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Mixed Convection Past A Vertical Heated Plate With Nano Fluids

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Abstract: An attempt has been made to analyse the heat transfer process past a vertical heated plate embedded in a Newtonian fluid-saturated sparsely packed porous medium in the presence of nanofluids. The boundary layer flow in the porous medium is governed by Brinkmann's extended Darcy model with nanofluids. The governing equations are coupled and highly non-linear in nature. The partial differential equations are converted to higher-order ordinary differential equations using similarity transformations. The higher-order ordinary differential equations are reduced to first-order ordinary differential equations and solved to obtain velocity and temperature graphs by shooting technique choosing the initial guess values. The distributions of velocity and temperature are analysed for various pertinent parameters along with nanofluids and it found that the volume fraction of the nanoparticle enhances the thermal conductivity thereby giving the better heat transfer. The results obtained under the limiting conditions were found to be in good agreement with existing ones.
Keywords: Boundary layer, mixed convection, nanofluids, variable fluid properties, vertical flat plate.

INTRODUCTION:

Convection is the method of transferring heat through the movement of molecules within fluids like gases and liquids. The initial transfer of heat between the object and the fluid occurs through conduction, but the bulk transfer of heat occurs due to the fluid's motion. Heat transfer is defined as the transmission of energy from one region to the other region due to temperature differences. Mixed convection is the heat transfer that occurs through external forces and as well as buoyancy forces, a combination of both forced and free convection. Such heat transfer which is a consequence of buoyancy effects has played a vital role in many industrial and engineering applications. Due to wide applications in engineering as post-accidental heat removal in nuclear reactors, solar collectors, drying processes, heat exchangers, geothermal energy extractions, petroleum reservoirs, chemical catalytic reactors and processes, blood flow in a tube, drying of porous solids, thermal insulation, enhanced oil recovery, packed-bed catalytic reactors, tidal waves, wind power geothermal reservoirs, gas turbines, thermal energy storage.

The mixed convection boundary layer flow on an impermeable vertical surface embedded in a saturated porous medium has been treated by Merkin [1]. Benenati and Brosilow [2], found that the permeability of a porous medium varies due to the variation of porosity from the wall to the interior of the porous medium. Chandrashekar and Namboodiri [3] have analysed the effectiveness of variable permeability of the porous medium on velocity distribution and heat transfer. Mohammadein and El-Shaer [4] have studied combined free and forced convective flow past a semi-infinite vertical plate embedded in a porous medium incorporating the variable permeability.

The importance of varying the fluid properties was more concentrated with a high porosity medium and continued the study by Pal and Shivakumara [5]. Effects of Variable fluid properties and MHD on mixed convection heat transfer from a vertically heated plate embedded in a sparsely packed porous medium were analyzed by Nalinakshi et al. [6]. The study of double-diffusive mixed convection with variable fluid properties is analyzed numerically by Nalinakshi et al. [7] and continued the study of Soret and Dufour's effects on mixed convection heat and mass transfer with variable fluid properties.

Nanofluids are fluids with solid-sized nanoparticles suspended in them and are described as a mixture of nanometer-sized particles in the base fluid. The nanoparticles are prepared from copper, aluminium, titanium oxide, gold and materials that are stable chemically. In recent times, heat transfer analysis in nanofluids has been the topic of extensive research due to the intensification of thermal properties in heat transfer processes. The base fluids are ordinary fluids, including water, toluene, ethylene glycol and mineral oils. The usage of the nanofluid as a cooling device will intrinsically enhance the thermal performance of the manufacturing processes and also reduce operating costs. Nanofluid possesses some special behaviour, by which it is very stable and does not have any additional problems such as erosion, sedimentation, non-Newtonian behaviour or additional pressure drop. This happens due to the tiny size and low volume fraction of nano-elements needed for the thermal conductivity improvement in a wide variety of industrial and engineering applications, the transfer of heat energy is a common and necessary process, and the enrichment of heating or cooling an industrial process can result in several benefits including energy savings, reduced processing time, and improved final product quality. However, the heat transfer efficiency also depends on the thermal performance of working fluids. Thus, Improving the thermal conductivity of working fluids is a major challenge for industrial applications. The nanofluids have tremendous applications such as engine cooling, solar water heating, cooling of electronic equipment, cooling of transformer oil, cooling of heat exchanging devices, chillers, refrigerator–freezers, nuclear reactors, space vehicles, formation and dispersion of fog, the distribution of temperature and moisture over agricultural fields and environmental pollution, due to their higher thermal conductivity and convective heat transfer rates. It has applications in cancer imaging and drug delivery for cancer therapeutics in the biomedical industry.

