

A swift approach to decode Integer programming problems using CMI's reduction method

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Abstract: A Rapid method for solving the Integer linear programming problem, refraining from the usage of Gomorian constraint and dual simplex method is evolved by utilizing the CMI Reduction Method. The CMI Reduction Method relieves the user from the formation of Gomorian constraint from the final iteration followed by rest other procedures. To derive the solution with the given constraints CMI Reduction Method is applied in the final iteration which leads to the desired solution within the short span. Keeping the students in mind as they face and feel lot of complications in solving Integer linear programming problems the CMI Reduction method is incorporated. As the complexities are rooted out, this method will get due significance in solving the problems with multiple constraints. This will impart comfort to the students and help them to avoid mistakes by doing repeated iterations and cross mapping. This Proposed method is illustrated with numerical examples.

Key words: Gomorian Constraint, CMI Reduction Method, Integer Linear Programming Problems, Optimal solution

I. REVIEW OF LITERATURE

Integer programming problem is invariably used in solving practical problems of real world. It plays a pivotal role in optimization of the economy, strategically planning many other real-life problems. Many research activities are going to take off to improve the available methods. The most popular technique in existence is Gomory's Fractional Cut Algorithm. It was first developed and introduced by R.E.Gomory [3] Adopting this algorithm necessitates the usage of new constraint and dual simplex method. As well as limitation on iterations cannot be predicted while using this method which is demerit of this Algorithm. As Integer Programming is getting its attention and wide usage in economic revolution there are more research activities around this and plenty of innovative methods are getting established. In recent time Mr.P. Pandian and Mr. Balakrishnan [5] have developed a concept in 'New method for a class of linear programming problems' and by applying this, 'A class of linear integer and a fractional programming' was solved. Also, Mr.P. Pandian along with Jayalakshmi [6] introduced a new approach for solving a class of pure integer programming problems. By studying all the existing methods and also from the above articles, we introduced a CMI Reduction Method for a integer linear programming problem which is better in various parameters than all the existing methods. This method is based on simpler mathematical concepts which are easy to understand and apply into problem of any of its kind. This efficient and time reduction technique for solving Integer programming problem will be a prevalent tool in the industry.

The following would be the advantage in Solving IPP using CMI Reduction Method,

1. Relieves from addition of new constraints
2. Redeems from usage of dual simplex
3. Refrains from creation of multiple iterations
4. Releases from complexity in the computation

II. CMI REDUCTION METHOD ALGORITHM

Step 1: Formulate the given problem in a standard form and solve it by existing simplex method.

Step 2: Verify that the optimum solution is an Integer or not in the simplex table. If Integer then the resultant solution will be the Optimum solution of the IPP problem. In case of non-integer move to the next step

Step 3: By using body matrix and the corresponding value of x_B in the table reformulate the equation in the form as below, $\sum_{n=1}^k a_{mn} x_n = x_{Bm}$ where $m=1, 2, \dots, k$ If

x_{Bm} is an integer retaining the equation in case of non-integer then, reconstruct

x_{Bm} as an integer $\pm$ fractional part ($0 < x < 1$), on which the following cases may arise

Case (i): If the decimal value in the fractional part lies between the interval $0.0 \leq x \leq 0.6$ then select that corresponding left integer $\pm$ fractional part, for the other interval the right integer $\pm$ fractional part

Case (ii): If in case of both equations with x_{Bm} of same value then one equation to be formed as right integer $\pm$ fractional part and the other equation as left integer $\pm$ fractional part. In Case of three equations, the third equation will be right integer $\pm$ fractional Part.

