TuijinJishu/Journal of Propulsion Technology
ISSN: 1001-4055
Vol. 44 No.6 (2023),

Forecasting Stock Price Using Brownian Motion Model

Dr Jyoti Singhal,

Associate Professor, Vijay Patil School of Management, DY Patil University, Navi Mumbai

1. Introduction

Brownian motion is a simple continuous stochastic process that is widely used in physics for the movements of a molecule of gas and same concept is used in finance to model random behavior of asset price that evolves. Brownian motion theory is applicable where behavior are random or asset's price fluctuate. Brownian motion gets its name from the botanist Robert Brown (1828) who observed in 1827 how particles of pollen suspended in water moved erratically on a microscopic scale.

The Brownian motion process can be used to explain the behavior of the risky asset prices as it is limiting case of random walk which is asset pricing model. The stock prices presented in the paper are equity which comes under the risky asset prices. Therefore, the Brownian motion is usually used to model a stock price. However, Brownian motion process has the independent increments property. This means that the present price must not affect the future price. In fact, the present stock price may influence the price at some time in the future. Hence, Brownian motion process may or may not suitable to explain the stock price. Another process, a fractional Brownian motion process, exhibits a long-range dependent property.We can use a fractional Brownian motion process to describe the behavior of stock price instead of Brownian motion process. The rate of return and volatility in general asset pricing models are normally the constant parameters for widely used Capital Asset Pricing Model. But, the rate of return and volatility in actual are not constant at any time. In the paper, these parameters are updated on timely basis by using the new information. The Tata Consultancy Services (TCS), Reliance Industries Limited and ICICI Bank Ltdstock prices are three cases in the paper. TheseThree stocks are chosen from National Stock Exchange (NSE) . The TCS is from It sector, RIL from the Manufacturing sector and ICICI Bank is from banking sector. The TCS, ICICI Bank and RIL stock price models are studied by using Brownian motion and fractional Brownian motion process to explain the uncertain behavior of stock market.The Brownian motion model with adaptive parameters (BMAP) and the fractional Brownian motion model with adaptive parameters (FBMAP) are analyzed on the ICICI Bank Ltd, RIL and TCS stock prices.

The paper is organized as follows.

Preliminaries on a fractional Brownian motion	Section 2
The estimation of the rate of return and volatility	Section 3
Explanation of BMAP and the FBMAP	Section 4
Final, concludes	Section 5

2. Fractional Brownian Motionand Brownian Model

Fractional Brownian motion (fBm) is a widely usedconcept for modeling various situations such as the level of water in a river, the temperature at a specific place, the empirical volatility of a stock, the price dynamics of electricity. It appears naturally in these phenomena because of its capability of explaining the dependence structure in real-life observations. This structure in fBm is represented by its Hurst parameter H. The randomness of the stock price is modeledby Brownian motion process. A stock price process $(St,\ t \geq 0)$ is represented by the stochastic differential equation (SDE) as shown in

$dSt = St\ (\mu dt + \sigma dWt)$ ....(1)

The process (Wt, $t \geq 0$) in (1) is a standard Brownian motion process. The stochastic differential equation (1) is driven by the Brownian motion process (Wt, $t \geq 0$). In the real world, μ and σ in (1) are not constant at any time. Hence these parameters in the paper are the adaptable parameters based on time.μ and σ are calculated on daily basis to make them variable to satisfy the preconditions of the stochasticmotion. In the paper, model (1) is called a Brownian motion model with adaptive parameters (BMAP). In practice, the dynamics of the stock price has a long-range dependence because of investor may use past experience of trade after a long time. The BMAP model in (1) is may not be suitable to describe the dynamics of stock price. Therefore, the fractional Brownian motion process is considered as a more accurate modelin the paper. The fractional Brownian motion process (,$t \geq 0$) with Hurst index H is a centered Gaussian process. If $H = 0.5$, then (,$t \geq 0$) is a standard Brownian motion process. If $H = 0.5/$, then (BHt , $t \geq 0$) is neither a semi-martingale nor a Markov process. For $H = 0.5/$case, the (,$t \geq 0$) is the long memory process. The (,$t \geq 0$) is represented in (2) by Mandelbrot and Van Ness [2]. Consider the following:

$BHt = 1\ \Gamma\ (1+\alpha)\ [Zt + Bt]$. .....(2)

The function $\Gamma(\cdot)$ is the gamma function. The process (Zt,≥ 0) is defined by