In the recent era, 3\heat transfer analysis in nanofluids has been the extensive research pursued by a very large group of researchers. The intensification of thermal properties in the heat transfer processes has attracted many researchers in the study of convection in nanofluids. Although heat transfer is a common and necessary process, which is reflected in many industrial processes, the heat transfer efficiency depends on the thermal performance of working fluids. Improving the thermal conductivity of such working fluids becomes a challenging task for industrial necessity, which can be overcome by the inspiration of suspended nanoparticles in a base fluid to increase thermal conductivity.

Nanofluids have tremendous applications such as engine cooling, solar water heating, cooling of electronic equipment, cooling of transformer oil, cooling of heat exchanging devices, in chillers, refrigerator-freezers, nuclear reactors, and space vehicles due to their high thermal performance and convective heat transfer rates. In 1965 Choi et al. [8] analyzed Enhancing Thermal conductivity of fluids with Nanoparticles, developments and applications of non-Newtonian flows. Wang et al. [9] thereafter studied theoretical and experimental investigations on the nanofluid heat transfer property by observing the thermal conductivity of nanoparticle fluid mixture. The convective transport in Nanofluids was studied by Buongiorno [10]. The boundary layer flow of a nanofluid past a vertical plate for Natural convection was analyzed by Kuznetsov et al. [11]. Das [12] studied the flow and heat transfer characteristics of nanofluids in a rotating frame. The suspended particle effect on nanofluid boundary layer flow past a stretching surface was analyzed by Gireesha et al. [13] The theory of nanofluid has been a field of active research area for about two decades. This fluid is a suspension of nanometer-sized solid particles and fibres in a convectional base fluid. Common base fluids used for nanofluids include water, toluene, oil, and mixtures of ethylene glycol. The choice of base fluid depends on the intended application. The solid particles used for a nanofluid are metallic solids viz., copper, aluminium, silver and gold; non-metallic solids viz. Silicon (SiO), alumina (Al2O3), metallic liquid and sodium. In recent years, the concept of nanofluids has been proposed as a means of enhancing heat transfer rates in liquids. Materials with nanometer-scale sizes possess unique physical and chemical properties. Some numerical and experimental studies on nanofluids include thermal conductivity studied by Abu-nada et al. [14]. They studied mixed convection in a nanofluid past a vertical plate in a saturated porous medium by incorporating the variation of permeability [15-18].

The study of a nanofluid-saturated porous medium by Nield and Kuznetsov [20], and Cheng and Minkowycz [19-21]. Khan and Aziz [22] have extended their work on the natural convective flow of a nanofluid over a vertical plate when the plate is subjected to a constant heat flux. Khan and Pop [23] have studied the boundary layer flow of a nanofluid past a stretching sheet. The free convection flow of nanofluid over an infinite vertical plate with damped thermal flux was studied by Zaib Un Nisa et al. [24] and Sravan Kumar et al. [25,26] have studied the convective mass transfer in the presence of MHD.

A collective Literature survey of many researchers gives the clarity that the heat transfer analysis has been made with the base fluids, porous medium with the fluid properties being constant initially and continued with the variability of the fluid properties for different geometries. Further, the heat transfer study has been made by a large number of authors suspending the nanoparticles with the base fluids that is the nanofluids and observed that the heat energy transfer is improved drastically in comparison with base fluid. However, the heat energy transport is not been initiated by any authors varying the fluid properties. Hence, the main objective of this work is to attempt to analyze the effects of variable fluid properties in the study of mixed convection boundary layer flow past a vertically heated plate embedded with nanofluids.

MATHEMATICAL FORMULATION AND SOLUTION :

We consider the steady mixed convection boundary layer flow past a vertical semi-infinite flat plate embedded in a porous medium filled with a nanofluid. The x-coordinate is measured along the plate from its leading edge, and the y-coordinate is normal to it. Let U_o be the velocity of the fluid in the upward direction and the gravitational field is acting in the downward direction as shown in Figure 1. The plate is maintained at a uniform temperature T_w which is always greater than the free stream values existing far from the plate (i.e., $T_w > T_\infty$).

