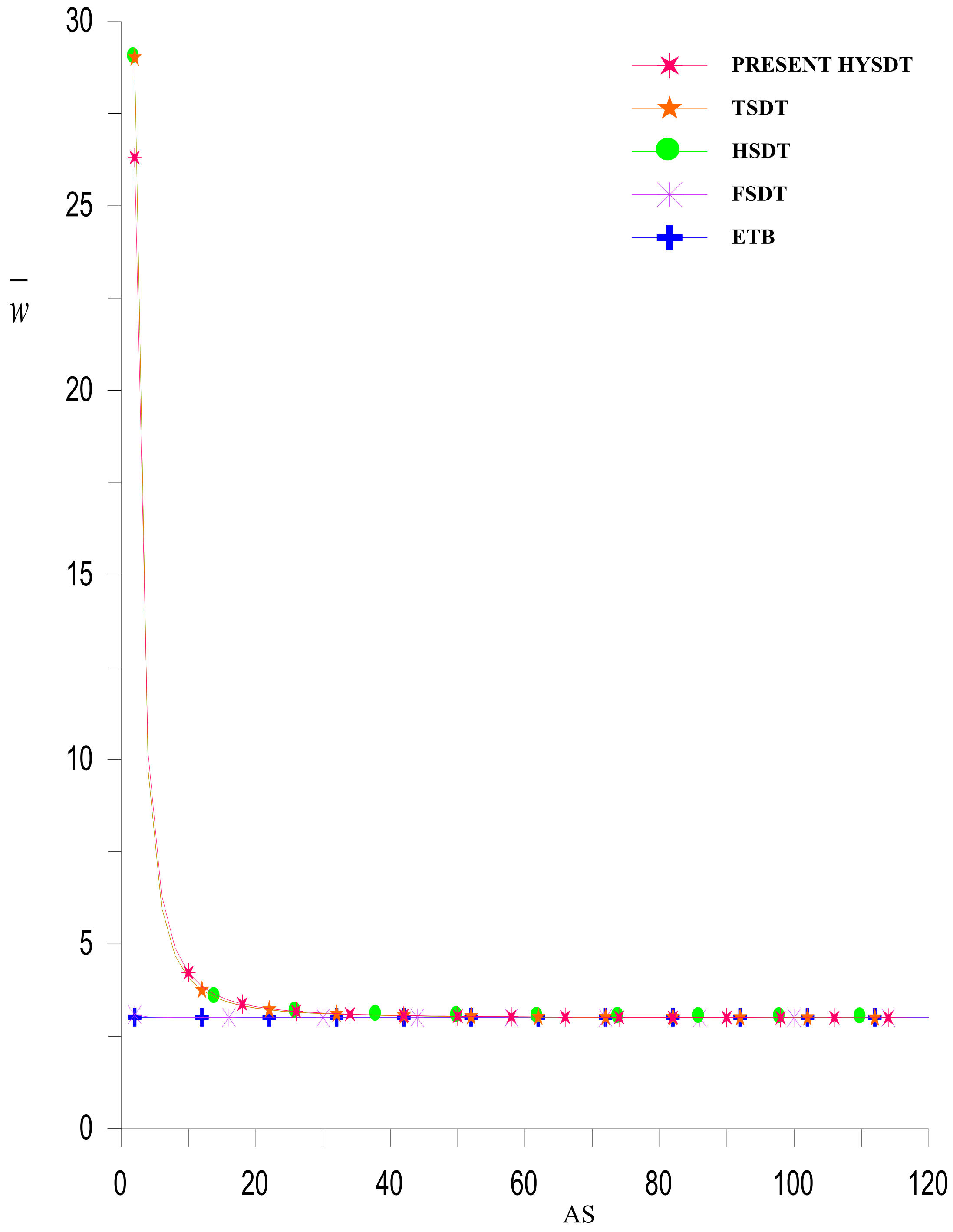

Figure 5: Variation of maximum transverse displacement for ply angle ($90^0/0^0/90^0$)