Step 4: In step-3, retain only integer value of the equations and omit the fractional parts as they are too low values which will not affect the result in finding the optimum solution. Then, rewrite the $\sum_{n=1}^k a_{mn} x_n = x_{Bm}$ where $m=1, 2, \dots, k$ as,

(i) For Two variables:

$$\sum_{n=1}^2 a_{mn} x_n = b_m \text{ where } m=1,2. \text{ (} b_m \text{-integer value)}$$

$$P = \text{Minimum of } \left\{ \frac{b_m}{a_{mn}}, m=1, 2 \right\}, n = 1, 2.$$

$$\text{which implies } x_n \in \{0, 1, 2 \dots .P\}, n = 1, 2. \text{ and } x'_n = \text{Minimum of } \left\{ \frac{b_m - a_{m1} x_n}{a_{m2}} \right\}, m=1, 2.$$

(ii) For Three variables:

$$\sum_{n=1}^3 a_{mn} x_n = b_m \text{ where } m=1, 2, 3 \quad P = \text{Minimum of } \left\{ \frac{b_m}{a_{mn}}, m=1, 2, 3 \right\}, n = 1, 2, 3.$$

After finding the value of x_n , move that variable in the equation from LHS to RHS and substitute the value of P in x_n , by this the 3 variable reduces to 2 variable equation and then we need solve by using 2 variables formulae.

Step 5: Substitute the values obtained from the 3 variables formulae in the given objective function of pure IPP and find out the maximum value of Z .

Step 6: Apply the optimum value in the constraints to check whether it satisfies or not. If not satisfied then apply the preceding value of P from x_n and verify if still not satisfied, then continue the procedure.

III NUMERICAL EXAMPLES

EXAMPLE –I

Solve the Integer Programming Problem

$$\text{Maximize } Z = 2x_1 + 2x_2$$

Subject to the constraints

$$5x_1 + 3x_2 \leq 8$$

$$2x_1 + 4x_2 \leq 8,$$

where $x_1, x_2 \geq 0$ and are Integers

Solution

Convert the problem into a standard form and solve it by simplex method. As the derived optimum solution is a non-integer, Proceed as per step 3 of the algorithm to form the below equations.

$$x_1 + 0x_2 = 4/7 \text{ (0-4/7)}$$

$$0x_1 + x_2 = 12/7 \text{ (2-2/7)}$$

By applying case(i) of step-3 from the algorithm, 4/7 and 12/7 is split into left integer - fractional part and right integer - fractional part respectively. The integer part is retained and the fractional part is discarded and the below equation is formed,

$$x_1 + 0x_2 = 0$$

$$0x_1 + x_2 = 2$$

By applying two variables formula,

$$P = \text{Minimum of } \{0/1, 0/0, 2/0, 2/1\} = 0 \text{ which is a coefficient of } x_1$$

$$\text{Therefore } x_1 = 0 \text{ and } x_2 = \text{Minimum of } \left\{ \frac{0-x_1}{0}, \frac{2-0x_1}{1} \right\} = 2$$

$$\text{Hence } x_1 = 0, x_2 = 2$$

On Substituting the above values in the objective function of given IPP, we get

$$x_1 = 0, x_2 = 2 \text{ \& } Z = 4$$

Hence the optimal solution is satisfied with constraints.

EXAMPLE -2:

Solve the Integer Programming Problem

$$\text{Maximize } Z = x_1 + 2x_2$$

Subject to the constraint

$$2x_2 \leq 7$$

$$x_1 + x_2 \leq 7$$

$$2x_1 \leq 11$$

where $x_1, x_2 \geq 0$ and are integers

Solution:

Formulate the given problem in a standard form and solve it by existing simplex method

From the obtained optimum table, solution is not an integer hence move down to step-3 in the algorithm.

$$0x_1 + x_2 = 7/2 \quad (3+1/2)$$

$$x_1 + 0x_2 = 7/2 \quad (4-1/2)$$

Proceed as per case(ii) in step-3 and step-4 in the algorithm, we get

$$0x_1 + x_2 = 3$$

$$x_1 + 0x_2 = 4$$

By applying two variables formula,

$$P = \text{Minimum of } \{3/0, 3/1, 4/1, 4/0\} = 3 \text{ which is in coefficient of } x_2$$

$$\text{Therefore } x_2 = 3 \in \{0, 1, 2, 3\} \text{ and } x_1 = \text{Minimum of } \left\{ \frac{3-x_2}{0}, \frac{4-0x_2}{1} \right\} = 4$$

$$\text{Hence } x_1 = 4, x_2 = 3$$

Substitute the above values in the objective function of given IPP, we get

$$x_1 = 4, x_2 = 3 \text{ \& } Z = 10$$

Hence the optimal solution is satisfied with constraints.