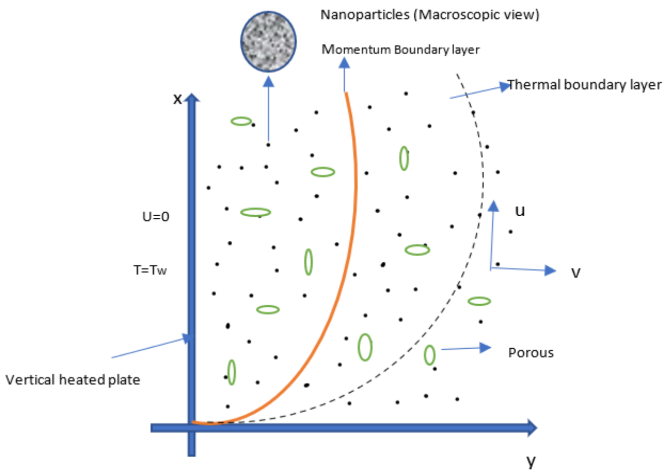

Figure 1: Physical configuration of the system

Table 1. Thermophysical properties of water and nanoparticles

Physical Properties	Water/base fluid	Cu(Copper)
$\rho\ (kg/m^3)$	997.1	8933
$c_p\ (J/kg.K)$	4179	385
$k\ (W/m.K)$	0.613	401
$\sigma\ (S/m)$	0.05	59.6×10^6

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$\rho_{nf}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=g(\rho g)_{nf}(T-T_{\infty})+\mu_{nf}\frac{\partial^2 u}{\partial y^2}+\mu_{nf}\frac{\varepsilon(y)}{K(y)}(U_0-u) \quad (2)$$

$$\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{k_{nf}}{(\rho c_p)_{nf}}\frac{\partial^2 T}{\partial y^2}+\frac{\mu_{nf}}{(\rho c_p)_{nf}}\left(\frac{\partial u}{\partial y}\right)^2 \quad (3)$$

The above governing equations need to be solved subject to the following boundary

$$u=0, v=0, T=T_w \ \text{ at } \ y=0 \quad (4)$$

$$u=U_0, v=0, T=T_{\infty} \ \text{ as } \ y\to\infty \quad (5)$$

We now introduce the following dimensionless variables f and θ as well as similarity variable η ([4]).

$$\eta=\left(\frac{y}{x}\right)\left(\frac{U_0 x}{\gamma}\right)^{\frac{1}{2}},\ \psi=\sqrt{\gamma U_0 x}\, f(\eta),\ \theta=\frac{T-T_{\infty}}{T_w-T_{\infty}} \quad (6)$$

Where, a prime represents differentiation with respect to η and $T=T_w$ is the plate temperature.

In equation (6) the stream function $\psi(x,y)$ is defined by $u=\frac{\partial\psi}{\partial y}$, $v=-\frac{\partial\psi}{\partial x}$, such that the continuity equation (1) is satisfied automatically and the velocity components are given by,

$$u=U_0 f'(\eta),\ v=-\frac{1}{2}\sqrt{\frac{\upsilon U_0}{x}}\left(f(\eta)-\eta f'(\eta)\right) \quad (7)$$

The nanofluid properties are given as ([8,9])

Where the density $\rho_{nf}=(1-\varphi)\rho_f+\varphi\rho_s$ (8)

Fluid viscosity of the nanofluid $\mu_{nf}=\frac{\mu_f}{(1-\varphi)^{2.5}}$ (9)

Heat Capacitance $(\rho c_p)_{nf}=(1-\varphi)(\rho c_p)_f+\varphi(\rho c_p)_s$ (10)

Thermal conductivity $k_{nf}=k_f\left(\frac{(k_s+2k_f)-2\varphi(k_f-k_s)}{(k_s+2k_f)+\varphi(k_f-k_s)}\right)$ (11)

Thermal diffusivity $\alpha_{nf}=\frac{k_{nf}}{(\rho c_p)_{nf}}$ (12)

φ is the nanoparticle volume fraction, ρ_f and ρ_s are the density of the base fluid and nano-particles respectively, c_{pf} and c_{ps} - specific heat of the base fluid and nanoparticles respectively and k_f and k_s are thermal conductivity of base fluid and nanoparticles respectively.

