

The Intuitionistic Fuzzy Critical Path and Characteristics of a Project Network using an N-Array Tree Representation

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Abstract:In project management, a critical path is the sequence of project network activities that add up to the largest overall duration, regardless of whether that longest duration has float or ergot. This determines the shortest time to complete the project. Total float (unused time) can exist within the critical path. The Critical Path Method(CPM) is one of the most frequently used and effective techniques in project planning. Fuzzy set theory has been proposed as an alternative methodology for measuring uncertainty related to project activity duration. In this paper, we identify the intuitionistic fuzzy critical path and project characteristics using an n-array tree representation. The total float of each activity is determined, and the earliest and latest times of each activity are found using total float with Intuitionistic Fuzzy Sets (IFS).This method is effective in finding project characteristics obtained using IFS and the intuitionistic fuzzy critical path when compared to existing methods.

Keywords: Project network; Critical Path; Intuitionistic Fuzzy Numbers, Fuzzy times

1. Introduction

The Critical Path Method (CPM) techniques have become widely recognised as valuable tools for planning and scheduling large projects. The primary goal of government agencies and industrial organizations is to plan their project to maximize resource utility and minimize overall costs. This type of management problem can be effectively addressed using the network technique called the critical path method. In reality, obtaining estimates of activity time is often difficult due to the uncertainty of information and variations in the management scenario. Therefore, conventional approaches tend to be less effective. Fuzzy set theory, proposed by Zadeh [1] can play a significant role in decision-making environments involving vagueness.

The fuzzy critical path is one of the useful methods in project planning and development. Network exploration is a method that determines several sequences of activities for a project and the project completion time. The main purpose of the critical path method is to identify critical activities on the critical paths. Liang and Han [2] used trapezoidal fuzzy numbers to characterize the fuzzy measures of activity times with linguistic values and proposed an algorithm for finding the fuzzy critical path of a project network. Rezvani[3] proposed the ranking of intuitionistic trapezoidal fuzzy numbers. Jayagowri and Geetharamani [4] proposed a metric distance ranking method to find the intuitionistic fuzzy critical path. In this method, they used it to find the intuitionistic fuzzy critical path without converting the intuitionistic fuzzy activity times to crisp numbers. Nagoor Gani et al. [5] analyzed the intuitionistic fuzzy critical path in a network diagram while searching for the intuitionistic fuzzy shortest path. Uhtra et al.[6] proposed the ranking of generalized intuitionistic fuzzy numbers. Jayagowri and Geetharamani [7] analysed the intuitionistic fuzzy critical path using intuitionistic triangular fuzzy numbers in a fuzzy network diagram. Sudha and Sophia Porchelvi [8] proposed an algorithm to perform intuitionistic fuzzy critical path analysis, the length of which is the triangular intuitionistic fuzzy number. Researchers [10-18] proposed fuzzy methods for performing intuitionistic fuzzy critical path analysis and its applications. In this paper, a new algorithm is proposed to find the intuitionistic fuzzy critical path and project characteristics of a project network using an n-array tree representation.

2. Preliminaries

In this section, we review some basic definitions related to Intuitionistic Fuzzy Sets and Intuitionistic Numbers [9].

2.1. Basic Definitions

This section covers various basic definitions as outlined in reference [9].

Definition 2.1 Fuzzy set : Let a non-empty fuzzy set A in x be characterized by a membership function $\mu_A(x)$ for all $x \in E$, where E is a non-empty set. The membership function associates with each point in E a real number in the interval $[0,1]$, and the value of $\mu_A(x)$ at x represents the ‘grade of membership’ of x in A .