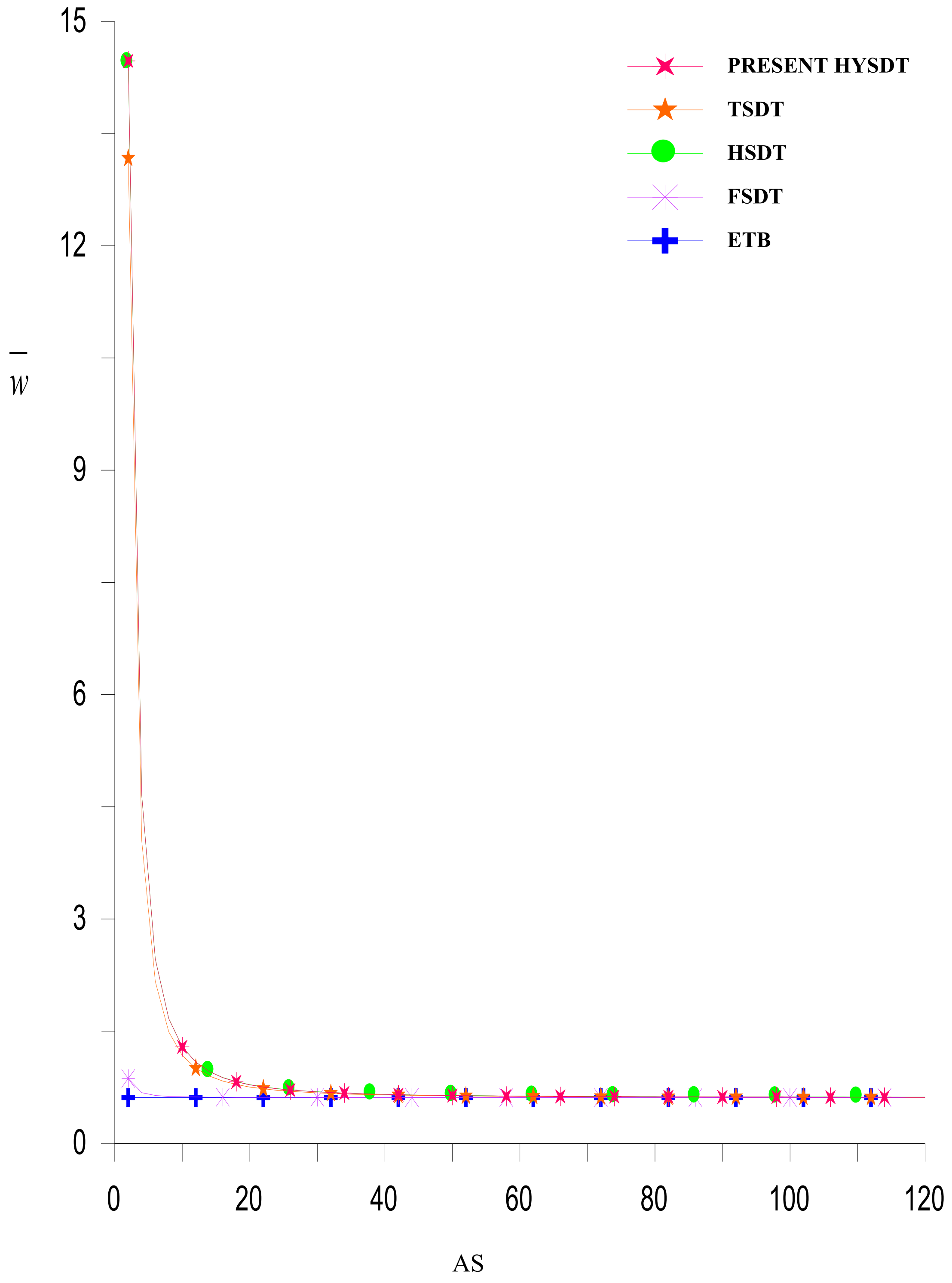

Figure 6: Variation of maximum transverse displacement for ply angle ($0^0/90^0/90^0/0^0$)