Following Chandrashekhar and Namboodiri [13], the variable permeability $k(\eta)$, the variable porosity $\varepsilon(\eta)$ are given by,

$$k(\eta)=k_0\left(1+de^{-\eta}\right) \quad (13)$$

$$\varepsilon(\eta)=\varepsilon_0\left(1+d^{*}e^{-\eta}\right) \quad (14)$$

Were k_0, ε_0 and are the permeability, and porosity at the edge of the boundary layer respectively, σ^* is the ratio of the thermal conductivity of solid to the conductivity of the fluid, d and d^* are treated as constants having values 3.0 and 1.5 respectively.

Substituting (6) – (12) in equations (2) and (3), we get the following transformed equations:

$$f''' + \frac{A_1 A_2}{2} f f'' + \frac{Gr}{R_e^2}\theta A_1 A_2 + \frac{(1-f')\left(1+d^* e^{-\eta}\right)}{R_e K\left(1+de^{-\eta}\right)} = 0 \qquad (15)$$

$$\theta'' + \frac{\Pr}{2}\frac{A_3}{A_4} f\theta' + Ec\Pr f''^2 \frac{1}{A_1 A_4} = 0 \qquad (16)$$

Were,

$$A_1 = \frac{1}{(1-\phi)^{2.5}}, A_2 = (1-\phi) + \phi\frac{\rho_s}{\rho_f}, A_3 = (1-\phi) + \phi\frac{\rho_s (c_p)_s}{\rho_f (c_p)_f}, A_4 = k_f\left[\frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)}\right]$$

$\Pr = \dfrac{\mu_f (c_p)_f}{k_f}$ is the Prandtl number, $\alpha^* = \mu / \bar{\mu}$ is the ratio of viscosities,

$E = \dfrac{U_0^2}{(c_p)_f (T_w - T_\infty)}$ is the Eckert number, $K = \dfrac{k_0}{x^2 \varepsilon_0}$ is the local permeability parameter,

$\mathrm{Re} = \dfrac{U_0 x}{\nu_f}$ is the local Reynolds number and $E = \dfrac{U_0^2}{(c_p)_f (T_w - T_\infty)}$ is the local Gasthof number.

The transformed boundary conditions are:

$$f = 0, f' = 0, \theta = 1 \text{ at } \eta = 0 \qquad (17)$$

$$f' = 1, \theta = 0 \text{ as } \eta \to \infty \qquad (18)$$

Once the velocity and temperature distributions are known, the skin friction and the rate of heat transfer can be calculated by

$$\tau = -\frac{1}{(1-\varphi)^{2.5}} f''(0) / \sqrt{\mathrm{Re}} \quad \text{and} \quad Nu = -\frac{k_{nf}}{k_f}\sqrt{\mathrm{Re}}\,\theta'(0)$$

Where τ is the skin friction and Nu is the Nusselt number.

NUMERICAL METHOD:

Equations (15) and (16) constitute a highly non-linear coupled boundary value problem (BVP) of third and second order respectively. A shooting technique is employed to solve the resulting nonlinear BVP. Thus, the coupled non-linear boundary value problem of third-order in f and second-order in θ has been reduced to a system of five simultaneous equations of first-order for five unknows as follows:

$$f_1' = f_2,\; f_2' = f_3,$$

$$f_3' = -\left(\frac{A_1 A_2}{2} f_1 f_3 + \frac{Gr}{R_e^2}\theta A_1 A_2 + \frac{(1-f_1)\left(1+d^* e^{-\eta}\right)}{R_e K\left(1+de^{-\eta}\right)}\right) \qquad (20)$$

$$f_4' = f_5,\; f_5' = -\left(\frac{1}{2}\Pr f_1 f_5 \frac{A_3}{A_4} + Ec\Pr f_3^2 \frac{1}{A_4 A_1}\right) \qquad (21)$$

Where $f_1 = f, f_2 = f', f_3 = f'', f_4 = \theta, f_5 = \theta'$ and a prime denotes differentiation with respect to η.

The boundary conditions now become

$$f_1 = 0, f_2 = 0, f_4 = 1 \quad \text{at} \quad \eta = 0 \tag{22}$$

$$f_2 = 1, f_4 = 0 \quad \text{as} \quad \eta \to \infty \tag{23}$$

RESULTS AND DISCUSSIONS:

In the present study of mixed convection past a vertical heated plate with nanofluids and variable fluid properties, the governing equations considered are highly coupled and non-linear partial differential equations. The equations are converted to higher-order ordinary differential equations using the similarity transformations. Higher order ODEs are then converted to first order ODE and the shooting technique procedure is applied using the Runge Kutta method.