$Zt = \int 0\ -\infty[(t - s)\alpha - (-s)\alpha\ ]dWs.$

The process (Bt, $t \geq 0$) is described by $Bt = \int t\ 0\ (t - s)\ dWs.$

The parameter $\alpha = H - 1/2$, where $H \in (0, 1)$ and (Wt, $t \geq 0$) is a standard Brownian motion process. The rate of return and volatility are not constant at any time. Hence, the paper also proposes the new approach of the asset pricing model. In this case, the driving process of model (1) is replaced by a fractional Brownian motion process. The rate of return and volatility are adaptive parameters. In this case, the model can be represented by the stochastic differential equation (SDE) as shown in

$dSt = St\ (\mu dt + \sigma\ )$... ...(3)

The parameters μ and σ in (3) are the rate of return and the volatility, respectively. The μ and σ are adaptive parameters the same as the previous model. The (,$t \geq 0$) is a fractional Brownian motion process. In the paper, model (3) is called a fractional Brownian motion model with adaptive parameters (FBMAP) Alos et al. [3] have proposed to use the process (Bt,$t \geq 0$) instead of (BHt , $t \geq 0$), since (Zt, $t \geq 0$) has absolutely continuous trajectory. So the process (Bt, $t \geq 0$) has long range dependence.

Hence, the model (3) is shown as

$dSt = St\ (\mu dt + \sigma dBt)$....... (4)

Thao gave An approximate approach to stochastic differential equation for fractional Brownian motion [4]. The process ($B\varepsilon t$, $t \geq 0$) is introduced. For every ε>0, the process ($B\varepsilon t$, $t \geq 0$) is defined by

$B\varepsilon t = \int\ t\ 0\ (t{-}s{+}\varepsilon)\ \alpha dWs$........ (5)

The process ($B\varepsilon t$, $t \geq 0$) is a semimartingale. Therefore, this process can be written as in $B\varepsilon t = \alpha\int\ t\ 0\ \varphi\varepsilon sds + \varepsilon\alpha Wt$, (6) where $\varphi\varepsilon t = \int t\ 0\ (t - s + \varepsilon){-}1dWs$. The process ($B\varepsilon t$, $t \geq 0$) converges to (Bt, $t \geq 0$) in $L2$ (Ω) when ε approaches to 0. This convergence is uniform with respect to $t \in [0, T]$. Hence, the model (4) can be considered as shown in

$dSt = St\ (\mu dt + \sigma dB\varepsilon t)$. ....(7)

3. **The Estimation of the Rate of Return and Volatility** In this paper, the ICICI Bank Ltd, RIL and TCSclosed prices are identified by two asset pricing models. In the BMAP, the driving process is Brownian motion. On the other hand, the driving process is fractional Brownian motion in the FBMAP. The parameters μ and σ in both models are adaptive parameters at any time. Under Brownian motion and Fractional Brownian motion theory μ and σ are constant parameters but in actual both are keep changing as the prices of stock markets are continuously changing. To make the Brownian motion and fractional Brownian motion assumptions more realistic we have used μ and σas adaptive parameters.

The ICICI Bank Ltd, RIL and TCSsimulated stock prices are compared with these empirical prices. The ICICI Bank Ltd, RIL and TCSempirical prices can be obtained from https://www.nseindia.com/products/content/equities/equities/eq_security.htm. The data of ICICI Bank Ltd, RIL and TCS empirical prices from April 01, 2017, to May 31, 2018, are used in the paper.These data are divided into two joint sets for two purposes. The first set (April 01, 2017, to May 31, 2017) is used to estimate the drift rate and volatility. The second set (June 01, 2017, to May 31, 2018) is used for model validations. The rate of return and volatility contained in the BMAP and the FBMAP are adaptive parameters based on time. Therefore, these parameters are not constant. In this section, the rate of return and volatility of the ICICI Bank Ltd, RIL and TCS stock prices are estimated. The ICICI Bank Ltd, RIL and TCSclosed prices from April 01, 2017, to May 31, 2017, are used to estimate μj and σj by using (8) and (9), respectively [5].

Consider the following:

