

Augmentation on Alternate Quadra Submerging Polar Fuzzy Graph and its Application

Anthoni Amali A¹ and J. Jesintha Rosline²

PG and Research Department of Mathematics, Auxilium College (Autonomous), Vellore – 632006, Affiliated to Thiruvalluvar University, Serkkadu, Tamil Nadu.

Abstract: - In this research artifact, we introduce the new fuzzy graph module Alternate Quadra Sub -merging Polar (AQSP) Fuzzy Graph which could be used to find the solution of uncertain subjects and occurrences. We present the sufficient conditions for each of the Augmentation to be precise and reliable. And we show that if any of these Augmentation is complete, then at least one of the factors is strong. We introduce some combinatoric operations of Augmentation of AQSP fuzzy graphs. The expansion of the AQSP Fuzzy graph with Direct, Strong, Tensor, Normal, Modular and Homomorphic Augmentations analysis is presented with the membership values and submerging level of confidence such as, $[-0.5, 0.5] \subset [-1, 1]$. The AQSP fuzzy graph Application paves the way to find the destination of persons Max- Min Self-esteem in the status of conflict feelings. The AQSP fuzzy graph module can be applied in the field of Human Science, Mathematical Psychology, Environmental Science and Medical Science.

Keywords: AQSP fuzzy graph, Direct, Strong, Tensor, Normal, Modular Augmentations.

1. Introduction

Knowledge is an unending adventure at the edge of uncertainty. This insight learning of uncertainty has innovated the Fuzzy set theory by Prof. Lotfi. A. Zadeh [1-2] in 1965 as a model of an implement to study uncertainty. The uncertain imprecision raised due to submerge conflict evolution which has an impact in determining the fuzzy level fixed membership grades alternatively. The main proposal of graph theory is invented with the problem of Konigsberg bridges and its application imprecise resolution by Mathematician Euler in 1736. In 1975 Rosenfeld [3-4] introduced the methods of fuzzy graphs and its logic with the qualitative data modeling of complex fuzzy graph relationship and systems of non - random uncertainty. But the first definition of fuzzy graph which is different from ordinary graph is defined by Kauffman[5] in 1973. Some remarks on fuzzy graphs explored by Bhattacharya[6]. K.T. Atanassov [7], Parvathi[8] presented Intuitionistic and interval valued fuzzy sets and graphs. Raashmanlou [9-10] investigated categorical properties in intuitionistic fuzzy graphs. On automorphism on fuzzy graphs, strong arcs in fuzzy graphs are explored by K.R. Bhutani, [11-12].

Currently, fuzzy graphs are used in many uncertain connected fields including Communication systems, Medical and Engineering Science and Network systems. Moderson, [13] Peng [14], and Nair discovered different types of operation on fuzzy graph, cycles and cocycles of fuzzy graph and fuzzy hypergraphs. M.S. Sunitha [15-16] Mathew[17] and Vijayakumar initiated different types of arcs, bridges, cut nodes, and complement of fuzzy graphs. In 1994, Zhang[18] originated the conception of bipolar fuzzy sets, bipolar valued fuzzy sets, as an extension of fuzzy sets with the membership degree of optimistic and undesirable restraints values lies within $[-1, 1]$. A. Nagoorgani, [19] K. Radha, and Malarvizhi examined on regular and irregular fuzzy graphs and domination in fuzzy graphs. Yang

[20], and Xu [21] proposed Pythagorean fuzzy sets. M.Akram,[22-23] W.A.Dudek, S.Samanta,[24-25] M.Pal presented bipolar fuzzy graphs, certain types of interval valued fuzzy graphs,m- polar fuzzy graphs and fuzzy line graphs and Pythagorean fuzzy graphs with applications. Poulik and Ghorai [26] introduced many indices in bipolar fuzzy graphs and explained their properties with applications.

Many real-life applications on fuzzy sets and fuzzy graphs such as Physical Sciences, Biological Sciences, Mathematical Sciences, genetic studies, image segmentations, data mining, airline routines, planning projects, and many more insights are developed in fuzzy field. Edification on fuzzy graph operations and possessions of Double layered fuzzy graphs and hesitant fuzzy graphs presented by T. Pathinathan [27], J. Jesintha Roseline [28]. The bipolar fuzzy graph framework cemented the way to differentiate the realisms in intuitionistic and m-polar graphs. In this AQSP module we deliberate the different types of Augment fuzzy graphs which pave the way to find the destination of mind influence factors in psychological area of different emotional, intelligent quotients and conflict approaches and behaviors.

2. Preliminaries

Definition.2.1: Fuzzy Graphs [3]

Let V is a vacant set. A uncertain - graph is $G: (\sigma, \mu)$ where σ is subset of V , μ is a relation on σ , where $\sigma: V \rightarrow [0,1]$ and the edge set $\mu: V \times V \rightarrow [0,1]$, $\mu(x, y) \leq \min(\mu(x), \mu(y)) \forall x, y \in V$. The fundamental crisp graph of fluffy graph $G: (\sigma, \mu)$ is denoted as $\tilde{U}$ of vertices and $\mu^* = E \in V \times V$.

Definition.2.3: Complete fuzzy graph [11]

A fuzzy graph $G: (\sigma, \mu)$ where σ is a fuzzy subset of V , μ is a relation on σ , $\sigma: V \rightarrow [0,1]$ and $\mu: V \times V \rightarrow [0,1]$ such that $\mu(x, y) = \wedge(\mu(x), \mu(y)) \forall x, y \in V$ with satisfying membership degree constrain in a vertex set is called as complete fuzzy graph.

Definition.2.3: Strong fuzzy graph [11]

A fuzzy graph $G: (\sigma, \mu)$ where σ is a fuzzy subset of V , μ is a relation on σ , $\sigma: V \rightarrow [0,1]$ and $\mu: V \times V \rightarrow [0,1]$ such as $\mu(x, y) = \wedge(\mu(x), \mu(y)) \forall x, y \in V \times V$ with satisfying membership degree constrain in edge set is called as strong fuzzy graph.

Definition 2.4: Intersection of two graphs [14]

The intersection of graphs $G_1 \cap G_2$ is defined as $(\sigma_1 \cap \sigma_2, \mu_1 \cap \mu_2)$ of the crisp graph G_1^* and G_2^* as follows,

- (i) $(\sigma_1 \cap \sigma_2)(a) = \sigma_1(a)$ if $a \in V_1 \cap \bar{V}_2$
- (ii) $(\sigma_1 \cap \sigma_2)(a) = \sigma_2(a)$ if $a \in V_2 \cap \bar{V}_1$
- (iii) $(\mu_1 \cap \mu_2)(a, b) = \mu_1(a, b)$ if $a, b \in E_1 \cap \bar{E}_2$
- (iv) $(\mu_1 \cap \mu_2)(a, b) = \mu_2(a, b)$ if $a, b \in E_2 \cap \bar{E}_1$.

Definition 2.5: Cartesian product of fuzzy graph [14]

Let $G_1 = (\sigma_1, \mu_1)$ and $G_2 = (\sigma_2, \mu_2)$ be the fuzzy graph with primitive vertex sets, V_1 and V_2 , the edging sets E_1 and E_2 correspondingly. Then the tensor product of G_1 and G_2 is a pair of $(\sigma_1 \times \sigma_2, \mu_1 \times \mu_2)$ with underlying vertex set,

$V_1 \times V_2 = \{(\delta_1, \tau_1) : \delta_1 \in V_1 \text{ and } \tau_1 \in V_2\}$, $E_1 \times E_2 = \{(\delta_1, \tau_1)(\delta_2, \tau_2) : \delta_1 = \delta_2, (\tau_1, \tau_2) \in E_2 \text{ or } \delta_1, \delta_2 \in E_1, \tau_1 = \tau_2\}$ with $(\sigma_1 \times \sigma_2)(\delta_1, \tau_1) = \sigma_1(\delta_1) \cap \sigma_2(\tau_1)$, where $\delta_1 \in V_1, \tau_1 \in V_2, (\mu_1 \times \mu_2)(\delta_1, \tau_1)(\delta_2, \tau_2) = \sigma_1(\delta_1) \cap \mu_2(\delta_1, \tau_1)$, if $\delta_1 = \delta$ and $(\tau_1, \tau_2) \in E_2 = \mu_1(\delta_1, \tau_1) \cap \sigma_2(\tau_1)$, if $\delta_1, \delta_2 \in E_1$ and $\tau_1 = \tau_2$.

3. An Alternate Quadra – Submerging Polar Fuzzy Graph (AQSP)

An Alternate Quadra – Submerging Polar Fuzzy Graph (AQSP) $G = (\sigma_{AQSP}, \mu_{AQSP})$

be an AQSP fuzzy graph with $G^* = (\sigma_{AQSP}^*, \mu_{AQSP}^*)$ is given as

$V = \{\sigma^P(x), \sigma^N(x), \rho^P(x), \rho^N(x)\}$ which is the value of AQSP nodes along

with the associate values of edges are given as

$E = V \times V = (\mu^P x, \mu^P y), (\mu^N x, \mu^N y), (\gamma^P x, \gamma^P y), (\gamma^N x, \gamma^N y)$

where $\sigma_P = V \rightarrow [0, 1]$, $\sigma_N = V \rightarrow [-1, 0]$,

$\rho_P = d | 0.5, \sigma^P(x) |$ and $\rho_N = -d | -0.5, \sigma^N(x) |$.

And it satisfies the following conditions which is given as,

(i) $(\mu^P(x), (\mu^P(y))) \leq \min(\sigma^P(x), \sigma^P(y))$ (ii) $(\mu^N(x), (\mu^N(y))) \geq \max(\sigma^N(x), \sigma^N(y))$

(iii) $(\gamma^P(x), (\gamma^P(y))) \leq \min(\rho^P(x), \rho^P(y))$ (iv) $(\gamma^N(x), (\gamma^N(y))) \geq \max(\rho^N(x), \rho^N(y))$

By definition, $\mu_P = V \times V \rightarrow [0, 1] \times [1, 0]$, $\mu_N = V \times V \rightarrow [-1, 0] \times [0, -1]$ and the submerging mappings and non – membership such as,

$\gamma_P = V \times V \rightarrow [0, 0.5] \times [0.5, 0]$, $\gamma_N = V \times V \rightarrow [-0.5, 0] \times [0, -0.5]$, which denotes the impact of

the alternate quadrant polarized fuzzy mapping. Also, it implies the result,

$-1 \leq \sigma^P(x) + \sigma^N(x) \leq 1$ and $|\rho^P(x) + \rho^N(x)| \leq 1$ with AQSP fuzzy membership function constrain, $0 \leq \sigma^P(x) + \sigma^N(x) + |\rho^P(x) + \rho^N(x)| \leq 2$. such that the uncertain status of submerging presumption, transform into its precise consistent level with fixation mid - value 0.5, which implies that level of confidence 0.5 in an AQSP as the valuable membership of its position which is real and valid in the fuzzification. An illustration of AQSP fluffy graph is given in Figure.1.