Definition 2.2 Triangular fuzzy number : A fuzzy number is represented with three points as follows $A = (a_1, a_2, a_3)$. This representation is interpreted as membership functions and holds the following conditions :

- (i) a_1 to a_2 is increasing function.
- (ii) a_2 to a_3 is decreasing function.
- (iii) $a_1 \leq a_2 \leq a_3$

The membership function μ_A is defined as follows :

$$\mu_A(x) = \begin{cases} 0 & \text{for } x < a_1, \\ \frac{x - a_1}{a_2 - a_1}, & \text{for } a_1 \leq x \leq a_2, \\ \frac{a_3 - x}{a_3 - a_2}, & \text{for } a_2 \leq x \leq a_3 \\ 0, & \text{for } x > a_3. \end{cases}$$

Definition 2.3 Intuitionistic fuzzy set : Let X be the universe of discourse. Then, an Intuitionistic Fuzzy Set (IFS) A in X is given by $A = \{ x, \mu_A(x), \nu_A(x) / x \in X \}$, where the functions $\mu_A : X \rightarrow [0,1]$ and $\nu_A : X \rightarrow [0,1]$ determine the degree of membership and non-membership of the element $x \in X$, respectively. For every $x \in X$, it holds that

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

Definition 2.4 Trapezoidal intuitionistic fuzzy number :

An Intuitionistic fuzzy set $A = (a_1, a_2, a_3, a_4); (b_1, b_2, b_3, b_4)$ is considered a trapezoidal intuitionistic fuzzy number if its membership function (μ_A) and non-membership function (ν_A) are given by :

$$\mu_A(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1}, & \text{for } a_1 \leq x \leq a_2, \\ 1, & \text{for } a_2 \leq x \leq a_3, \\ \frac{a_4 - x}{a_4 - a_3}, & \text{for } a_3 \leq x \leq a_4. \end{cases}$$

$$\nu_A(x) = \begin{cases} \frac{x - b_1}{b_2 - b_1}, & \text{for } b_1 \leq x \leq b_2, \\ 1, & \text{for } b_2 \leq x \leq b_3, \\ \frac{x - b_4}{b_3 - b_4}, & \text{for } b_3 \leq x \leq b_4. \end{cases}$$

Definition 2.5 Arithmetic operations of trapezoidal intuitionistic fuzzy number:

If $\tilde{A} = (a_1, a_2, a_3, a_4); (a'_1, a'_2, a'_3, a'_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4); (b'_1, b'_2, b'_3, b'_4)$

are two intuitionistic trapezoidal fuzzy numbers then :

- (i) Addition ($\oplus$)

$$\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4); (a'_1 + b'_1, a'_2 + b'_2, a'_3 + b'_3, a'_4 + b'_4)$$

(ii) Subtraction ($\ominus$)

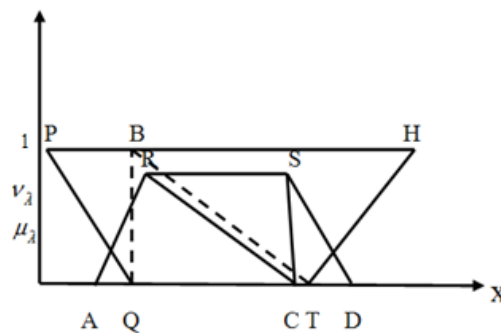
$$\tilde{A} \ominus \tilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1); (a'_1 - b'_4, a'_2 - b'_3, a'_3 - b'_2, a'_4 - b'_1)$$

2.3 The Ranking Functions for Trapezoidal Intuitionistic Fuzzy Numbers

The ranking function for trapezoidal intuitionistic fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4); (a'_1, a'_2, a'_3, a'_4)$ is illustrated in Fig.1. It utilizes both membership and non-membership functions :

1. For the membership function : $R(\tilde{A}) = \left(\frac{a_1 + 2a_2 + 2a_3 + a_4}{186} \right)$
2. For the non-membership function : $R(\tilde{A}) = \left(\frac{a'_2 + a'_3}{2} \right) + \left(\frac{a'_4 - a'_3 - a'_2 + a'_1}{6} \right)$

These functions are designed to incorporate both the membership and non-membership aspects of the trapezoidal intuitionistic fuzzy number.