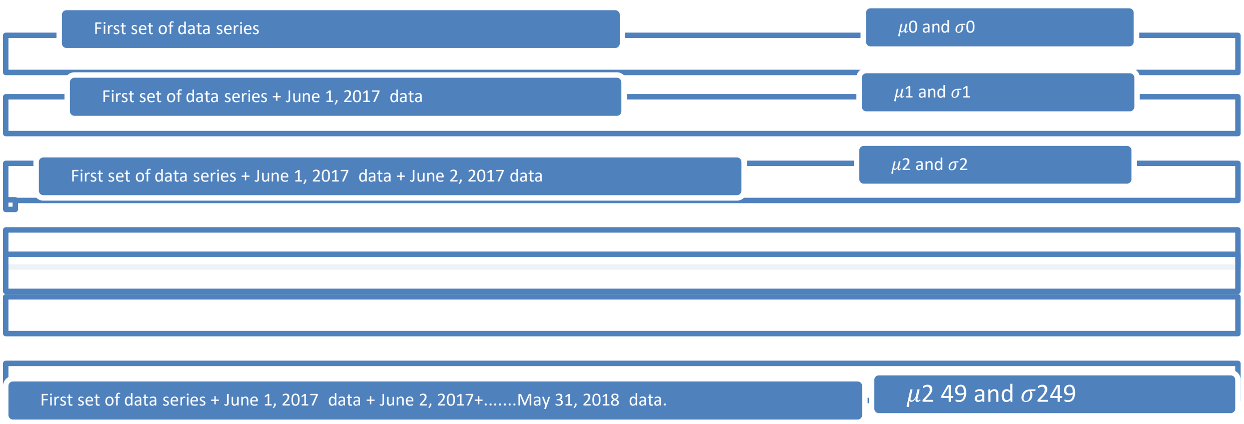
$\mu j = 252/M(\sum_{i=1}^{M} Ri,)$ (8)

$\sigma j = \sqrt{252/M-1 \sum_{i=1}^{M} (Ri - \bar{R})^2}$, (9)

WhereRi is the return of stock price which can be computed by $Ri = (Si+1 - Si)/Si$, $\bar{R}$ is the average of return Ri, and M is the number of returns. The parameters μj and σj are estimated by the set of data as shown in Figure 1. In this figure, the data from April 01, 2017, to May 31, 2017, are used to estimate the initial $\mu 0$ and $\sigma 0$.The data from April 01, 2017, to June 1, 2017, are used to estimate $\mu 1$ and $\sigma 1$. The data from April 01, 2017, to June 2, 2017, are used to estimate $\mu 2$ and $\sigma 2$, The stock market is closed on the weekend. Therefore, the closed prices on June 3, and June 4, 2017, are not available. Using the same procedure, the data from April 01, 2017, to May 31, 2018 , are used to estimate $\mu 249$ and $\sigma 249$.

Price on first set of data series	Estimators for $\mu 0$ and $\sigma 0$
(April 01, 2017, to May 31, 2017)	
Price for second day and estimators of $\mu 0$ and $\sigma 0$	Estimators $\mu 1$ and $\sigma 1$
Price for last day of data series	Estimators μj and σj

Figure **1:The flowchart to estimate the parameters μj and σj:**



From the estimation results, the parameters μj and σj of the ICICI Bank Ltd, RIL, and TCS are shown in Figures 2 ,3 and 4.

4. Stock Prices Mathematical Models

4.1. **Brownian Motion Model with Adaptive Parameters (BMAP).** The BMAP can be considered by the SDE as shown in (1). The rate of return μ and the volatility σ are adaptive parameters and can be estimated using the flowchart in Figure 1. In the paper, the Euler discretization method is applied to solve the SDE. The solution of the discretized form of the SDE (1) is denoted by Sj. Therefore, the Euler discretization form of (1) can be written in

$Sj+1 = Sj + \mu jSj\Delta t + \sigma jSj\Delta Wj$, (10) where j is the time index ($j = 0, . . . , N$), N is the number of datasets, Δt is a sampling time, and μj and σj are estimated in the previous section. For the paper, N is equal to 249 and Δt is set to 1/252. The initial value $S0$ is equal to the stock price onthe first day of the data set. The term ΔWj can be approximated by

$$\Delta Wj = Zj \sqrt{\Delta t}. \qquad (11)$$

The random variable Zj is the standard normally distributed random variable with mean = 0 and variance = 1. It is generated by Excel using the method of Box and Muller [8]. The ICICI Bank Ltd, RIL, and TCS stock prices calculated from the BMAP are simulated throughExcel. These simulated data are compared with the second set of data on empirical prices for model validation. **The average relative percentage error (ARPE)** as given in (12) is the accuracy index in the paper. APRE denotes the accuracy of the model. APRE we are calculating the deviation of the model or simulated price from the actual price. Lower the deviation better the model. We compared BMAP and FBMAP models based on ARPE and concluded that which is the better model. Consider the following:

$$\text{ARPE} = 1/N(^{N} \sum_{i=1} \text{I}(Xi - \text{Y}i)\text{I}/Xi \times 100) .... (12)$$

where N is the number of datasets, Xi is the empirical price (market price), and Yi is the model price (simulated price). For the simulation results by the BMAP, Figure 4 shows the empirical prices compared with the prices simulated by the BMAP for a given path of Brownian motion process. In the paper, the date period for simulation is between June 01, 2017 to May 31, 2018.