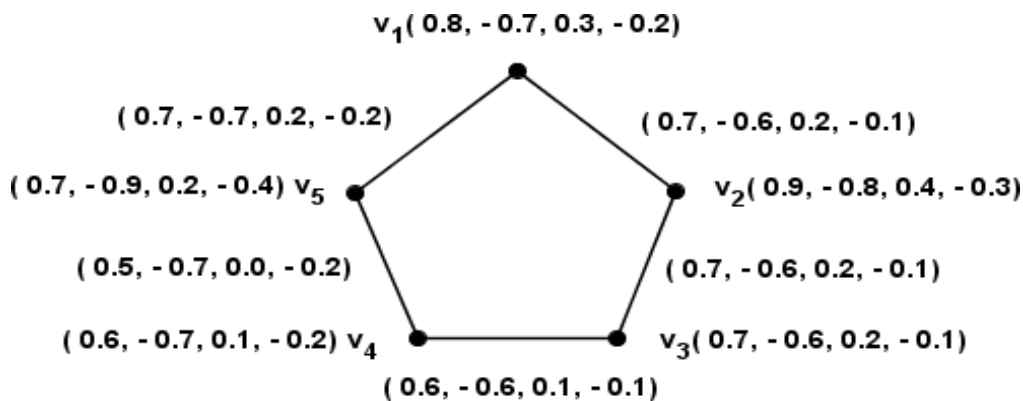


Figure. 1. AQSP Fuzzy Graph = $(\sigma_{AQSP}, \mu_{AQSP})$

4. Interface on Level of fixation and Submerging Polar Co – ordinates:

Consider a situation in which the value is uncertain. Suppose that the mid – values $\{[0,0], [0,0.5], [0.5,0.5], [0.5,0]\}$ and $\{[0,0], [-0.5,0], [-0.5, -0.5], [0, -0.5]\}$ is positive and negative membership of conflicts in human thoughts processes, then we accept that the uncertain value belongs to referential set of uncertainty alternate quadrant which is denoted as the Fuzzy set of $\{[0,0], [0,1], [1,1], [1,0]\}$ and $\{[0,0], [-1,0], [-1, -1], [0, -1]\}$. In many situations encountered by scientists and psychologists that it is possible to locate the value inside a level of fixation AQSP Fuzzy graph. This can be defined as $[-0.5,0] \subseteq [-1,0]$, $[-0.5,0.5] \subseteq [-1,1]$ and $[0,0.5] \subseteq [0,1]$, $[0.5,0.5] \subseteq [1,1]$ which shows that the transformation status of uncertain position to certain position of subjective and relative quantity.

Generally, the membership degree of elements $[-0.5,0.5] \subseteq [-1,1]$ are finite, then the AQSP Fuzzy graph is useful, or even necessary to consider that of maximum values $[-1,1]$. In other cases, instead of considering a closed interval, we consider open and closed interval by using the max – min values as $] - 0.5,0.5]$ or $(-0.5, 0.5]$: open at the left then we have the following as open at the right $[-0.5,0.5[$ or $[-0.5,0.5)$ and $] - 0.5,0.5[$ or $(-0.5,0.5)$ which is open at the left and at the right. Note that the membership values $x \in [-0.5,0.5] \subseteq [-1,1]$ denotes the fact that ‘x’ is an uncertain value. Alternate AQSP Fuzzy graph using sub – merging level of fixation satisfies the concrete decision of the human behavior. This fixed level of self-reliance in fuzzy graph specifies all the conceivable partition membership degree of attainment to certain level of destiny in alternate Quadra submerging polar distinguishing functions in fuzzy graph which is consistent. This obsession level will be an instrument to find the reliable level of presupposition with much information in AQSP Fuzzy graphs.

5. Augments on AQSP Fuzzy Graphs:

Definition 5.1: Direct Augment of AQSP Fuzzy Graphs

If $\sigma_1 = \{\sigma_1^P(x), \sigma_1^N(x), \rho_1^P(x), \rho_1^N(x)\}$ and $\sigma_2 = \{\sigma_2^P(x), \sigma_2^N(x), \rho_2^P(x), \rho_2^N(x)\}$ be two vertex sets of G_1 and G_2 . Let the edge sets of sets of G_1 and G_2 are given as,

$$\mu_1 = \{(\mu_1^P(\delta_1), \mu_1^P(\delta_2)), (\mu_1^N(\delta_1), \mu_1^N(\delta_2)), (\gamma_1^P(\delta_1), \gamma_1^P(\delta_2)), (\gamma_1^N(\delta_1), \gamma_1^N(\delta_2))\} \text{ and}$$

$$\mu_2 = \{(\mu_2^P(\tau_1), \mu_2^P(\tau_2)), (\mu_2^N(\tau_1), \mu_2^N(\tau_2)), (\gamma_2^P(\tau_1), \gamma_2^P(\tau_2)), (\gamma_2^N(\tau_1), \gamma_2^N(\tau_2))\}.$$

Let $G_1 = \{\varphi_1, (\sigma_1, \rho_1)^P, (\sigma_1, \rho_1)^N, (\mu_1, \gamma_1)^P, (\mu_1, \gamma_1)^N\}$ and $G_2 = \{\varphi_2, (\sigma_2, \rho_2)^P, (\sigma_2, \rho_2)^N, (\mu_2, \gamma_2)^P, (\mu_2, \gamma_2)^N\}$, be the AQSP fuzzy graphs, with the condition $\varphi = \varphi_1 \times \varphi_2$, where φ is non - empty set. The Direct Augment of two AQSP fuzzy graphs as $G_1 \times G_2$.

$$G_1 \times G_2 = \{\varphi, ((\sigma_1, \rho_1)^P \times (\sigma_2, \rho_2)^P), ((\sigma_1, \rho_1)^N \times (\sigma_2, \rho_2)^N), ((\mu_1, \gamma_1)^P \times (\mu_2, \gamma_2)^P), ((\mu_1, \gamma_1)^N \times (\mu_2, \gamma_2)^N)\}.$$

And the membership and non - membership values of the Direct Augment of AQSP fuzzy graphs vertex sets of $G_1 \times G_2$ are defined by using the Figure.2,

$$\sigma_1^P(\delta_1) \times \sigma_2^P(\tau_1) = \sigma_1^P(\delta_1) \wedge \sigma_2^P(\tau_1), \quad \rho_1^P(\delta_1) \times \rho_2^P(\tau_1) = \rho_1^P(\delta_1) \wedge \rho_2^P(\tau_1)$$

$$\sigma_1^N(\delta_1) \times \sigma_2^N(\tau_1) = \sigma_1^N(\delta_1) \vee \sigma_2^N(\tau_1), \quad \rho_1^N(\delta_1) \times \rho_2^N(\tau_1) = \rho_1^N(\delta_1) \vee \rho_2^N(\tau_1)$$

If $\delta_1, \delta_2 \in E_1$ and $\tau_1, \tau_2 \in E_2$ then the Direct Augment of AQSP fuzzy membership and non-membership values of edges are defined as,

$$\mu_1^P \times \mu_2^P((\delta_1, \tau_1), (\delta_2, \tau_2)) \leq \mu_1^P(\delta_1, \delta_2) \wedge \mu_2^P(\tau_1, \tau_2),$$

$$\gamma_1^P \times \gamma_2^P((\delta_1, \tau_1), (\delta_2, \tau_2)) \leq \gamma_1^P(\delta_1, \delta_2) \wedge \gamma_2^P(\tau_1, \tau_2),$$

$$\mu_1^N \times \mu_2^N((\delta_1, \tau_1), (\delta_2, \tau_2)) \geq \mu_1^N(\delta_1, \delta_2) \vee \mu_2^N(\tau_1, \tau_2),$$

$$\gamma_1^N \times \gamma_2^N ((\delta_1, \tau_1), (\delta_2, \tau_2)) \geq \gamma_1^N (\delta_1, \delta_2) \vee \gamma_2^N (\tau_1, \tau_2),$$

with the conditions, $(\mu, \gamma)_{(E_1 \times E_2)} = \{((\delta_1, \tau_1), (\delta_2, \tau_2)) : (\delta_1, \delta_2) \in E_1, (\tau_1, \tau_2) \in E_2, \delta_1 \in \varphi_1, \tau_1 \in \varphi_2\}$.

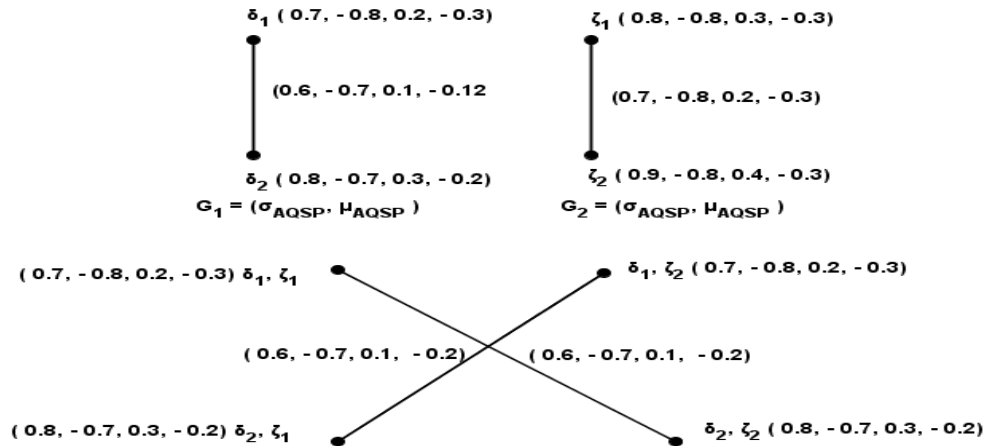


Figure. 2. $(G_1 \times G_2) = (\sigma_{AQSP}, \mu_{AQSP})$

Theorem 5.2: Let $G_1 = \{\varphi_1, (\sigma_1, \rho_1)^P, (\sigma_1, \rho_1)^N, (\mu_1, \gamma_1)^P, (\mu_1, \gamma_1)^N\}$ and

$G_2 = \{\varphi_2, (\sigma_2, \rho_2)^P, (\sigma_2, \rho_2)^N, (\mu_2, \gamma_2)^P, (\mu_2, \gamma_2)^N\}$ be the Direct AQSP Fuzzy graphs,

then we prove that $G_1 \times G_2$ is also a Direct Augment of AQSP Fuzzy graphs with the

necessary and sufficient condition $\rho^P(x) = d \mid 0.5, \sigma^P(u) \mid$ and $\rho^P(y) = d \mid 0.5, \sigma^P(v) \mid$.