The initiation of the shooting technique in solving the system of first-order ordinary differential equations with boundary conditions, given in equations (22) and (23).

Table 2: Results for $f''(0)$ *and* $-\theta'(0)$ for selected fixed values of , $\Pr$, $\frac{Gr}{\mathrm{Re}^2}$, K and φ for UP and VP cases: M=0

Pr	$\frac{Gr}{\mathrm{Re}^2}$	K	ϕ	Uniform Permeability (UP)		Variable Permeability (VP)	
				$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$
6.2	0.0	0.1	0.02	0.588001	0.653333	0.602331	0.653303
	0.2	0.1	0.02	1.228601	0.805780	1.016561	0.757780
	2.0	0.1	0.02	1.324464	0.833119	1.323464	0.853119
6.2	0.2	0.1	0.0	0.597862	0.600123	0.611483	0.632659
		0.1	0.01	0.589838	0.645647	0.639313	0.678076
		0.1	0.02	0.577864	0.476893	0.688132	0.556896
6.2	0.2	1	0.02	0.598452	0.603456	0.600281	0.653333
		2	0.02	0.623454	0.634565	0.654681	0.677463
		3	0.02	0.764536	0.603546	0.842689	0.722621

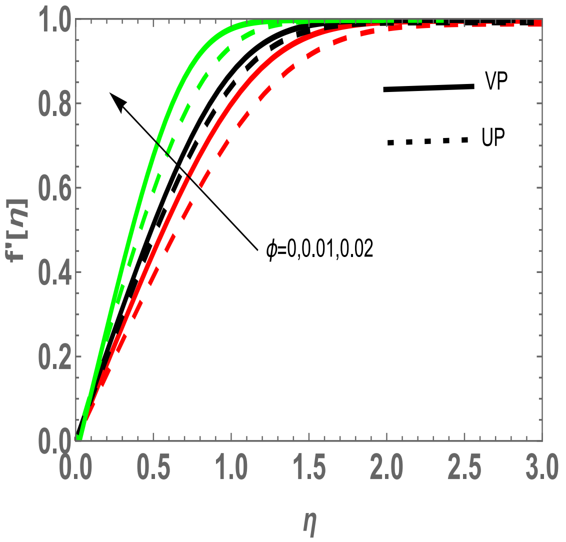

FIGURE 2. Velocity profile for various ϕ

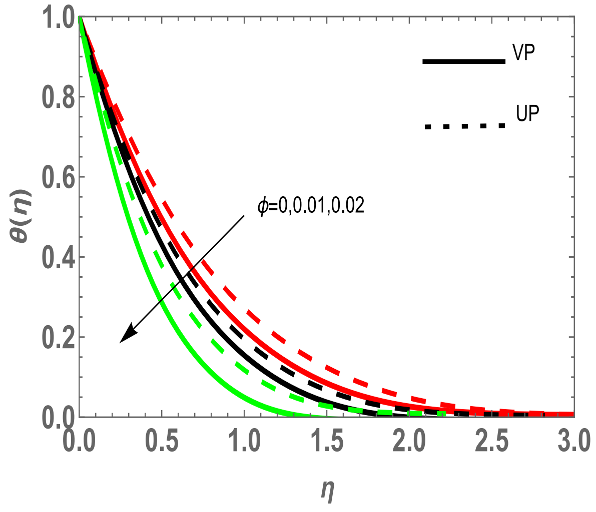

FIGURE 3. Temperature profile for various ϕ

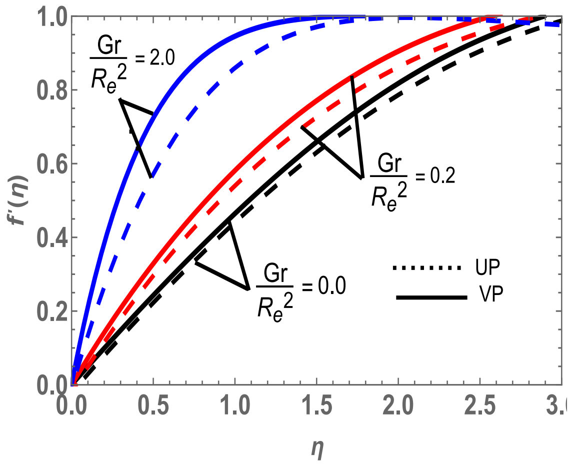

FIGURE 4. Velocity profile for various Gr/Re^2

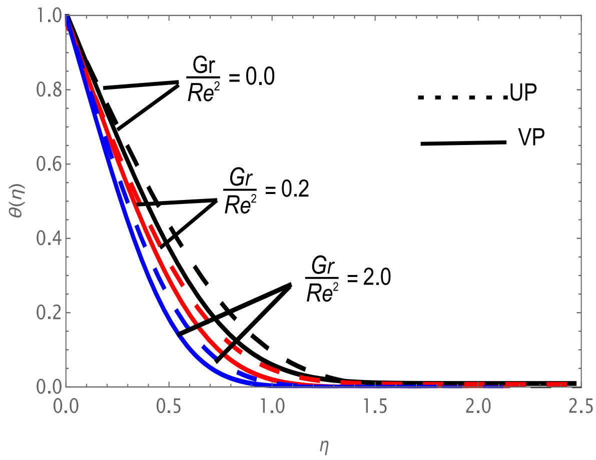

FIGURE 5. Temperature profile for various Gr/Re^2

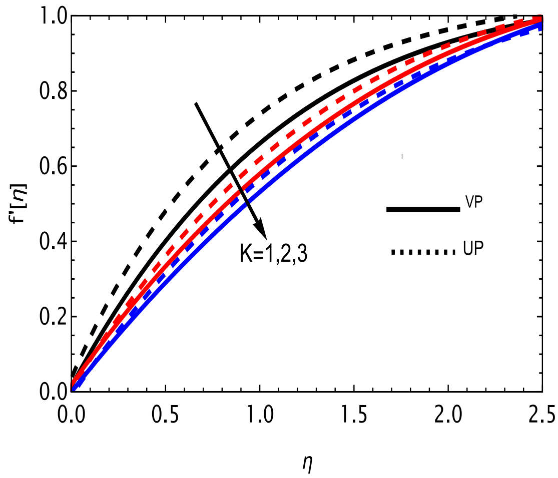

FIGURE 10. Velocity profile for various values of K

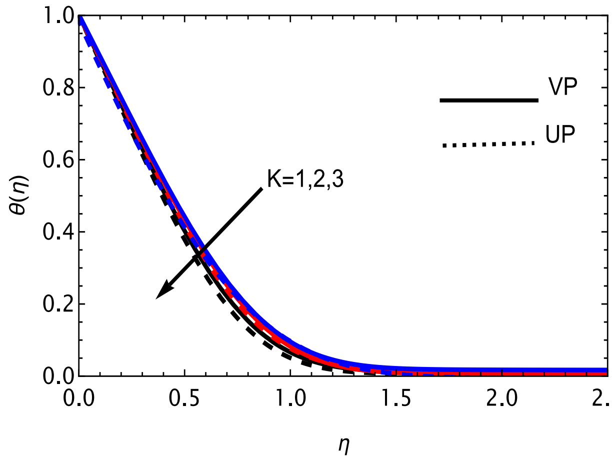

FIGURE 11. Temperature profile for various values of K