4.2. Fractional Brownian Motion Model with Adaptive Parameters (FBMAP). The FBMAP can be described by SDE (5). The rate of return μ and the volatility σ in this model are variable parameters depending on time. The μ and σ can be estimated using the block diagram as shown in Figure 1. The SDE (57) is solved by using the Euler discretization method. Therefore, the Euler discretization form of (5) can be written in

$$S\varepsilon j{+}1 = S\varepsilon j + \mu j S\varepsilon j \Delta t + \sigma j S\varepsilon j \, [\alpha \varphi j \Delta t + \Delta \ \ ] .... (13)$$

Figure 2: The historical drift rate μj andvolatility $\sigma j$for closed prices prediction from June, 2017 to May 2018 TCS.

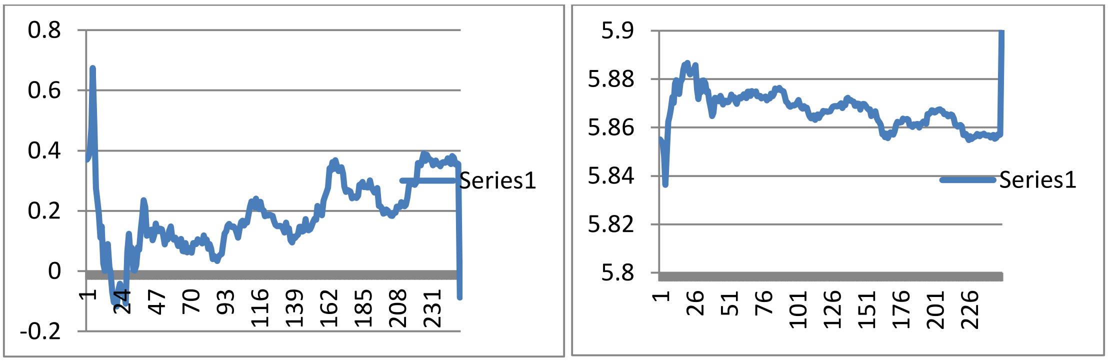


Figure 3:The historical drift rate μj andvolatility σj for closed prices prediction from June, 2017 to May 2018 for Reliance

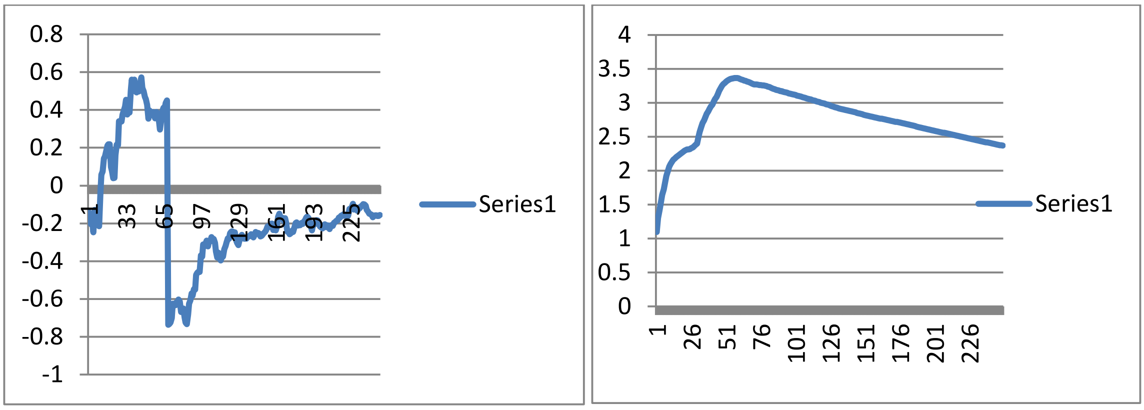
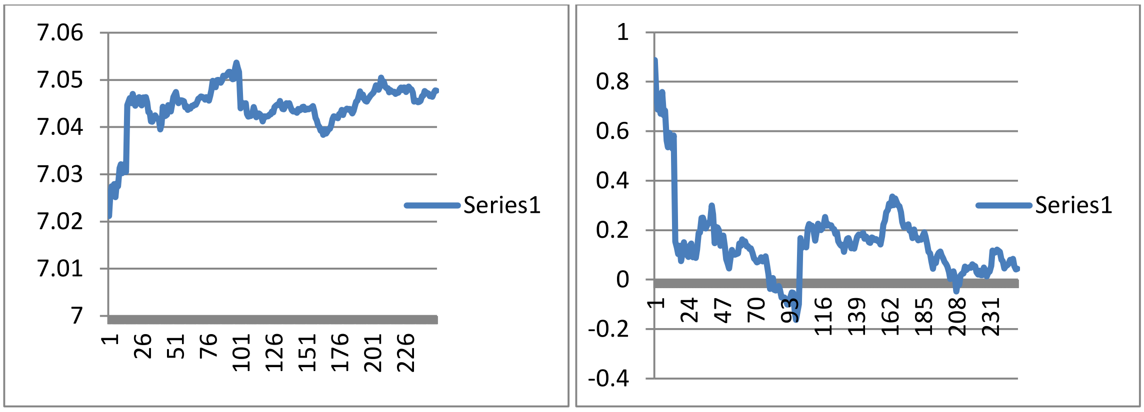