Proof: Consider, the AQSP fuzzy graphs, $G_1 = \{\varphi_1, (\sigma_1, \rho_1)^P, (\sigma_1, \rho_1)^N, (\mu_1, \gamma_1)^P, (\mu_1, \gamma_1)^N\}$ and $G_2 = \{\varphi_2, (\sigma_2, \rho_2)^P, (\sigma_2, \rho_2)^N, (\mu_2, \gamma_2)^P, (\mu_2, \gamma_2)^N\}$ are Direct AQSP fuzzy graphs, then we prove the following result of AQSP fuzzy graphs. where $\delta_1 \in \varphi_1$ and $\tau_1 \in \varphi_2$.

$$\mu_1^P(\delta_1, \delta_2) = \sigma_1^P(\delta_1) \wedge \sigma_1^P(\delta_2),$$

$$\gamma_1^P(\delta_1, \delta_2) = \rho_1^P(\delta_1) \wedge \rho_1^P(\delta_2),$$

$$\mu_2^P(\tau_1, \tau_2) = \sigma_2^P(\tau_1) \wedge \sigma_2^P(\tau_2),$$

$$\gamma_2^P(\tau_1, \tau_2) = \rho_2^P(\tau_1) \wedge \rho_2^P(\tau_2),$$

similarly for non-membership values are given as,

$$\mu_1^N(\delta_1, \delta_2) = \sigma_1^N(\delta_1) \vee \sigma_1^N(\delta_2),$$

$$\gamma_1^N(\delta_1, \delta_2) = \rho_1^N(\delta_1) \vee \rho_1^N(\delta_2),$$

$$\mu_2^N(\tau_1, \tau_2) = \sigma_2^N(\tau_1) \vee \sigma_2^N(\tau_2),$$

$$\gamma_2^N(\tau_1, \tau_2) = \rho_2^N(\tau_1) \vee \rho_2^N(\tau_2),$$

For all the values of $b_1, b_2 \in E_1$ and $c_1, c_2 \in E_2$, now by the definition of Direct Augment AQSP fuzzy graph we have the following membership and non – membership vales,

$$\mu_1^P \times \mu_2^P ((\delta_1, \tau_1), (\delta_2, \tau_2)) \leq \mu_1^P (\delta_1, \delta_2) \wedge \mu_2^P (\tau_1, \tau_2),$$

$$\leq \sigma_1^P(\delta_1) \wedge \sigma_2^P(\tau_1) \wedge \sigma_1^P(\delta_2) \wedge \sigma_2^P(\tau_2),$$

$$\leq \sigma_1^P \times \sigma_2^P(\delta_1, \tau_1) \wedge \sigma_1^P \times \sigma_2^P(\delta_2, \tau_2),$$

for the submerging AQSP fuzzy graphs,

$$\gamma_1^P \times \gamma_2^P((\delta_1, \tau_1), (\delta_2, \tau_2)) \leq \gamma_1^P(\delta_1, \delta_2) \wedge \gamma_2^P(\tau_1, \tau_2),$$

$$\leq \rho_1^P(\delta_1) \wedge \rho_2^P(\tau_1) \wedge \rho_1^P(\delta_2) \wedge \rho_2^P(\tau_2),$$

$$\leq \rho_1^P \times \rho_2^P(\delta_1, \tau_1) \wedge \rho_1^P \times \rho_2^P(\delta_2, \tau_2),$$

Now by the definition of Direct Augment AQSP fuzzy graph we have the result of the non - membership values of $\delta_1, \delta_2 \in E_1$ and $\tau_1, \tau_2 \in E_2$.

$$\mu_1^N \times \mu_2^N((\delta_1, \tau_1), (\delta_2, \tau_2)) \geq \mu_1^N(\delta_1, \delta_2) \vee \mu_2^N(\tau_1, \tau_2),$$

$$\geq \sigma_1^N(\delta_1) \vee \sigma_2^N(\tau_1) \vee \sigma_1^N(\delta_2) \vee \sigma_2^N(\tau_2),$$

$$\geq \sigma_1^P \times \sigma_2^P(\delta_1, \tau_1) \vee \sigma_1^P \times \sigma_2^P(\delta_2, \tau_2),$$

$$\gamma_1^N \times \gamma_2^N((\delta_1, \tau_1), (\delta_2, \tau_2)) \geq \gamma_1^N(\delta_1, \delta_2) \vee \gamma_2^N(\tau_1, \tau_2),$$

$$\geq \rho_1^N(\delta_1) \vee \rho_2^N(\tau_1) \vee \rho_1^N(\delta_2) \vee \rho_2^N(\tau_2),$$

$$\geq \rho_1^P \times \rho_2^P(\delta_1, \tau_1) \vee \rho_1^P \times \rho_2^P(\delta_2, \tau_2),$$

Hence $G_1 \times G_2$ is also a direct Augment of AQSP fuzzy graph is proved.

Definition 5.3: Strong and complete Augment of AQSP Fuzzy Graphs

Let $\sigma_1 = \{\sigma_1^P(x), \sigma_1^N(x), \rho_1^P(x), \rho_1^N(x)\}$ and $\sigma_2 = \{\sigma_2^P(x), \sigma_2^N(x), \rho_2^P(x), \rho_2^N(x)\}$

be vertex sets of G_1 and G_2 . Let $\mu_1 = \{(\mu_1^P(\delta_1), \mu_1^N(\delta_1)), (\mu_1^P(\delta_2), \mu_1^N(\delta_2)), (\gamma_1^P(\delta_1), \gamma_1^N(\delta_1)), (\gamma_1^P(\delta_2), \gamma_1^N(\delta_2))\}$ and

$\mu_2 = \{(\mu_2^P(\tau_1), \mu_2^N(\tau_1)), (\mu_2^P(\tau_2), \mu_2^N(\tau_2)), (\gamma_2^P(\tau_1), \gamma_2^N(\tau_1)), (\gamma_2^P(\tau_2), \gamma_2^N(\tau_2))\}$ be the edge sets of G_1 and G_2 .

We denote the Strong and complete Augment of two AQSP fuzzy graphs as, $G_1 \bullet G_2$.

$G_1 = \{\varphi_1, (\sigma_1, \rho_1)^P, (\sigma_1, \rho_1)^N, (\mu_1, \gamma_1)^P, (\mu_1, \gamma_1)^N\}$ and $G_2 = \{\varphi_2, (\sigma_2, \rho_2)^P, (\sigma_2, \rho_2)^N, (\mu_2, \gamma_2)^P, (\mu_2, \gamma_2)^N\}$ be given with the condition $\varphi = \varphi_1 \times \varphi_2$, where φ is non - empty set. Then, we define.

$$G_1 \bullet G_2 = \{\varphi, ((\sigma_1, \rho_1)^P \bullet (\sigma_2, \rho_2)^P), ((\sigma_1, \rho_1)^N \bullet (\sigma_2, \rho_2)^N), ((\mu_1, \gamma_1)^P \bullet (\mu_2, \gamma_2)^P), ((\mu_1, \gamma_1)^N \bullet (\mu_2, \gamma_2)^N)\}$$

And the strong and complete AQSP fuzzy graphs edges are defined as,

$$(\mu, \gamma)_{(E_1 \bullet E_2)} = \{((\delta_1, \tau_1), (\delta_2, \tau_2)) : \delta_1 \in \varphi_1, \tau_1, \tau_2 \in E_2\} \cup \{((\delta_1, \tau_1), (\delta_2, \tau_2)) : \delta_1, \delta_2 \in E_1, \tau_1 \in \varphi_2\} \cup \{((\delta_1, \tau_1), (\delta_2, \tau_2)) : \delta_1, \delta_2 \in E_1, \tau_1, \tau_2 \in E_2\}.$$

And the fuzzy association values of strong and complete AQSP fuzzy graphs vertices are given in the following results,

$$\sigma_1^P(\delta_1) \bullet \sigma_2^P(\tau_1) = \sigma_1^P(\delta_1) \wedge \sigma_2^P(\tau_1), \quad \rho_1^P(\delta_1) \bullet \rho_2^P(\tau_1) = \rho_1^P(\delta_1) \wedge \rho_2^P(\tau_1)$$

$$\sigma_1^N(\delta_1) \bullet \sigma_2^N(\tau_1) = \sigma_1^N(\delta_1) \vee \sigma_2^N(\tau_1), \quad \rho_1^N(\delta_1) \bullet \rho_2^N(\tau_1) = \rho_1^N(\delta_1) \vee \rho_2^N(\tau_1)$$

Where $\delta_1 \in \varphi_1$ and $\tau_1 \in \varphi_2$.