• Fig. 1: Trapezoidal Intuitionistic Fuzzy Number

Let $\tilde{A} = (a_1, a_2, a_3, a_4); (a'_1, a'_2, a'_3, a'_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4); (b'_1, b'_2, b'_3, b'_4)$ be two intuitionistic fuzzy numbers.

If $R(\tilde{A}) > R(\tilde{B})$ then $(\tilde{A}) > (\tilde{B})$

If $R(\tilde{A}) < R(\tilde{B})$ then $(\tilde{A}) < (\tilde{B})$

If $R(\tilde{A}) = R(\tilde{B})$ then $(\tilde{A}) = (\tilde{B})$

3. Intuitionistic Fuzzy Critical Path and Characteristics of a Project Network with N-Array Tree

Representation

The intuitionistic fuzzy project network was described using n-array tree representation. The method of constructing the n-array tree representation is explained by considering the intuitionistic fuzzy project network shown in Fig. 2 and the n-array tree representation shown in Fig. 3. From the n-array tree representation, it is clear that there are four paths, namely 1-4-5-6, 1-3-5-6, 1-4-6, 1-2-6.

3.1 Computation of Intuitionistic Fuzzy Total Float of Each Activity

The intuitionistic fuzzy earliest and latest times for an activity (i, j) and its total float times can be expressed as functions of the lengths of some intuitionistic fuzzy paths. The intuitionistic fuzzy earliest starting time for (i, j) is the length of the longest intuitionistic fuzzy path from 1 to i, and its latest starting time is the difference between the length of the longest path from 1 to n and the length of the longest path from i to n. The intuitionistic total float of each activity is obtained by taking the difference between latest starting time and earliest start time of each activity.

3.2. Computation of Intuitionistic Fuzzy Earliest Times

Intuitionistic trapezoidal fuzzy earliest time for the starting node is considered as $(0,0,0,0;0,0,0,0)$. The intuitionistic fuzzy earliest time of node j in the activity (i,j) for the membership function and non-membership function is obtained by taking the maximum and minimum sums of the intuitionistic fuzzy earliest time and intuitionistic fuzzy time of activity (i,j) , respectively. The intuitionistic fuzzy earliest times of each event is calculated using the node weights.