Figure 4: The historical drift rate μj andvolatility σj for closed prices prediction from June, 2017 to May 2018 for ICICI Bank.



In (13), the $S\varepsilon j$ is the discretized solution of the SDE (5).j, Δt, and N have the same meaning as the BMAP case. The parameter $\alpha = H - 0.5$, where H is Hurst index and $H \in (0, 1)$.

Brownian motion is generated from a defined Hurst exponent, where Hurst exponent is 0< H <.5, the random process Hurst exponentwill be a long memory process unlike BMAP model. Data sets like this can be referred to as fractional Brownian motion (abbreviated FBM). Fractional Brownian motion can be generated by a variety of methods, including spectral synthesis using either the Fourier transform or the wavelet transform but we used Hurst exponent in this model.The estimation of Hurst exponent is shown in Section 4.2.1[7]. The parameters μj and σj are used the same as those of the BMAP case. In the paper, N, ε, and Δt are set equal to 249, 0.005, and 1/252, respectively.The initial value $S\varepsilon$ 0 is equal to stock price at April1, 2017. The term $\Delta W j$ can be generated same as BMAP case. The term φj in (13) can be calculated by [14]

$\varphi j = \sqrt{j\Delta t N} N{-}1 \sum k{=}0 \, (t - kj\Delta t N + \varepsilon) \, \alpha{-}1 \, Zk.$ (14)

The random variable Zk in (14) is the standard normally distributed random variable with mean = 0 and variance = 1. It is generated in excel byBox and Muller method.

For model validation, excel simulated data are compared with the empirical prices from June 1, 2017 to May 31, 2018. The accuracy index in this case also uses the average relative percentage error (ARPE) as calculated by (12) in earlier case.

4.2.1. Parameter Estimation. The parameters likeα for RIL, TCS and ICICI bank stock prices are the unknown values. Therefore, the estimation of these parameters is proposed in this section. The parameter α is calculated by $\alpha = H - 0.5$. Firstly, α is varied from −0.5 to 0.5 with step size equal to 0.1.sovalues are between 0 to .49 in this model.

The ICICI Bank Ltd, RIL and TCS empirical prices and simulated price by BMAP Date June 1, 2018 to May 31,2018 are used to calculate ARPE.