If $\delta_1, \delta_2 \in E_1$ and $\tau_1, \tau_2 \in E_2$ then the strong and complete Augment of AQSP fuzzy membership and non - membership values of edge sets are defined as below,

$$\mu_1^P \bullet \mu_2^P ((\delta_1, \tau_1), (\delta_2, \tau_2)) = \sigma_1^P(\delta_1) \wedge \mu_2^P(\tau_1, \tau_2),$$

$$\mu_1^P \bullet \mu_2^P ((\delta_1, \tau_1), (\delta_2, \tau_2)) = \mu_1^P(\delta_1, \delta_2) \wedge \mu_2^P(\tau_1, \tau_2),$$

Similarly for submerging AQSP membership value of edge is given as

$$\gamma_1^P \bullet \gamma_2^P ((\delta_1, \tau_1), (\delta_2, \tau_2)) = \rho_1^P(\delta_1) \wedge \gamma_2^P(\tau_1, \tau_2),$$

$$\gamma_1^P \bullet \gamma_2^P ((\delta_1, \tau_1), (\delta_2, \tau_2)) = \gamma_1^P(\delta_1, \delta_2) \wedge \gamma_2^P(\tau_1, \tau_2),$$

similarly for non-membership value of edge is defined as in the following,

$$\mu_1^N \bullet \mu_2^N ((\delta_1, \tau_1), (\delta_2, \tau_2)) = \sigma_1^N(\delta_1) \vee \mu_2^N(\tau_1, \tau_2),$$

$$\mu_1^N \bullet \mu_2^N ((\delta_1, \tau_1), (\delta_2, \tau_2)) = \mu_1^N(\delta_1, \delta_2) \vee \mu_2^N(\tau_1, \tau_2),$$

Similarly for submerging AQSP non - membership value of edge is given as

$$\gamma_1^N \bullet \gamma_2^N ((\delta_1, \tau_1), (\delta_2, \tau_2)) = \rho_1^N(\delta_1) \wedge \gamma_2^N(\tau_1, \tau_2),$$

$$\gamma_1^N \bullet \gamma_2^N ((\delta_1, \tau_1), (\delta_2, \tau_2)) = \gamma_1^N(\delta_1, \delta_2) \vee \gamma_2^N(\tau_1, \tau_2),$$

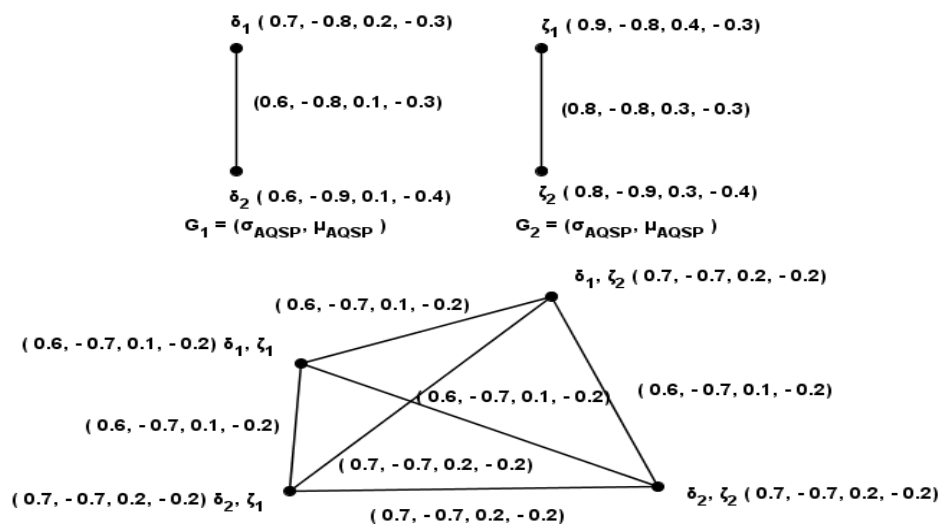


Figure. 3. $(G_1 \bullet G_2) = (\sigma_{AQSP}, \mu_{AQSP})$

Theorem 5.4: Let $G_1 = \{\varphi_1, (\sigma_1, \rho_1)^P, (\sigma_1, \rho_1)^N, (\mu_1, \gamma_1)^P, (\mu_1, \gamma_1)^N\}$ and $G_2 = \{\varphi_2, (\sigma_2, \rho_2)^P, (\sigma_2, \rho_2)^N, (\mu_2, \gamma_2)^P, (\mu_2, \gamma_2)^N\}$ be the strong and complete Augment of AQSP fuzzy graphs. Then we prove that $G_1 \bullet G_2$ is also strong and complete AQSP fuzzy graph with the condition.

$$G_1 \bullet G_2 = \{((\sigma_1, \rho_1)^P, (\sigma_1, \rho_1)^N) \bullet ((\sigma_2, \rho_2)^P, (\sigma_2, \rho_2)^N), ((\mu_1, \gamma_1)^P, (\mu_1, \gamma_1)^N) \bullet ((\mu_2, \gamma_2)^P, (\mu_2, \gamma_2)^N)\}.$$

Proof: The strong and complete augment of AQSP fuzzy graph proves that its every single pair of nodes are neighboring. If $((\delta_1, \tau_1), (\delta_2, \tau_2)) \in E$ then the strong and complete Augment of AQSP fuzzy edges with the associate values are defined as,

$$\mu_1^P \bullet \mu_2^P ((\delta_1, \tau_1), (\delta_2, \tau_2)) = \sigma_1^P(\delta_1) \wedge \mu_2^P(\tau_1, \tau_2),$$

$$= \sigma_1^P(\delta_1) \wedge \sigma_2^P(\tau_1) \wedge \sigma_1^P(\delta_1) \wedge \sigma_2^P(\tau_2),$$

$$= \sigma_1^P \bullet \sigma_2^P(\delta_1, \tau_1) \wedge \sigma_1^P \bullet \sigma_2^P(\delta_1, \tau_2),$$

for submerging AQSP fuzzy graphs,

$$\begin{aligned}\gamma_1^P \bullet \gamma_2^P ((\delta_1, \tau_1), (\delta_2, \tau_2)) &= \rho_1^P(\delta_1) \wedge \gamma_2^P(\tau_1, \tau_2), \\ &= \rho_1^P(\delta_1) \wedge \rho_2^P(\tau_1) \wedge \rho_1^P(\delta_1) \wedge \rho_2^P(\tau_2), \\ &= \rho_1^P \bullet \rho_2^P(\delta_1, \tau_1) \wedge \rho_1^P \bullet \rho_2^P(\delta_1, \tau_2),\end{aligned}$$

Now for non – membership values we have,

$$\begin{aligned}\mu_1^N \bullet \mu_2^N ((\delta_1, \tau_1), (\delta_2, \tau_2)) &= \sigma_1^N(\delta_1) \wedge \mu_2^N(\tau_1, \tau_2), \\ &= \sigma_1^N(\delta_1) \wedge \sigma_2^N(\tau_1) \wedge \sigma_1^N(\delta_1) \wedge \sigma_2^N(\tau_2), \\ &= \sigma_1^N \bullet \sigma_2^N(\delta_1, \tau_1) \wedge \sigma_1^N \bullet \sigma_2^N(\delta_1, \tau_2),\end{aligned}$$

for submerging AQSP fuzzy graphs,

$$\begin{aligned}\gamma_1^N \bullet \gamma_2^N ((\delta_1, \tau_1), (\delta_2, \tau_2)) &= \sigma_1^N(\delta_1) \vee \gamma_2^N(\tau_1, \tau_2), \\ &= \rho_1^N(\delta_1) \vee \rho_2^N(\tau_1) \vee \rho_1^N(\delta_1) \vee \rho_2^N(\tau_2), \\ &= \rho_1^N \bullet \rho_2^N(\delta_1, \tau_1) \vee \rho_1^N \bullet \rho_2^N(\delta_1, \tau_2),\end{aligned}$$

If $(\delta_1, \tau_1), (\delta_2, \tau_1)) \in E$

$$\begin{aligned}\mu_1^P \bullet \mu_2^P ((\delta_1, \tau_1), (\delta_2, \tau_1)) &= \mu_1^P(\delta_1, \delta_2) \wedge \sigma_2^P(\tau_1), \\ &= \sigma_1^P(\delta_1) \wedge \sigma_2^P(\tau_1) \wedge \sigma_1^P(\delta_2) \wedge \sigma_2^P(\tau_1), \\ &= \sigma_1^P \bullet \sigma_2^P(\delta_1, \tau_1) \wedge \sigma_1^P \bullet \sigma_2^P(\delta_2, \tau_1),\end{aligned}$$

similarly, for submerging AQSP we have,

$$\begin{aligned}\gamma_1^P \bullet \gamma_2^P ((\delta_1, \tau_1), (\delta_2, \tau_1)) &= \gamma_1^P(\delta_1, \delta_2) \wedge \rho_2^P(\tau_1), \\ &= \rho_1^P(\delta_1) \wedge \rho_2^P(\tau_1) \wedge \rho_1^P(\delta_2) \wedge \rho_2^P(\tau_1), \\ &= \rho_1^P \bullet \rho_2^P(\delta_1, \tau_1) \wedge \rho_1^P \bullet \rho_2^P(\delta_2, \tau_1),\end{aligned}$$

If $(\delta_1, \tau_1), (\delta_2, \tau_1)) \in E$, then the non – membership values are given as

$$\begin{aligned}\mu_1^N \bullet \mu_2^N ((\delta_1, \tau_1), (\delta_2, \tau_1)) &= \mu_1^N(\delta_1, \delta_2) \vee \sigma_2^N(\tau_1), \\ &= \sigma_1^N(\delta_1) \vee \sigma_2^N(\tau_1) \vee \sigma_1^N(\delta_2) \vee \sigma_2^N(\tau_1), \\ &= \sigma_1^N \bullet \sigma_2^N(\delta_1, \tau_1) \vee \sigma_1^N \bullet \sigma_2^N(\delta_2, \tau_1),\end{aligned}$$

for submerging AQSP fuzzy graphs,

$$\begin{aligned}\gamma_1^N \bullet \gamma_2^N ((\delta_1, \tau_1), (\delta_2, \tau_1)) &= \gamma_1^N(\delta_1, \delta_2) \vee \rho_2^N(\tau_1), \\ &= \rho_1^N(\delta_1) \vee \rho_2^N(\tau_1) \vee \rho_1^N(\delta_2) \vee \rho_2^N(\tau_1), \\ &= \rho_1^N \bullet \rho_2^N(\delta_1, \tau_1) \vee \rho_1^N \bullet \rho_2^N(\delta_2, \tau_1),\end{aligned}$$

If $((\delta_1, \tau_1), (\delta_2, \tau_2)) \in E$, then

$$\begin{aligned}\mu_1^P \bullet \mu_2^P ((\delta_1, \tau_1), (\delta_2, \tau_2)) &= \mu_1^P(\delta_1, \delta_2) \wedge \mu_2^P(\tau_1, \tau_2), \\ &= \sigma_1^P(\delta_1) \wedge \sigma_1^P(\tau_1) \wedge \sigma_2^P(\delta_2) \wedge \sigma_2^P(\tau_2),\end{aligned}$$

$$= \sigma_1^P \bullet \sigma_2^P(\delta_1, \tau_1) \wedge \sigma_1^P \bullet \sigma_2^P(\delta_2, \tau_2),$$