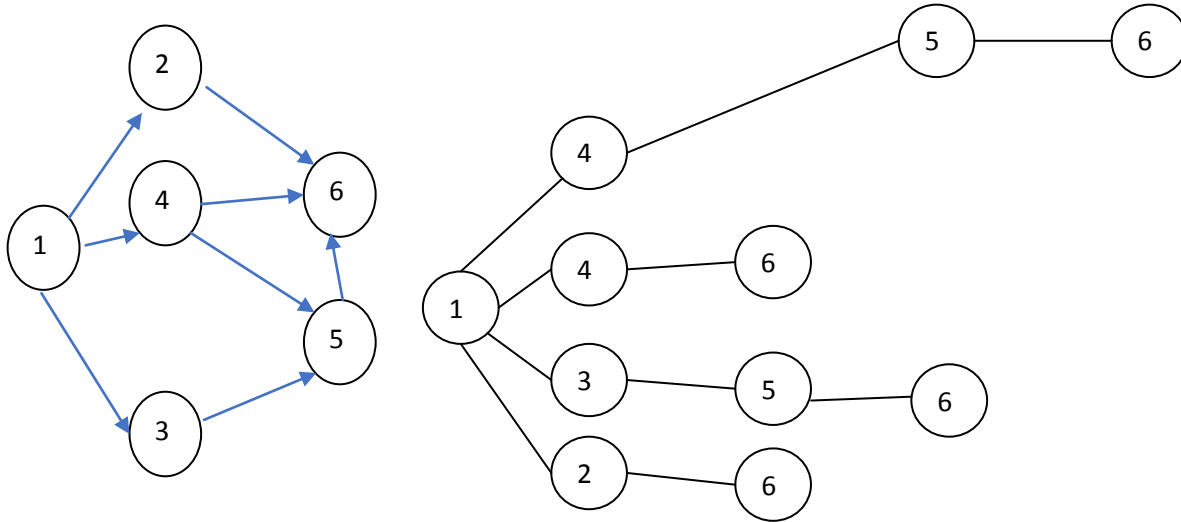


Fig.2 : Intuitionistic Trapezoidal Fuzzy project network

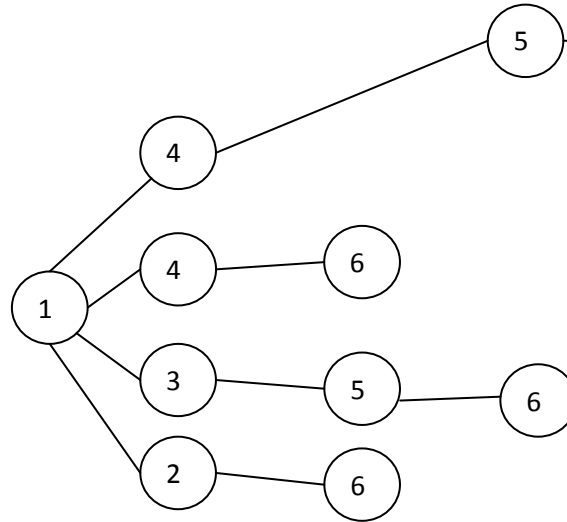


Fig. 3 : N-array tree representation of Intuitionistic Trapezoidal fuzzy project network

3.3 Computation of Intuitionistic Fuzzy Latest Times

In the case of intuitionistic trapezoidal fuzzy numbers, new operation $*$ is introduced to calculate the intuitionistic fuzzy latest times. Let $\tilde{A} = (a_1, a_2, a_3, a_4); (a'_1, a'_2, a'_3, a'_4)$ and

$\tilde{B} = (b_1, b_2, b_3, b_4); (b'_1, b'_2, b'_3, b'_4)$ be two intuitionistic trapezoidal fuzzy numbers.

The new operation ' $*$ ' is defined as $\tilde{A} * \tilde{B} = (a_1 + b_4, a_2 + b_3, a_3 + b_2, a_4 + b_1)$ for membership and $\tilde{A} * \tilde{B} = (a'_1 - b'_4, a'_2 - b'_3, a'_3 - b'_2, a'_4 - b'_1)$ for non-membership.

In this method, the fuzzy intuitionistic latest start time for membership is defined as the total float time multiplied by the earliest start time of each activity (i,j) . For non-membership, the fuzzy intuitionistic latest start time is the difference between the fuzzy intuitionistic total float time and the fuzzy intuitionistic earliest start time of each activity (i,j) . The fuzzy intuitionistic latest finish time for membership is defined as the latest start time multiplied by the intuitionistic fuzzy time of each activity (i,j) . For non-membership, the fuzzy intuitionistic latest finish time is the difference between the fuzzy intuitionistic latest start time and the intuitionistic fuzzy time of each activity (i,j) . In this context, the term 'multiplied' refers to the $*$ operation.

3.4 Defuzzification of Total Float and Criticality Degree of Each Activity

Let $\tilde{A} = (a_1, a_2, a_3, a_4); (a'_1, a'_2, a'_3, a'_4)$ be an intuitionistic trapezoidal fuzzy number representing total float of an activity.

The defuzzification of $\tilde{A}$ is $\frac{(a_1+2a_2+2a_3+a_4)}{6}; \frac{a'_1+2a'_2+2a'_3+a'_4}{6}$.

Criticality degree of $\tilde{A}$ is $\frac{(-a_1)}{(a_2+a_3+a_4)}; \frac{(-a'_1)}{(a'_2+a'_3+a'_4)}$.

3.5 Defuzzified Value and Criticality Degree of Each Path in an Intuitionistic Fuzzy

Project Network

Let P be the set of all possible paths in the intuitionistic fuzzy project network. The defuzzified value of each path is calculated as the sum of all total floats of each activity present in the path. The intuitionistic fuzzy critical path is defined to be the path in P with the smallest defuzzified value. The criticality degree of each path is defined as the minimum of all criticality degrees of activities present in the path belongs to P.