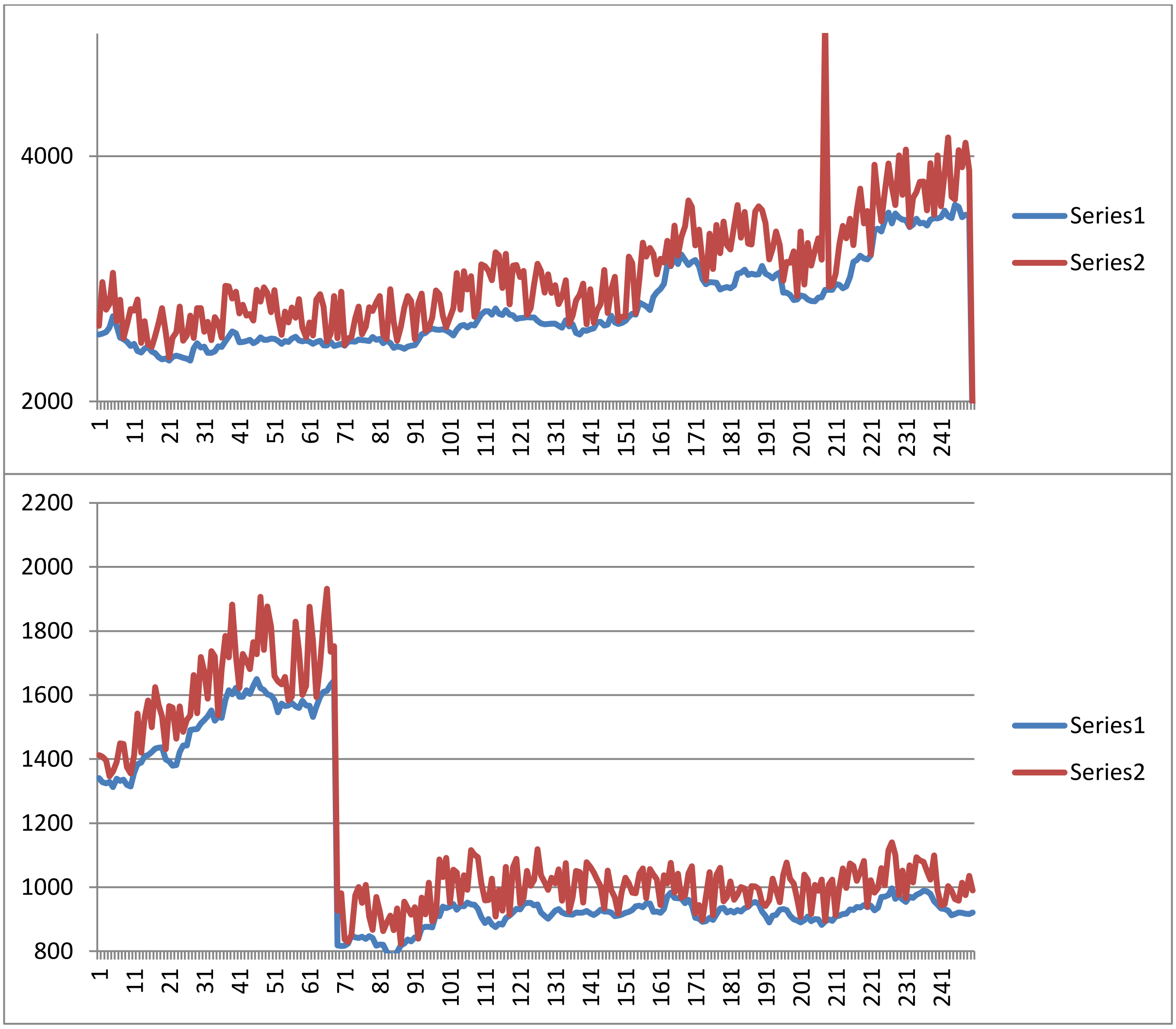
Figure5: The simulation results using the BMAP and empirical share price .

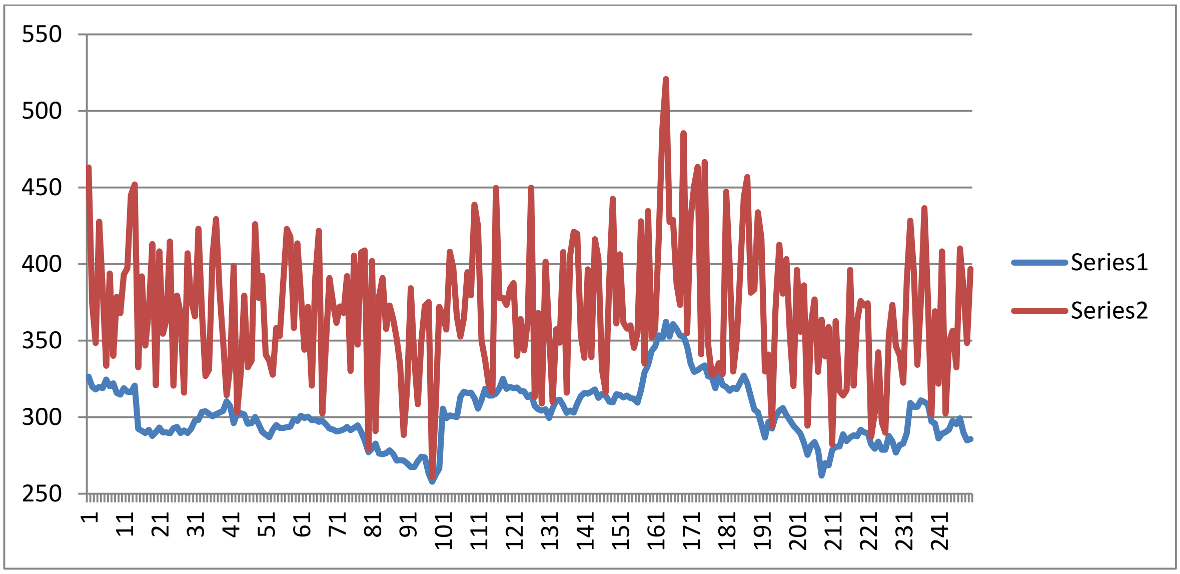
Series 1: Empirical time series   Series 2: Simulated Time series