For the non – membership values are given as

$$\mu_1^N \bullet \mu_2^N((\delta_1, \tau_1), (\delta_2, \tau_2)) = \mu_1^N(\delta_1, \delta_2) \vee \mu_2^N(\tau_1, \tau_2),$$

$$= \sigma_1^N(\delta_1) \vee \sigma_1^N(\tau_1) \vee \sigma_2^N(\delta_2) \vee \sigma_2^N(\tau_2),$$

$$= \sigma_1^N \bullet \sigma_2^N(\delta_1, \tau_1) \vee \sigma_1^N \bullet \sigma_2^N(\delta_2, \tau_2),$$

for submerging AQSP fuzzy graphs we have,

$$\gamma_1^P \bullet \gamma_2^P((\delta_1, \tau_1), (\delta_2, \tau_2)) = \gamma_1^P(\delta_1, \delta_2) \wedge \gamma_2^P(\tau_1, \tau_2),$$

$$= \rho_1^P(\delta_1) \wedge \rho_1^P(\tau_1) \wedge \rho_2^P(\delta_2) \wedge \rho_2^P(\tau_2),$$

$$= \rho_1^P \bullet \rho_2^P(\delta_1, \tau_1) \wedge \rho_1^P \bullet \rho_2^P(\delta_2, \tau_2),$$

and the non – membership values are given as for submerging AQSP fuzzy graphs we have,

$$\gamma_1^N \bullet \gamma_2^N((\delta_1, \tau_1), (\delta_2, \tau_2)) = \gamma_1^N(\delta_1, \delta_2) \vee \gamma_2^N(\tau_1, \tau_2),$$

$$= \rho_1^N(\delta_1) \vee \rho_1^N(\tau_1) \vee \rho_2^N(\delta_2) \vee \rho_2^N(\tau_2),$$

$$= \rho_1^N \bullet \rho_2^N(\delta_1, \tau_1) \vee \rho_1^N \bullet \rho_2^N(\delta_2, \tau_2),$$

Hence $G_1 \bullet G_2$ is strong and complete Augment in AQSP fuzzy graph is proved by using Figure.3.

Definition 5.5: Tensor Strictly Strong Augment of AQSP Fuzzy Graphs

Let $\sigma_1 = \{\sigma_1^P(x), \sigma_1^N(x), \rho_1^P(x), \rho_1^N(x)\}$ and $\sigma_2 = \{\sigma_2^P(x), \sigma_2^N(x), \rho_2^P(x), \rho_2^N(x)\}$ be two AQSP fuzzy vertex sets of G_1 and G_2 . Let $\mu_1 = \{(\mu_1^P(\delta_1), \mu_1^P(\delta_2)), (\mu_1^N(\delta_1), \mu_1^N(\delta_2)), (\gamma_1^P(\delta_1), \gamma_1^P(\delta_2)), (\gamma_1^N(\delta_1), \gamma_1^N(\delta_2))\}$ and

$$\mu_2 = \{(\mu_2^P(\tau_1), \mu_2^P(\tau_2)), (\mu_2^N(\tau_1), \mu_2^N(\tau_2)), (\gamma_2^P(\tau_1), \gamma_2^P(\tau_2)), (\gamma_2^N(\tau_1), \gamma_2^N(\tau_2))\}$$

be the two edge sets of G_1 and G_2 . We denote the Tensor Strictly Strong Augment of two AQSP fuzzy graphs as

$G_1 \otimes G_2$. The AQSP fuzzy graphs be considered as $G_1 = \{\varphi_1, (\sigma_1, \rho_1)^P, (\sigma_1, \rho_1)^N, (\mu_1, \gamma_1)^P, (\mu_1, \gamma_1)^N\}$ and $G_2 = \{\varphi_2, (\sigma_2, \rho_2)^P, (\sigma_2, \rho_2)^N, (\mu_2, \gamma_2)^P, (\mu_2, \gamma_2)^N\}$ with the condition $\varphi = \varphi_1 \otimes \varphi_2$, we define the Tensor Strictly Strong Augment of AQSP fuzzy graphs such as,

$$G_1 \otimes G_2 = \{\varphi, ((\sigma_1, \rho_1)^P \otimes (\sigma_2, \rho_2)^P), ((\sigma_1, \rho_1)^N \otimes (\sigma_2, \rho_2)^N), ((\mu_1, \gamma_1)^P \otimes (\mu_2, \gamma_2)^P), ((\mu_1, \gamma_1)^N \otimes (\mu_2, \gamma_2)^N)\}.$$

And the associate values of the Tensor Strictly

Strong Augment in AQSP fuzzy vertex set is defined by using the Figure. 4.

$$\sigma_1^P(\delta_1) \otimes \sigma_2^P(\tau_1) = \sigma_1^P(\delta_1) \wedge \sigma_2^P(\tau_1), \quad \rho_1^P(\delta_1) \otimes \rho_2^P(\tau_1) = \rho_1^P(\delta_1) \wedge \rho_2^P(\tau_1)$$

$$\sigma_1^N(\delta_1) \otimes \sigma_2^N(\tau_1) = \sigma_1^N(\delta_1) \vee \sigma_2^N(\tau_1), \quad \rho_1^N(\delta_1) \otimes \rho_2^N(\tau_1) = \rho_1^N(\delta_1) \vee \rho_2^N(\tau_1)$$

Where $\delta_1 \in \varphi_1$ and $\tau_1 \in \varphi_2$.

If $\delta_1, \delta_2 \in E_1$ and $\tau_1, \tau_2 \in E_2$ then the Tensor Strictly Strong Augment of AQSP fuzzy membership

and non - membership values of edges are defined as,

$$\mu_1^P \otimes \mu_2^P((\delta_1, \tau_1), (\delta_2, \tau_2)) = \mu_1^P(\delta_1, \delta_2) \wedge \mu_2^P(\tau_1, \tau_2),$$

$$\gamma_1^P \otimes \gamma_2^P((\delta_1, \tau_1), (\delta_2, \tau_2)) = \gamma_1^P(\delta_1, \delta_2) \wedge \gamma_2^P(\tau_1, \tau_2),$$

$$\mu_1^N \otimes \mu_2^N ((\delta_1, \tau_1), (\delta_2, \tau_2)) = \mu_1^N (\delta_1, \delta_2) \vee \mu_2^N (\tau_1, \tau_2),$$

$$\gamma_1^N \otimes \gamma_2^N ((\delta_1, \tau_1), (\delta_2, \tau_2)) = \gamma_1^N (\delta_1, \delta_2) \vee \gamma_2^N (\tau_1, \tau_2).$$

with the constraints $(\mu, \gamma)_{(E_1 \otimes E_2)} = \{((\delta_1, \tau_1), (\delta_2, \tau_2)) : (\delta_1, \delta_2) \in E_1, (\tau_1, \tau_2) \in E_2\}$

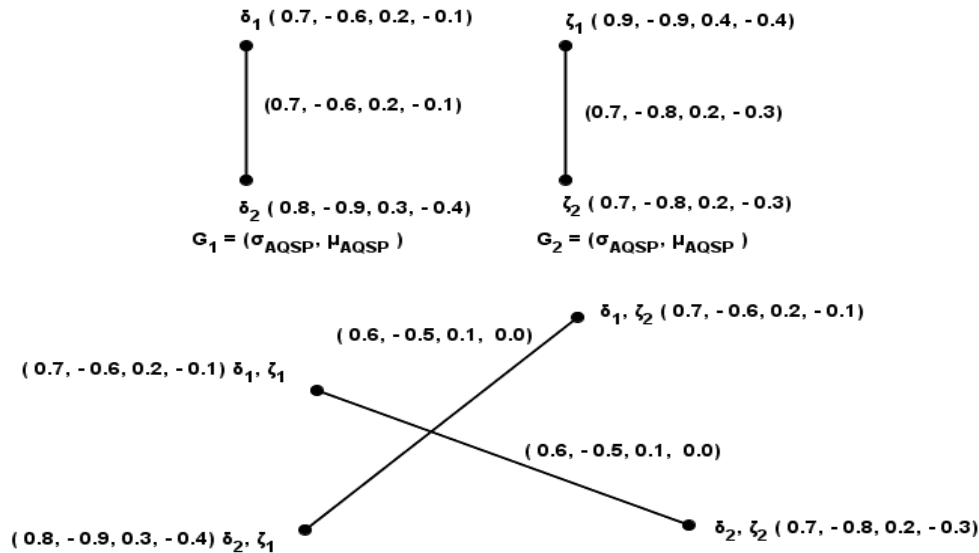


Figure. 4 $(G_1 \otimes G_2) = (\sigma_{AQSP}, \mu_{AQSP})$

Definition 5.6: Normal Augment of AQSP Fuzzy Graphs

Let $\sigma_1 = \{\sigma_1^P(x), \sigma_1^N(x), \rho_1^P(x), \rho_1^N(x)\}$ and $\sigma_2 = \{\sigma_2^P(x), \sigma_2^N(x), \rho_2^P(x), \rho_2^N(x)\}$ be two AQSP fuzzy vertex sets of G_1 and G_2 . Let $\mu_1 = \{(\mu_1^P(\delta_1), \mu_1^P(\delta_2)), (\mu_1^N(\delta_1), \mu_1^N(\delta_2)), (\gamma_1^P(\delta_1), \gamma_1^P(\delta_2)), (\gamma_1^N(\delta_1), \gamma_1^N(\delta_2))\}$ and