The results of computation are presented through graphs from Figure 2-9 and in table 1 and 2. The values of pertinent parameters and thermo-physical properties of base fluid (water) and nanoparticle (copper) have been considered. The volume fraction φ is chosen in the range of $0 \le \phi \le 0.02$ and the velocity and temperature profiles for the same are shown for both Uniform Permeability (UP) and Variable Permeability (VP).

The values of d and d^* is presumed as 0 for uniform permeability and 3.0, 1.5 respectively for variable permeability. When the base fluid mixes with a nanofluid containing copper nanoparticles at different volume fractions, the velocity and temperature graphs exhibit certain changes. Figures 2 and 3 display the effect of the volume fraction of the nanoparticles on the velocity and temperature profile. It is noticed that with the increasing volume fraction of Cu nanoparticles, the velocity of the nanofluid decreases. The presence of nanoparticles can lead to an increase in fluid viscosity, resulting in reduced flow velocity. Whereas the reverse trend is observed in the case of temperature profiles, because the addition of Cu nanoparticles tends to enhance the thermal conductivity of the nanofluid and hence the thermal boundary layer thickness increases. As a result, the temperature gradient decreases and the fluid can dissipate heat more efficiently. This is an understandable fact that heat transfer is better in variable permeability cases than compared to uniform permeability.