Figure (A): TCS data from June 1, 2017 to May 31, 2018

Figure (B): RIL data from June 1, 2017 to May 31, 2018

Figure (C): ICICI Bank data from June 1, 2017 to May 31, 2018





Comparison of Accuracy Index between BMAP and FBMAP:

For a given standard Brownian motion sample path, table 1 show that themean and standard deviation of average relative percentage error (ARPE) of TCS for BMAP 9.33909% and 10.83591 FBMAP -10.4236.425% and 6.138%,ICICI ban for BMAP 20.69% and 12.96% FBMAP 11.31103% and 10.962%, Reliance for BMAP 8.87333%and 5.1189% for FBMAP 4.48815% and 4.703874% error (ARPE) of the FBMAP is smaller than those of the BMAP in case of all three stocks. In each path, the ARPE is computed from both models.

The comparison results between the BMAP and the FBMAP :Table 1.

stock	model	Average of ARPE	Standard deviation ARPE
TCS	BMAP	9.33909	10.83591
	FBMAP	6.42315	6.138126
ICICI Bank	BMAP	20.69960	12.96826
	FBMAP	11.31103	10.962739
Reliance	BMAP	8.87333	5.11899
	FBMAP	4.48815	4.703874

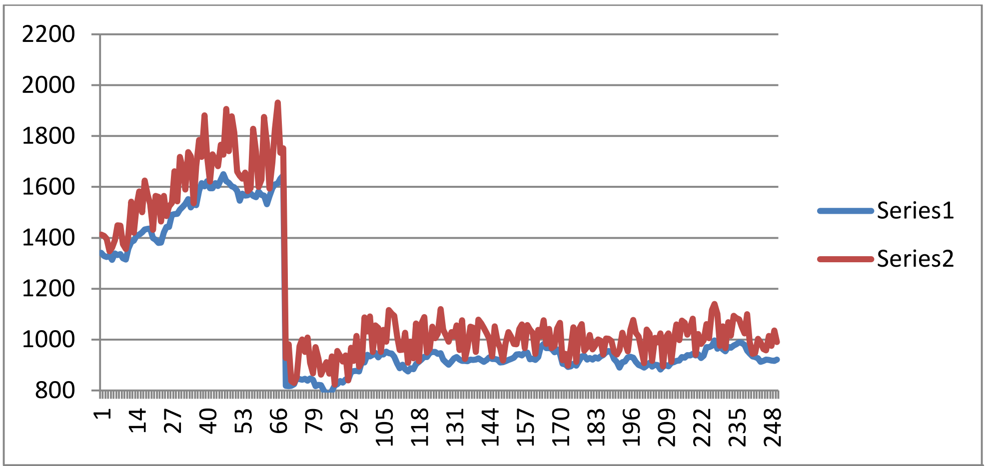
It is observed that the average ARPE of the FBMAP is less than the average ARPE of the BMAP. As well as the standard deviation of ARPE from the FBMAP is smaller compared with the BMAP. Figure 6: The simulation results using the FBMAP. It provides the small ARPE compared with the BMAP in case of TCS, ICICI Bank and Reliance.

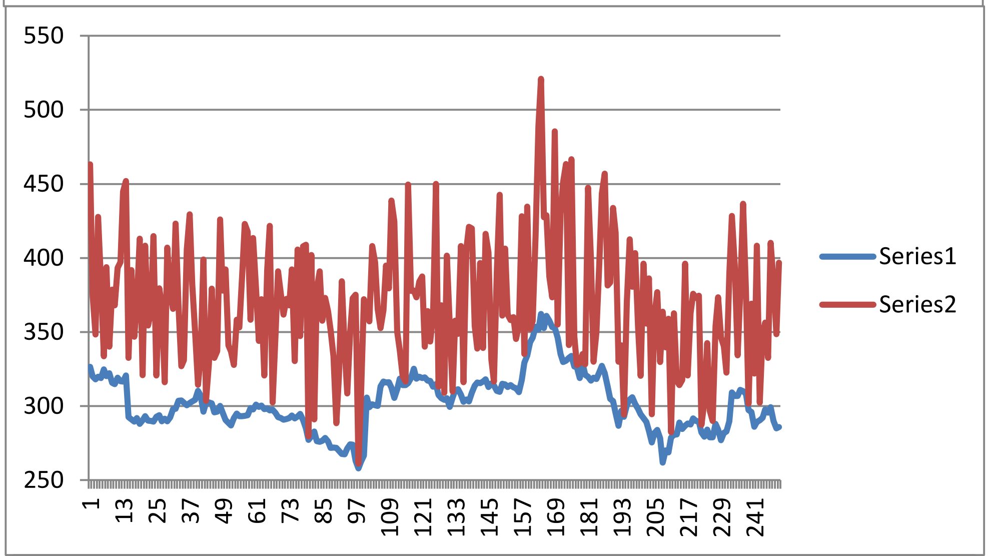
Series 1: Empirical time series   Series 2: Simulated Time series