$\mu_2 = \{(\mu_2^P(\tau_1), \mu_2^P(\tau_2)), (\mu_2^N(\tau_1), \mu_2^N(\tau_2)), (\gamma_2^P(\tau_1), \gamma_2^P(\tau_2)), (\gamma_2^N(\tau_1), \gamma_2^N(\tau_2))\}$ be the two edge sets of G_1 and G_2 . We denote the Normal Augment of two AQSP fuzzy graphs as $G_1 * G_2$. The AQSP fuzzy graphs,

$$G_1 = \{\varphi_1, (\sigma_1, \rho_1)^P, (\sigma_1, \rho_1)^N, (\mu_1, \gamma_1)^P, (\mu_1, \gamma_1)^N\} \text{ and } G_2 = \{\varphi_2, (\sigma_2, \rho_2)^P, (\sigma_2, \rho_2)^N, (\mu_2, \gamma_2)^P, (\mu_2, \gamma_2)^N\}.$$

we define the Normal Augment of AQSP fuzzy graphs with the condition $\varphi = \varphi_1 * \varphi_2$, such as,

$$G_1 * G_2 = \{\varphi, ((\sigma_1, \rho_1)^P * (\sigma_2, \rho_2)^P), ((\sigma_1, \rho_1)^N * (\sigma_2, \rho_2)^N), ((\mu_1, \gamma_1)^P * (\mu_2, \gamma_2)^P), ((\mu_1, \gamma_1)^N * (\mu_2, \gamma_2)^N)\}.$$

And the membership, non - membership values of the Normal Augment of AQSP edge set is defined with the condition by using the Figure.5,

$$(\mu, \gamma)_{(E_1 * E_2)} = \{((\delta_1, \tau_1), (\delta_2, \tau_2)) : \delta_1 = \delta_2, \tau_1, \tau_2 \in E_2, \delta_1, \delta_2 \in E_1, \tau_1 = \tau_2 \text{ or } \delta_2 \in E_1,$$

$$\tau_1 = \tau_2 \text{ or } \delta_1, \delta_2 \in E_1, \tau_1, \tau_2 \in E_2\},$$

The membership, non - membership values of the Normal Augment of AQSP fuzzy graph vertex set is defined in the following results,

$$\sigma_1^P(\delta_1) * \sigma_2^P(\tau_1) = \sigma_1^P(\delta_1) \wedge \sigma_2^P(\tau_1), \quad \rho_1^P(\delta_1) * \rho_2^P(\tau_1) = \rho_1^P(\delta_1) \wedge \rho_2^P(\tau_1)$$

$$\sigma_1^N(\delta_1) * \sigma_2^N(\tau_1) = \sigma_1^N(\delta_1) \vee \sigma_2^N(\tau_1), \quad \rho_1^N(\delta_1) * \rho_2^N(\tau_1) = \rho_1^N(\delta_1) \vee \rho_2^N(\tau_1)$$

Where $\delta_1 \in \varphi_1$ and $\tau_1 \in \varphi_2$.

If $\delta_1 = \delta_2$ and $\tau_1, \tau_2 \in E_2$ then we have

$$\mu_1^P * \mu_2^P((\delta_1, \tau_1), (\delta_2, \tau_2)) = \sigma_1^P(\delta_1) \wedge \mu_2^P(\tau_1, \tau_2),$$

$$\gamma_1^P * \gamma_2^P((\delta_1, \tau_1), (\delta_2, \tau_2)) = \rho_1^P(\delta_1) \wedge \gamma_2^P(\tau_1, \tau_2),$$

$$\mu_1^N * \mu_2^N((\delta_1, \tau_1), (\delta_2, \tau_2)) = \sigma_1^N(\delta_1) \vee \mu_2^N(\tau_1, \tau_2),$$

$$\gamma_1^N * \gamma_2^N((\delta_1, \tau_1), (\delta_2, \tau_2)) = \rho_1^N(\delta_1) \vee \gamma_2^N(\tau_1, \tau_2),$$

If $\tau_1 = \tau_2$ and $\delta_1, \delta_2 \in E_1$, then we have

$$\mu_1^P * \mu_2^P((\delta_1, \tau_1), (\delta_2, \tau_2)) = \mu_1^P(\delta_1, \delta_2) \wedge \sigma_2^P(\tau_1),$$

$$\mu_1^N * \mu_2^N((\delta_1, \tau_1), (\delta_2, \tau_2)) = \mu_1^N(\delta_1, \delta_2) \vee \sigma_2^N(\tau_1),$$

$$\gamma_1^P * \gamma_2^P((\delta_1, \tau_1), (\delta_2, \tau_2)) = \gamma_1^P(\delta_1, \delta_2) \wedge \rho_2^P(\tau_1),$$

$$\gamma_1^N * \gamma_2^N((\delta_1, \tau_1), (\delta_2, \tau_2)) = \gamma_1^N(\delta_1, \delta_2) \vee \rho_2^N(\tau_1), \text{ Where } \delta_1 \in \varphi_1 \text{ and } \tau_1 \in \varphi_2.$$

If $\delta_1, \delta_2 \in E_1$ and $\tau_1, \tau_2 \in E_2$, then we have

$$\mu_1^P * \mu_2^P((\delta_1, \tau_1), (\delta_2, \tau_2)) = \mu_1^P(\delta_1, \delta_2) \wedge \mu_2^P(\tau_1, \tau_2),$$

$$\mu_1^N * \mu_2^N((\delta_1, \tau_1), (\delta_2, \tau_2)) = \mu_1^N(\delta_1, \delta_2) \vee \mu_2^N(\tau_1, \tau_2),$$

$$\gamma_1^P * \gamma_2^P((\delta_1, \tau_1), (\delta_2, \tau_2)) = \gamma_1^P(\delta_1, \delta_2) \wedge \gamma_2^P(\tau_1, \tau_2),$$

$$\gamma_1^N * \gamma_2^N((\delta_1, \tau_1), (\delta_2, \tau_2)) = \gamma_1^N(\delta_1, \delta_2) \vee \gamma_2^N(\tau_1, \tau_2), \text{ Where } \delta_1 \in \varphi_1 \text{ and } \tau_1 \in \varphi_2.$$

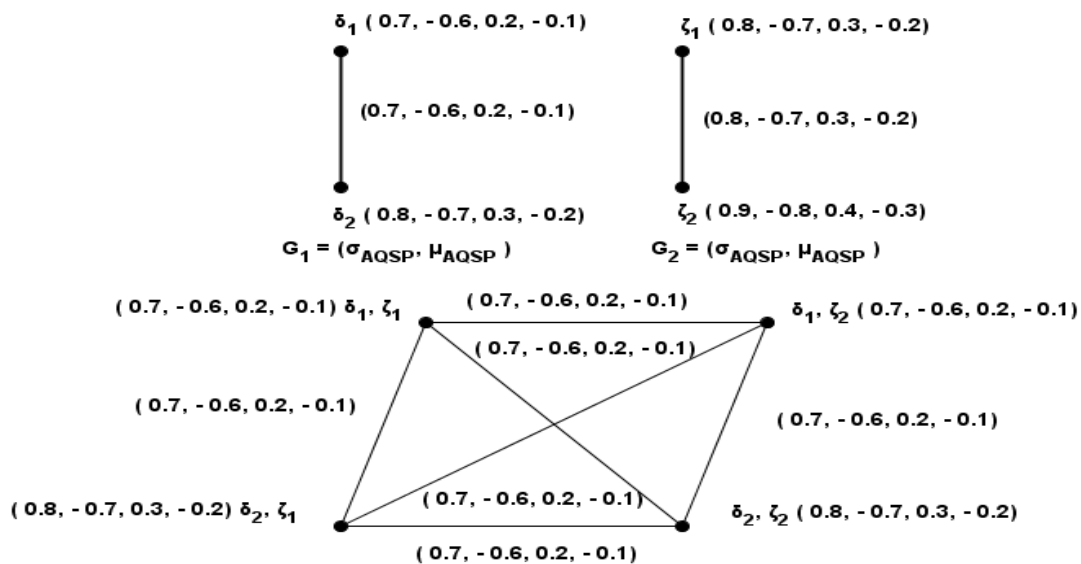


Figure. 5. $(G_1 * G_2) = (\sigma_{AQSP}, \mu_{AQSP})$

Definition 5.7: Modular Augment of AQSP Fuzzy Graphs

Let $\sigma_1 = \{\sigma_1^P(x), \sigma_1^N(x), \rho_1^P(x), \rho_1^N(x)\}$ and $\sigma_2 = \{\sigma_2^P(x), \sigma_2^N(x), \rho_2^P(x), \rho_2^N(x)\}$ be two AQSP fuzzy vertex sets of G_1 and G_2 . Let $\mu_1 = \{(\mu_1^P(\delta_1), \mu_1^P(\delta_2)), (\mu_1^N(\delta_1), \mu_1^N(\delta_2)), (\gamma_1^P(\delta_1), \gamma_1^P(\delta_2)), (\gamma_1^N(\delta_1), \gamma_1^N(\delta_2))\}$ and

$\mu_2 = \{(\mu_2^P(\tau_1), \mu_2^P(\tau_2)), (\mu_2^N(\tau_1), \mu_2^N(\tau_2)), (\gamma_2^P(\tau_1), \gamma_2^P(\tau_2)), (\gamma_2^N(\tau_1), \gamma_2^N(\tau_2))\}$ be the two edge sets of G_1 and G_2 . We define the Modular Augment of AQSP fuzzy graphs as $G_1 \square G_2$.

Then $G_1 = \{\varphi_1, (\sigma_1, \rho_1)^P, (\sigma_1, \rho_1)^N, (\mu_1, \gamma_1)^P, (\mu_1, \gamma_1)^N\}$ and $G_2 = \{\varphi_2, (\sigma_2, \rho_2)^P, (\sigma_2, \rho_2)^N, (\mu_2, \gamma_2)^P, (\mu_2, \gamma_2)^N\}$.