3.6 Procedure for Finding Intuitionistic Fuzzy Critical Path and Project Characteristics of Fuzzy Project Network Using N-Array Tree Representation

Step 1 : Construct a network G(V,E) where V is the set of vertices, and E is the set of arcs.

Here, G is an acyclic digraph, and the length of each arc is represented as intuitionistic trapezoidal fuzzy numbers.

Step 2 : Find the intuitionistic fuzzy critical path obtained by comparing path lengths and obtain the intuitionistic path float of each path.

Step 3 : Calculate the total float of each activity for membership and non-membership functions.

Step 4: Calculate the fuzzy earliest occurrence time of each node i.

Step 5: Calculate the fuzzy earliest start time of each activity for membership and non-membership functions.

Step 6 : Calculate the Latest start time and latest finishing time for membership and non-membership functions.

Step 7 : Calculate the defuzzified value and critical degree of the total float for each activity for membership and non-membership functions.

Step 8 : Calculate the defuzzified value and criticality degree of each path with membership and non-membership functions.

The path with criticality degree 1 is a critical path.

4. Illustration of the Proposed Method for an Intuitionistic Trapezoidal Fuzzy Project Network with Membership Functions and Non-Membership Functions

To illustrate the proposed method, we consider a fuzzy project network in Fig. 2 with activity times shown in Table 1 .

Table 1 : Activity Times of an IFS Network with Membership and Non-membership Functions

Activity	Activity time (Trapezoidal Intuitionistic fuzzy number)
1-2	(12,14,16,18) ; (10,15,16,18)
1-3	(4,10,14,18) ; (6, 10, 12,14)
1-4	(8,16,20,24); (9,13,15,17)
2-6	(5,10,15,20); (12,14,15,17)
3-5	(12,13,15,18); (16,18,20,22)
4-5	(6,10,16,24); (1,2,5,7)
4-6	(13,15,17,20); (11,15,18,19)
5-6	(5,7,9,15) ; (6,10,12,14)

Step 1 : Project network with Intuitionistic trapezoidal fuzzy numbers are represented in Fig.4 and Table 1.

Step 2 : Fuzzy paths:1-4-5-6,1-3-5-6,1-4-6,1-2-6

Fuzzy path lengths: (19,33,45,63); (16,25,32,38) ,(21,30,38,51);(28,38,44,50),
(21,31,37,44); (20,28,33,36), (17,24,31,38); (22,29,31,35)

Hence, the fuzzy critical path is 1-4-5-6.

Fuzzy project duration is (19,33,45,63);(16,25,32,38)

Path float of 1-4-5-6 : (-44,-12,12,44); (-22,-7,7,22)

Path float of 1-3-5-6 : (-32,-5,15,42);(-34,-19,-6,-10)

Path float of 1-4-6 : (-25,-4,14,42);(-20,-8,4,18)

Path float of 1-2-6 : (-19,2,21,46); (-19,-6,3,16).

Step 3-7 : We calculated the intuitionistic trapezoidal fuzzy times and criticality degree for each activity in a project network in step 1 and presented the results in Table 2.

Step 8 : Wecalculated the defuzzified value and criticality degree of each path in intuitionistic fuzzy project network.