In mixed convection flow over a vertical plate with nanofluid, the mixed parameter plays a significant role in determining the cooling properties, which measures the ratio of buoyancy forces to viscous forces. When the plate is heated, the mixed convection parameter is <<0 and when it is cooled it is >>0. The presence of nanofluid, which is a suspension of nanoparticles in a base fluid can further enhance the cooling effect. The fluid's effective thermal conductivity is often increased by nanoparticles, which improves heat transfer.

In Figure 4 the parameter is positive, the combination of buoyancy-driven natural convection and improved thermal conductivity of the nanofluid contribute to the increased cooling of the vertical plate.

The mixed convection parameter that could cause a greater temperature difference between the plate and the fluid, resulting in a lower temperature profile for the cases of UP and VP, the temperature is found to be less in VP as compared to UP. It is also observed that the effect of VP is more significant on temperature distribution for higher values as seen in Figure 5.

The impact of porosity parameter on the velocity and temperature profile as shown in Figures 6 and 7 for both UP and VP cases. Permeability refers to the ability of a porous media to allow liquids to pass through it. Due to the increase of porosity, the velocity field increases. Also, the temperature profiles reduce with increasing porosity parameter.

The Ec refers to the relationship between kinetic energy and enthalpy. It converts the conservation of kinetic energy into internal energy by the work done against the viscous fluid stress. In Figure 8, it is observed that the heat gets transferred from the sheet to the nanofluid, which increases the temperature of the fluid. Figure 9 shows the increase in skin friction with an increase in K.

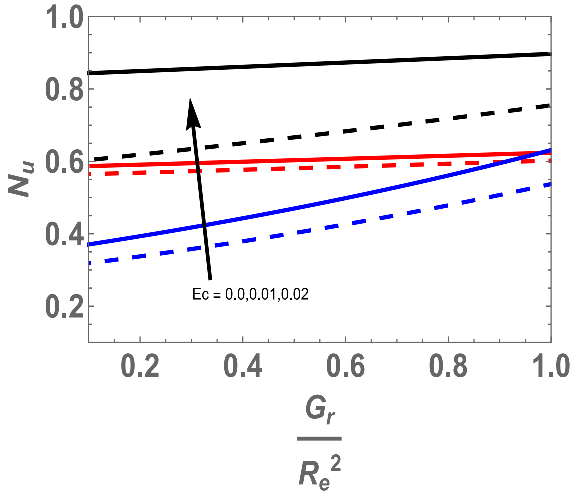

FIGURE 8. Nusselt number for various values of Ec

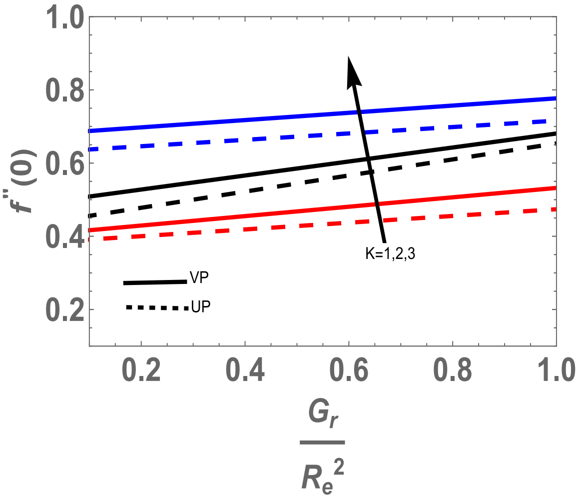

FIGURE 9. Skin friction for various values of K

CONCLUSION:

The mixed convection past a vertical plate has been studied in the presence of water-based nanoparticles and is numerically investigated. The effect of governing parameters such as nanoparticle volume fraction, permeability parameters, mixed parameters and the corresponding graphs are plotted and the following conclusions can be made from the study.

- The increasing volume fraction of Cu nanoparticles, the velocity of the nanofluid decreases whereas viscosity increases, temperature tends to enhance the thermal conductivity of the nanofluid.
- In a mixed convection vertical plate, increasing the mixed convection parameter faster will be the cooling in velocity and temperature.
- For higher the values of K velocity increases as the temperature decreases.

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