EXAMPLE -3:

Solve the Integer Programming Problem

$$\text{Minimize } Z = x_1 - 3x_2 + 2x_3$$

Subject to the constraints

$$3x_1 - x_2 + 2x_3 \leq 7$$

$$-2x_1 + 4x_2 \leq 12$$

$$-4x_1 + 3x_2 + 8x_3 \leq 10 \text{ where } x_1, x_2, x_3 \geq 0 \text{ and are integers}$$

Solution

Convert the given LPP into a standard maximisation problem and then determine the optimum solution by using simplex method. From the optimum table, the obtained optimum solution is not an integer hence move down to step-3. Reformulate the equations from the optimum table

$$x_1 + 0x_2 + 0x_3 = 78/25 \quad (3+3/25)$$

$$0x_1 + 0x_2 + x_3 = 11/10 \quad (1+1/10)$$

$$0x_1 + x_2 + 0x_3 = 114/25 \quad (4+14/25)$$

Proceed as per case(i) in step-3 and step-4 in the algorithm, we get

$$x_1 + 0x_2 + 0x_3 = 3$$

$$0x_1 + 0x_2 + x_3 = 1$$

$$0x_1 + x_2 + 0x_3 = 4$$

By applying three variables formula,

$$P = \text{Minimum of } \{3/1, 3/0, 3/0, 1/0, 1/1, 1/0, 4/0, 4/0, 4/1\} = 1 \text{ which is in coefficient of } x_3$$

Therefore $x_3 = 1 \in \{0,1\}$

$$x_1 + 0x_2 = 3 - 0x_3$$

$$0x_1 + 0x_2 = 1 - x_3$$

$$0x_1 + x_2 = 4 - 0x_3$$

If $x_3 = 1$, then the 2nd equation is meaningless and it is reduced to 2 variables

$$x_1 + 0x_2 = 3$$

$$0x_1 + x_2 = 4$$

= Minimum of $\{3/1, 3/0, 4/0, 4/1\} = 3$ which is in coefficient of x_1

Therefore $x_1 = 3 \in \{0,1,2,3\}$

$$\text{and } x_2 = \text{Minimum of } \left\{ \frac{3-x_1}{0}, \frac{4-0x_1}{1} \right\} = 4$$

Hence $x_3 = 1, x_2 = 4, x_1 = 3$

Substitute the above values in the objective function of given IPP, we get

$$x_3 = 1, x_2 = 4, x_1 = 3 \text{ \& Min } Z = -7$$

Hence the optimal solution is satisfied with constraints.

IV RESULTS & COMPARISONS

The process to find solutions using the CMI Reduction method is much simpler as no additional constraints or dual simplex methods are used. These elimination of additional steps achieves an invariable time saving which is more essential in our precise world. This will drive more interest for students as complications are reduced to a larger extend.

EXAMPLE No	IPP Method (Existing Method)	CMI Reduction method (New Approach)
1	Z=4	Z=4
2	Z=10	Z=10
3	Z=-7	Z=-7

V.CONCLUSION

The CMI Reduction method used in finding solutions for the pure Integer programming problem overcomes the usage of Gomorian constraint by which the problem gets simpler. Here introduction of additional constraints and dual simplex are not used to find the integer solution. Hence the efficiency of problem solving is improved and eliminates possibility of error occurrence due to repeated iteration. Hence by adopting this method the users will get comfort and excitement in solving the pure integer programming problem with much ease.

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