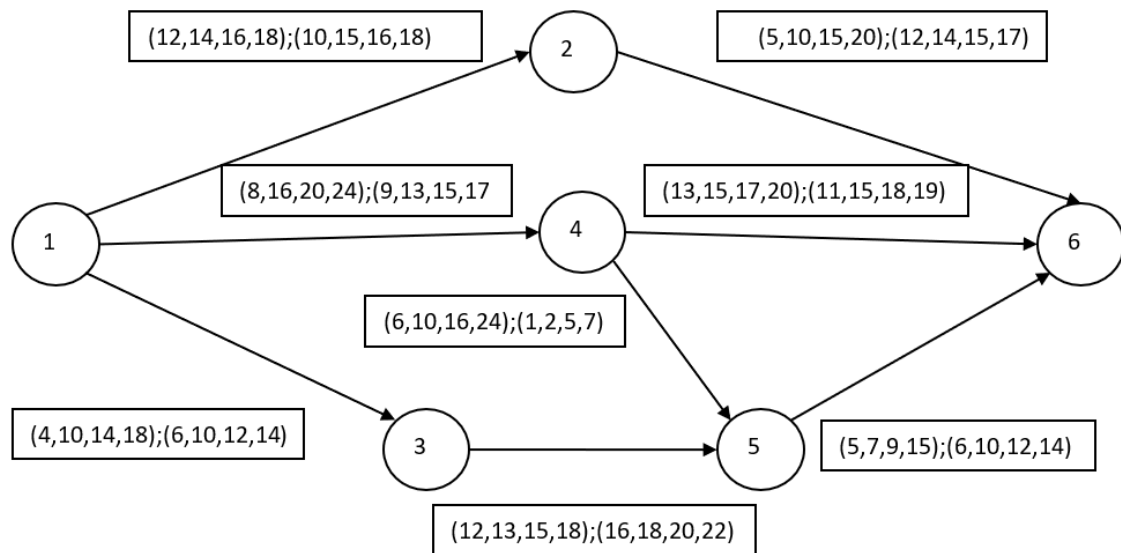


Fig 4. Project network with activity times

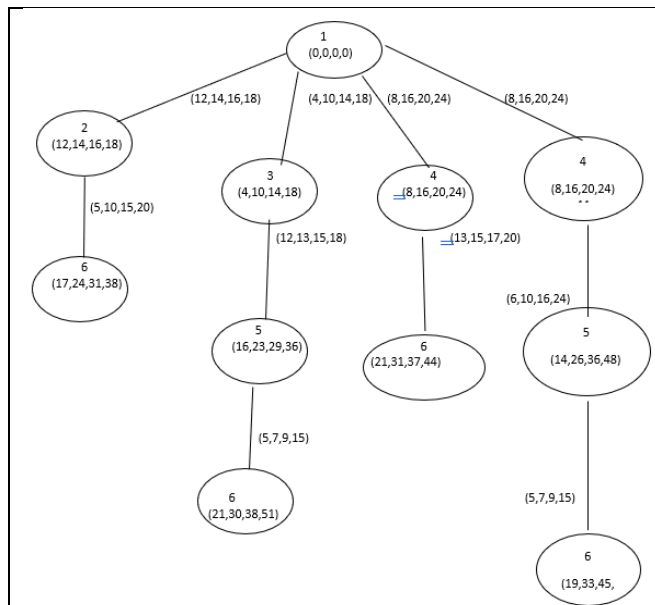


Fig. 5. N-array tree representation for the project network (membership in IFS)

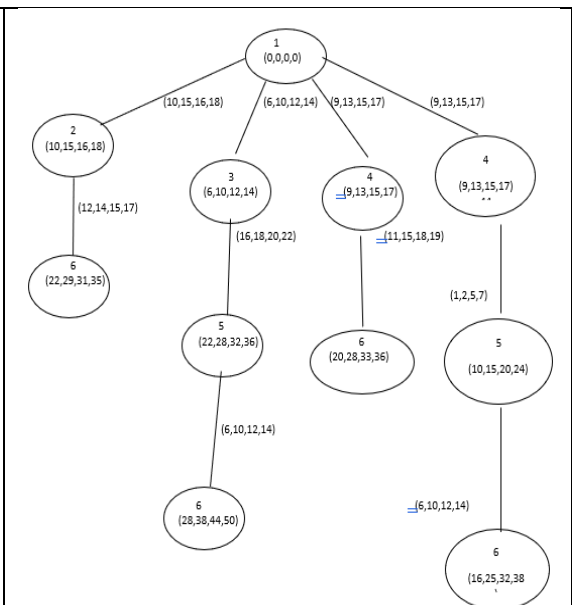


Fig. 6. N-array tree representation for the project network (non-membership in IFS)