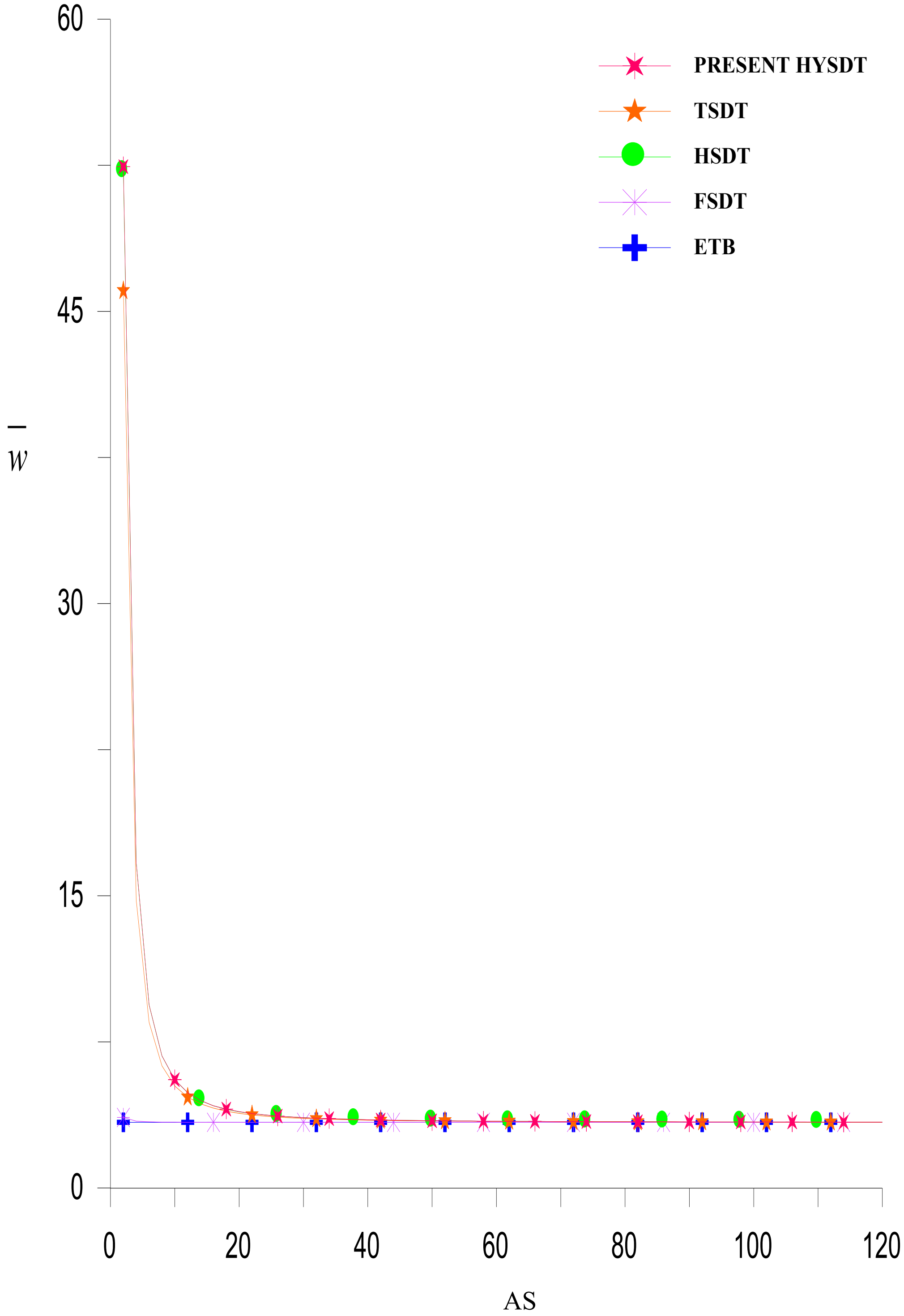

Figure 7: Variation of maximum transverse displacement for ply angle ($90^0/0^0/0^0/90^0$)

3. Conclusion

1. Comparable to other shear deformation theories, the current theory provides realistic findings for this displacement component.

2. The transverse displacement predicted by the current theory and those predicted by other higher order shear deformation theories agree quite well.

3. According to the current theory, the axial stress and how it is distributed across the thickness.

References

[1] Ali J.S.M., Bhaskar K., Varadan T.K. (1999). A new theory for accurate thermal/mechanical flexural analysis of symmetric laminated plates. Composite Structures 45, No.3, 227-232.

[2] Ali Reza Daneshmehr, Mostafa Mohammad Abadi1 and Amir Rajabpoor (2013).Thermal effect on static bending, vibration and buckling of Reddy beam based on modified couple stress theory. Applied Mechanics and Materials Vol. 332, 331-338.

[3] Anderse, Lars., Nielsen, R.K. (2000). Elastic beams in three dimensions. Scientific Publications.

[4] Arcangelo Messina and Kostas P. Soldatos (1999). Influence of edge boundary conditions on the free vibrations of cross-ply laminated circular cylindrical panels. Journal of Acoustic Society of America106 (5), 2608-2620.

[5] Arya H., Shimpi R. P., Naik N. K. (2002). A zig-zag model for laminated composite beams. Composite Structures. Vol. 56, 21–24.