$G_1 \square G_2 = \{\varphi, ((\sigma_1, \rho_1)^P \square (\sigma_2, \rho_2)^P), ((\sigma_1, \rho_1)^N \square (\sigma_2, \rho_2)^N), ((\mu_1, \gamma_1)^P \square (\mu_2, \gamma_2)^P), ((\mu_1, \gamma_1)^N \square (\mu_2, \gamma_2)^N)\}$. And the Modular Augment of AQSP fuzzy graphs edge set is given as ,

$(\mu, \gamma)_{E_1 \square E_2} = \{((\delta_1, \tau_1), (\delta_2, \tau_2)): \delta_1, \delta_2 \in E_1, \tau_1, \tau_2 \in E_2 \cup \delta_1, \delta_2 \notin E_1, \tau_1, \tau_2 \notin E_2\}$, With the condition of associate values of nodes and edges are given as,

$$\sigma_1^P(\delta_1) \square \sigma_2^P(\tau_1) = \sigma_1^P(\delta_1) \wedge \sigma_2^P(\tau_1), \quad \rho_1^P(\delta_1) \square \rho_2^P(\tau_1) = \rho_1^P(\delta_1) \wedge \rho_2^P(\tau_1)$$

$$\sigma_1^N(\delta_1) \square \sigma_2^N(\tau_1) = \sigma_1^N(\delta_1) \vee \sigma_2^N(\tau_1), \quad \rho_1^N(\delta_1) \square \rho_2^N(\tau_1) = \rho_1^N(\delta_1) \vee \rho_2^N(\tau_1)$$

If $\delta_1, \delta_2 \in E_1$ and $\tau_1, \tau_2 \in E_2$, then we have

$$\mu_1^P \square \mu_2^P((\delta_1, \tau_1), (\delta_2, \tau_2)) = \mu_1^P(\delta_1, \delta_2) \wedge \mu_2^P(\tau_1, \tau_2),$$

$$\mu_1^N \square \mu_2^N((\delta_1, \tau_1), (\delta_2, \tau_2)) = \mu_1^N(\delta_1, \delta_2) \vee \mu_2^N(\tau_1, \tau_2),$$

$$\gamma_1^P \square \gamma_2^P((\delta_1, \tau_1), (\delta_2, \tau_2)) = \gamma_1^P(\delta_1, \delta_2) \wedge \gamma_2^P(\tau_1, \tau_2),$$

$$\gamma_1^N \square \gamma_2^N((\delta_1, \tau_1), (\delta_2, \tau_2)) = \gamma_1^N(\delta_1, \delta_2) \vee \gamma_2^N(\tau_1, \tau_2), \text{ Where } \delta_1 \in \varphi_1 \text{ and } \tau_1 \in \varphi_2.$$

If $\delta_1, \delta_2 \notin E_1$ and $(\tau_1, \tau_2) \notin E_2$,

$$\sigma_1^P(\delta_1) \wedge \sigma_1^P(\delta_2) \wedge \sigma_2^P(\tau_1) \wedge \sigma_2^P(\tau_2),$$

$$\sigma_1^N(\delta_1) \vee \sigma_1^N(\delta_2) \vee \sigma_2^N(\tau_1) \vee \sigma_2^N(\tau_2),$$

$$\rho_1^P(\delta_1) \wedge \rho_1^P(\delta_2) \wedge \rho_2^P(\tau_1) \wedge \rho_2^P(\tau_2),$$

$$\rho_1^N(\delta_1) \vee \rho_1^N(\delta_2) \vee \rho_2^N(\tau_1) \vee \rho_2^N(\tau_2),$$

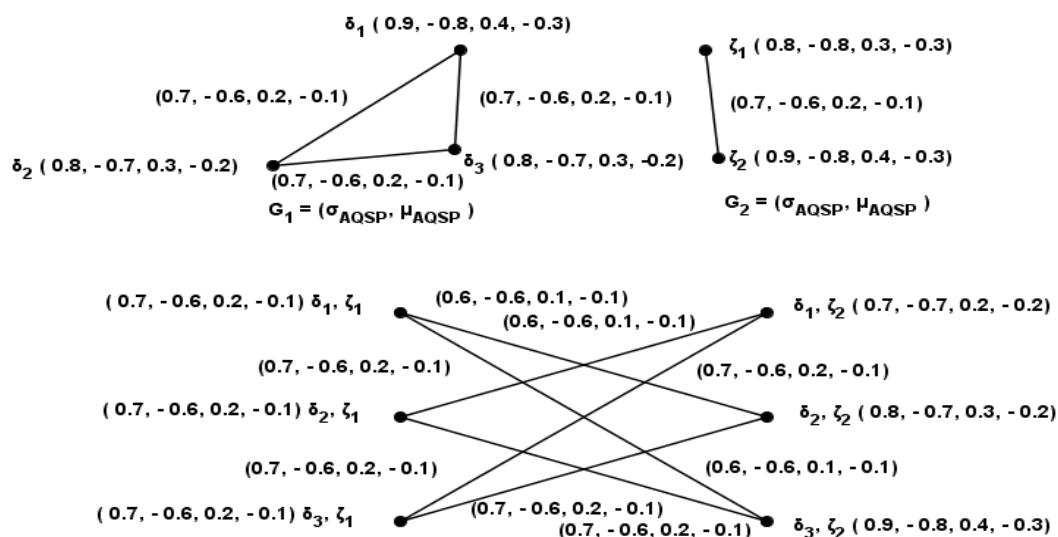


Figure. 5. $(G_1 \square G_2) = (\sigma_{AQSP}, \mu_{AQSP})$

Definition 5. 8: Homomorphic Augment of AQSP Fuzzy Graphs

Let $\sigma_1 = \{\sigma_1^P(x), \sigma_1^N(x), \rho_1^P(x), \rho_1^N(x)\}$ and $\sigma_2 = \{\sigma_2^P(x), \sigma_2^N(x), \rho_2^P(x), \rho_2^N(x)\}$ be two AQSP vertex sets of G_1 and G_2 . Let $\mu_1 = \{\mu_1^P(u_1, u_2), \mu_1^N(u_1, u_2), \gamma_1^P(u_1, u_2), \gamma_1^N(u_1, u_2)\}$ and

$\mu_2 = \{\mu_2^P(v_1, v_2), \mu_2^N(v_1, v_2), \gamma_2^P(v_1, v_2), \gamma_2^N(v_1, v_2)\}$ be the edge sets of G_1 and G_2 . We denote the Homomorphic Augment of two AQSP fuzzy graphs as $G_1 \circ G_2$. The AQSP fuzzy graphs,

$G_1 = \{\varphi_1, (\sigma_1, \rho_1)^P, (\sigma_1, \rho_1)^N, (\mu_1, \gamma_1)^P, (\mu_1, \gamma_1)^N\}$ and $G_2 = \{\varphi_2, (\sigma_2, \rho_2)^P, (\sigma_2, \rho_2)^N, (\mu_2, \gamma_2)^P, (\mu_2, \gamma_2)^N\}$ be given with the condition $\varphi = \varphi_1 \times \varphi_2$, where φ is non - empty set. We define Homomorphic Augment of AQSP fuzzy graphs G_1 and G_2 is a pair of combinatoric functions with the condition,

$$G_1 \circ G_2 = [\varphi, ((\sigma_1, \rho_1)^P \circ (\sigma_2, \rho_2)^P), ((\sigma_1, \rho_1)^N \circ (\sigma_2, \rho_2)^N), ((\mu_1, \gamma_1)^P \circ (\mu_2, \gamma_2)^P), ((\mu_1, \gamma_1)^N \circ (\mu_2, \gamma_2)^N)].$$

The edges of Homomorphic Augment of two AQSP fuzzy graphs membership and non – membership values are given as,

$$(\mu, \gamma)_{(E_1 \circ E_2)} = \{((u_1, v_1), (u_2, v_2)): (u_1 = u_2), (v_1, v_2) \in E_2 \text{ or } u_1, u_2 \in E_1, v_1, v_2 \text{ not in } E_2\},$$

with the condition of the AQSP fuzzy graphs vertex set,

$$((\sigma_1 \circ \sigma_2)^P(u_1, v_1)) = (\sigma_1)^P(u_1) \wedge (\sigma_2)^P(v_1), ((\rho_1 \circ \rho_2)^P(u_1, v_1)) = (\rho_1)^P(u_1) \wedge (\rho_2)^P(v_1)$$

$$((\sigma_1 \circ \sigma_2)^N(u_1, v_1)) = (\sigma_1)^N(u_1) \vee (\sigma_2)^N(v_1), ((\rho_1 \circ \rho_2)^N(u_1, v_1)) = (\rho_1)^N(u_1) \vee (\rho_2)^N(v_1),$$

where $u_1 = u_2$ and $(v_1, v_2) \in E_2$. The AQSP Homomorphic edge set is given as,

If $u_1 = u_2$ and $(v_1, v_2) \in E_2$

$$(\mu_1 \circ \mu_2)^P((u_1, v_1), (u_2, v_2)) = (\sigma_1)^P(u_1) \wedge (\mu_2)^P(v_1, v_2),$$

$$(\gamma_1 \circ \gamma_2)^P((u_1, v_1), (u_2, v_2)) = (\rho_1)^P(u_1) \wedge (\gamma_2)^P(v_1, v_2),$$

$$(\mu_1 \circ \mu_2)^N((u_1, v_1), (u_2, v_2)) = (\sigma_1)^N(u_1) \vee (\mu_2)^N(v_1, v_2),$$

$$(\gamma_1 \circ \gamma_2)^N((u_1, v_1), (u_2, v_2)) = (\rho_1)^N(u_1) \vee (\gamma_2)^N(v_1, v_2).$$

If $(u_1, u_2) \in E_1$ and (v_1, v_2) not in E_2

$$(\mu_1 \circ \mu_2)^P((u_1, v_1), (u_2, v_2)) = (\mu_1)^P(u_1, u_2) \wedge ((\sigma_2)^P(v_1) \wedge (\sigma_2)^P(v_2)),$$

$$(\gamma_1 \circ \gamma_2)^P((u_1, v_1), (u_2, v_2)) = (\gamma_1)^P(u_1, u_2) \wedge ((\rho_2)^P(v_1) \wedge (\rho_2)^P(v_2)), \text{ similarly, we have,}$$

$$(\mu_1 \circ \mu_2)^N((u_1, v_1), (u_2, v_2)) = (\mu_1)^N(u_1, u_2) \vee ((\sigma_2)^N(v_1) \vee (\sigma_2)^N(v_2)),$$

$$(\gamma_1 \circ \gamma_2)^N((u_1, v_1), (u_2, v_2)) = (\gamma_1)^N(u_1, u_2) \vee ((\rho_2)^N(v_1) \vee (\rho_2)^N(v_2)),$$