Table 2 : Intuitionistic Trapezoidal Fuzzy Times of a Project Network

Activity	Total Float Time	Earliest Start Time	Earliest Finish Time	Latest Start Time	Latest Finish Time	Defuzzified Value of Total Float	Criticality Degree
1-2	(-19,2,21,46); (-19,-6,3,16)	(0,0,0,0); (0,0,0,0)	(12,14,16,18); (10,15,16,18)	(-19,2,21,46); (-19,-6,3,16)	(-1,18,35,58); (-37,-22,-12,6)	12.1; -1.5	0.27; 1.46
1-3	(-32,-5,15,42); (-34,-19,-6,10)	(0,0,0,0); (0,0,0,0)	(4,10,14,18); (6,10,12,14)	(-32,-5,15,42); (-34,-19,-6,10)	(-14,9,25,46); (-48,-31,-16,4)	5; -12.3	0.61; -2.26
1-4	(-44,-12,12,44); (-22,-7,7,22)	(0,0,0,0); (0,0,0,0)	(8,16,20,24); (9,13,15,17)	(-44,-12,12,44); (-22,-7,7,22)	(-20,8,28,52); (-39,-22,-6,13)	0; 0	1; 1
2-6	(-19,2,21,46); (-19,-6,3,16)	(12,14,16,18); (10,15,16,18)	(17,24,31,38); (16,25,32,38)	(-1,18,35,58); (-37,-22,-12,6)	(19,33,45,63); (-54,-37,-26,-6)	12.1; 1.5	0.27; 1.46
3-5	(-32,-5,15,42); (-34,-19,-6,10)	(4,10,14,18); (6,10,12,14)	(16,23,29,36); (10,15,20,24)	(-14,9,25,46); (-48,-29,-16,4)	(4,24,38,58); (-70,-49,-34,-12)	5; -12.3	0.61, -2.26
4-5	(-44,-12,12,44); (-22,-7,7,22)	(8,16,20,24); (9,13,15,17)	(14,26,36,48); (10,15,20,24)	(-20,8,28,52); (-39,-22,-14,13)	(4,24,38,58); (-46,-27,-16,12)	0; 0	1; 1
4-6	(-25,-4,14,42); (-20,-8,4,18)	(8,16,20,24); (10,15,20,24)	(21,31,37,44); (16,25,32,38)	(-1,16,30,50); (-37,-23,-9,9)	(19,33,45,63); (-56,-41,-24,-2)	6.1; 1.6	0.48, 1.42

5-6	(-44,- 12,12,44); (-22,- 7,7,22)	(14,26,36,48); (16,25,32,38)	(19,33,45,63); (16,25,32,38)	(4,24,38,58); (-46,-27,- 8,12)	(19,33,45,63); (-60,-39,- 18,6)	0; 0	1; 1
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Table 3 : Defuzzified Value and Criticality Degree of each Path in Intuitionistic Fuzzy Project Network

Path	Defuzzified Value	Criticality Degree
1-4-5-6	0	1
1-3-5-6	-12.3	-2.26
1-4-6	-1.6	1
1-2-6	-1.5	1.46

5.Conclusion

The present paper proposes an n-array tree representation to identify the Intuitionistic Fuzzy Critical Path and project characteristics in a fuzzy project network. Here, a new method using Intuitionistic Trapezoidal Fuzzy numbers has been introduced for finding the critical path in a fuzzy project network. We have calculated fuzzy total float for each path in the fuzzy project network to determine the critical path using an n-array tree representation method. In this method, fuzzy total float and path float have been calculated without the need to find the fuzzy earliest and latest times, providing a means to identify the Intuitionistic Fuzzy Critical Path in an Intuitionistic Fuzzy Project Network.

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