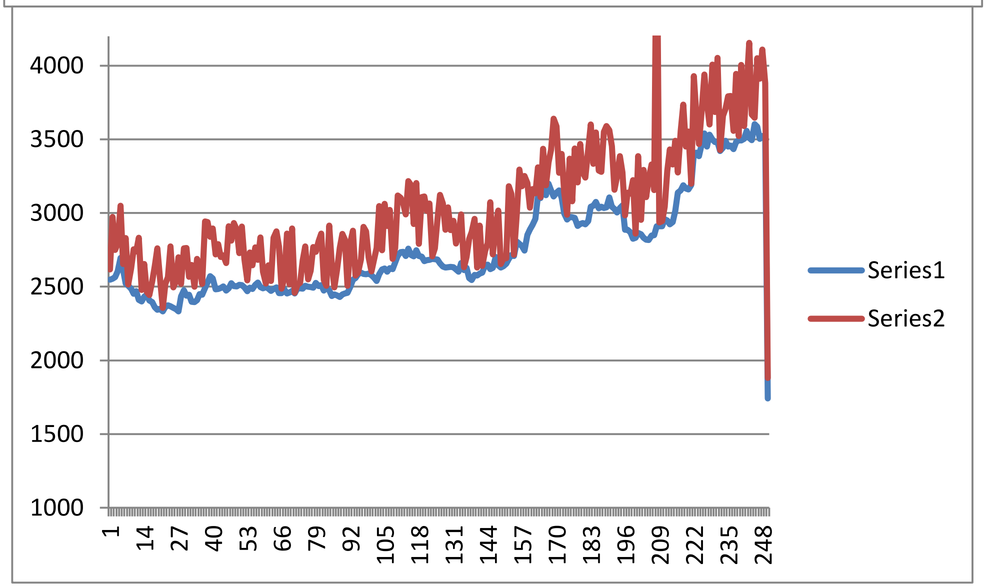
Figure (A): RIL data from June 1, 2017 to May 31, 2018

Figure (B): ICICI Bank data from June 1, 2017 to May 31, 2018

Figure (C): TCS data from June 1, 2017 to May 31, 2018







4. Conclusion

This model has analysed Two asset pricing models. One is the Brownian motion model with adaptive parameters called BMAP and another one is the fractional Brownian motion model with adaptive parameters called FBMAP. The Mean and volatility of rate of return in both models are adaptive at any time. The driven force in the BMAP is Brownian motion, while the driven process in the FBMAP is a fractional Brownian motion. The BMAP and the FBMAP are applied to simulate the RIL, TCS, and ICICI bank empirical stock prices.The simulated prices from both models are compared with the empirical prices and the chart is includedin the paper as Figure4 (A),(B),(C) respectively. The accuracy index ARPE is calculated for both models and the comparative study is shown in Table 1. Table 1 concludes that the standard deviation is very low for simulated data for FBMAP in comparison of BMAP. The less standard deviation shows that FBMAP model values are near to actual values. If we use FBMAP instead of BMAP we will get more accurate results. The lowest standard deviation is of Reliance therefore FBMAP is best suitable model for Reliance.

References

[1] Wiener, Norbert (1923), Differential space, Journal of Mathematical Physics 2, 131-17

[2] B. B. Mandelbrot and J. W. van Ness, “Fractional Brownian motions, fractional noises and applications,” SIAM Review, vol. 10, pp. 422–437, 1968.

[3] E. Alos, O. Mazet, and D. Nualart, “Stochastic calculus with ` respect to fractional Brownian motion with Hurst parameter lesser than 1/2,” Stochastic Processes and their Applications, vol. 86, no. 1, pp. 121–139, 2000.

[4] T. H. Thao, “An approximate approach to fractional analysis for finance,” Nonlinear Analysis: Real World Applications, vol. 7, no. 1, pp. 124–132, 2006.

[5] P. Wilmott, Paul Wilmott on Quantitative Finance, John Wiley & Sons, Chichester, UK, 2006. [6] R. Seydel, Tools for Computational Finance, Springer, Berlin, Germany, 2002.

[6] T. H. Thao and T. T. Nguyen, “Fractal Langevin equation,” Vietnam Journal of Mathematics, vol. 30, no. 1, pp. 89–96, 2002.