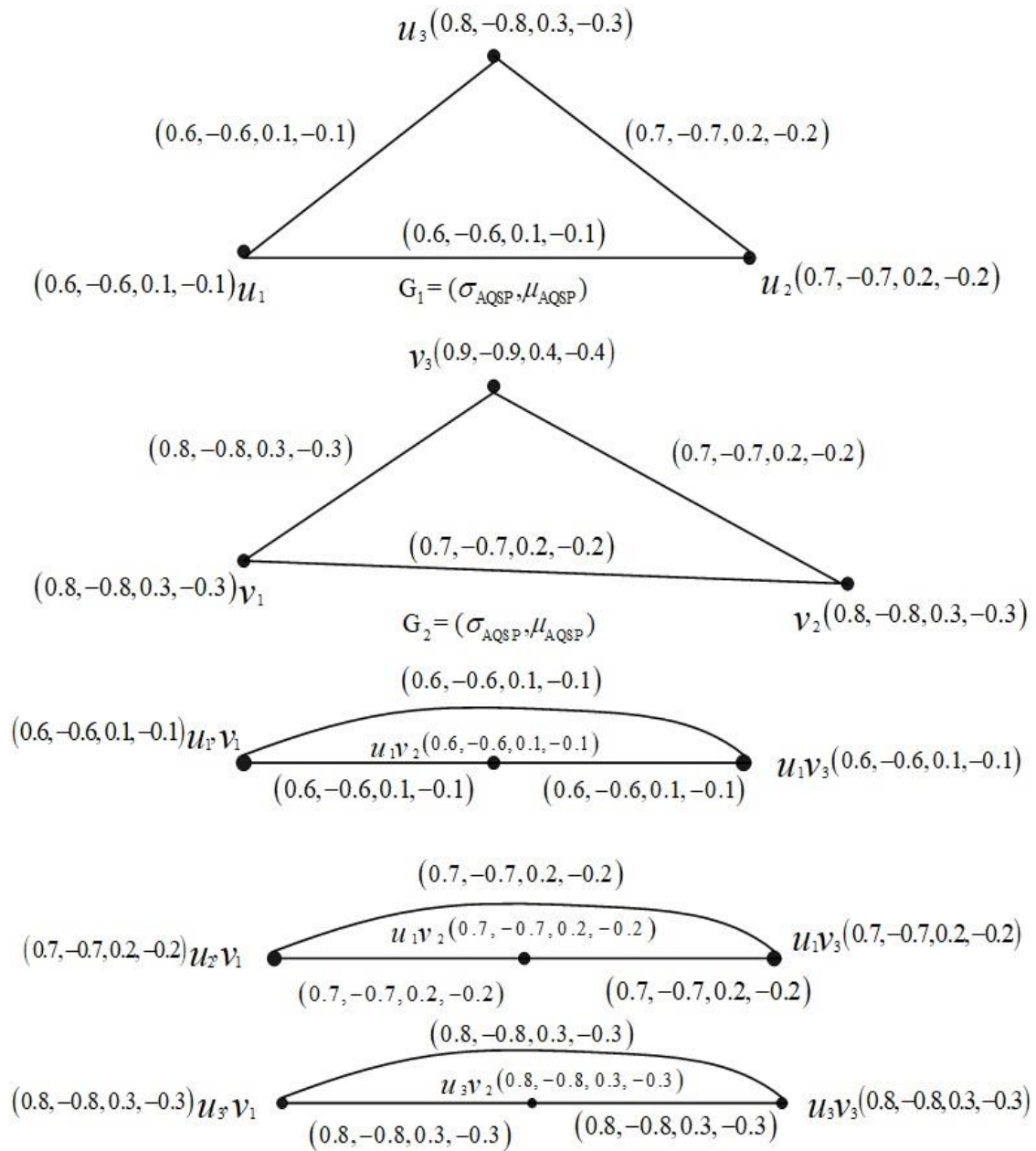


Figure. 6. $(G_1 \circ G_2) = (\sigma_{AQSP}, \mu_{AQSP})$

Theorem 5.9: Let $G = (\sigma_{AQSP}, \mu_{AQSP})$ be a Self complementary strong AQSP Fuzzy graph, and then prove that,

- (i) $\sum_{x \neq y} \mu^P(x, y) = \frac{1}{2} \sum_{x \neq y} (\sigma^P(x), \sigma^P(y))$, (ii) $\sum_{x \neq y} \mu^N(x, y) = \frac{1}{2} \sum_{x \neq y} (\sigma^N(x), \sigma^N(y))$
 (iii) $\sum_{x \neq y} \gamma^P(x, y) = \frac{1}{2} \sum_{x \neq y} (\rho^P(x), \rho^P(y))$, (iv) $\sum_{x \neq y} \gamma^N(x, y) = \frac{1}{2} \sum_{x \neq y} (\rho^N(x), \rho^N(y))$

Proof: Let $G = (\sigma_{AQSP}, \mu_{AQSP})$ be a Self complementary strong AQSP Fuzzy graph. Then there exists an isomorphism $\varphi : \sigma \rightarrow \sigma$, $\varphi : \rho \rightarrow \rho$ such that $\bar{\sigma}^P \varphi(x) = \varphi^P(x), \bar{\sigma}^N \varphi(x) = \varphi^N(x)$

$\forall x \in \sigma$ and $\bar{\mu}^P(\varphi(x), \varphi(y)) = \varphi^P(x, y), \bar{\mu}^N(\varphi(x), \varphi(y)) = \varphi^N(x, y) \forall (x, y) \in \mu$ and for submerging function we have the following, $\bar{\rho}^P \varphi(x) = \varphi^P(x), \bar{\rho}^N \varphi(x) = \varphi^N(x)$

$\forall x \in \rho$ and $\bar{\gamma}^P(\varphi(x), \varphi(y)) = \varphi^P(x, y), \bar{\gamma}^N(\varphi(x), \varphi(y)) = \varphi^N(x, y) \forall (x, y) \in \gamma$.

Now by definition of $\bar{G} = (\bar{\sigma}_{AQSP}, \bar{\mu}_{AQSP})$, we have the following results,

$$(i) \bar{\mu}^P(\varphi(x), \varphi(y)) = \bar{\sigma}^P(\varphi(x)) \wedge \bar{\sigma}^P(\varphi(y)) - \mu^P(\varphi(x), \varphi(y))$$

$$(ie) \mu^P(x, y) = \sigma^P(x) \wedge \sigma^P(y) - \mu^P(\varphi(x), \varphi(y))$$

$$(ie) \sum_{x \neq y} \mu^P(x, y) + \sum_{x \neq y} \mu^P(\varphi(x), \varphi(y)) = \sum_{x \neq y} (\sigma^P(x) \wedge \sigma^P(y))$$

$$(ie) 2 \sum_{x \neq y} \mu^P(x, y) = \sum_{x \neq y} (\sigma^P(x) \wedge \sigma^P(y))$$

$$\therefore \sum_{x \neq y} \mu^P(x, y) = \frac{1}{2} \sum_{x \neq y} (\sigma^P(x) \wedge \sigma^P(y))$$

$$(ii) \bar{\mu}^N(\varphi(x), \varphi(y)) = \bar{\sigma}^N(\varphi(x)) \vee \bar{\sigma}^N(\varphi(y)) - \mu^N(\varphi(x), \varphi(y))$$

$$(ie) \mu^N(x, y) = \sigma^N(x) \vee \sigma^N(y) - \mu^N(\varphi(x), \varphi(y))$$

$$(ie) \sum_{x \neq y} \mu^N(x, y) + \sum_{x \neq y} \mu^N(\varphi(x), \varphi(y)) = \sum_{x \neq y} (\sigma^N(x) \vee \sigma^N(y))$$

$$(ie) 2 \sum_{x \neq y} \mu^N(x, y) = \sum_{x \neq y} (\sigma^N(x) \vee \sigma^N(y))$$

$$\therefore \sum_{x \neq y} \mu^N(x, y) = \frac{1}{2} \sum_{x \neq y} (\sigma^N(x) \vee \sigma^N(y))$$

$$(iii) \bar{\gamma}^P(\varphi(x), \varphi(y)) = \bar{\rho}^P(\varphi(x)) \wedge \bar{\rho}^P(\varphi(y)) - \gamma^P(\varphi(x), \varphi(y))$$

$$(ie) \gamma^P(x, y) = \rho^P(x) \wedge \rho^P(y) - \gamma^P(\varphi(x), \varphi(y))$$

$$(ie) \sum_{x \neq y} \gamma^P(x, y) + \sum_{x \neq y} \gamma^P(\varphi(x), \varphi(y)) = \sum_{x \neq y} (\rho^P(x) \wedge \rho^P(y))$$

$$(ie) 2 \sum_{x \neq y} \gamma^P(x, y) = \sum_{x \neq y} (\rho^P(x) \wedge \rho^P(y))$$

$$\therefore \sum_{x \neq y} \gamma^P(x, y) = \frac{1}{2} \sum_{x \neq y} (\rho^P(x) \wedge \rho^P(y))$$

$$(iv) \bar{\gamma}^N(\varphi(x), \varphi(y)) = \bar{\rho}^N(\varphi(x)) \vee \bar{\rho}^N(\varphi(y)) - \gamma^N(\varphi(x), \varphi(y))$$

$$(ie) \gamma^N(x, y) = \rho^N(x) \vee \rho^N(y) - \gamma^N(\varphi(x), \varphi(y))$$

$$(ie) \sum_{x \neq y} \gamma^N(x, y) + \sum_{x \neq y} \gamma^N(\varphi(x), \varphi(y)) = \sum_{x \neq y} (\rho^N(x) \vee \rho^N(y))$$

$$(ie) 2 \sum_{x \neq y} \gamma^N(x, y) = \sum_{x \neq y} (\rho^N(x) \vee \rho^N(y))$$

$$\therefore \sum_{x \neq y} \gamma^N(x, y) = \frac{1}{2} \sum_{x \neq y} (\rho^N(x) \vee \rho^N(y))$$

Hence the proof is completed.

Theorem 5.10: Let $G = (\sigma_{AQSP}, \mu_{AQSP})$ be a AQSP fuzzy graph, then the automorphism groups of

$G = (\sigma_{AQSP}, \mu_{AQSP})$ and $\bar{G} = (\bar{\sigma}_{AQSP}, \bar{\mu}_{AQSP})$ are identical.

Proof: Let $\varphi \in \tau(G)$ then $\varphi \in \tau(\bar{G})$ for $\varphi^P : \sigma^P \rightarrow \sigma^P$, $\varphi^N : \sigma^N \rightarrow \sigma^N$ is bijective,

$$(i) \bar{\sigma}^P(\varphi^P(x)) = \sigma^P(\varphi^P(x)) = \sigma^P(x) = \bar{\sigma}^P(x)$$

$$\bar{\mu}^P(\varphi^P(x), \varphi^P(y)) = (\sigma^P(\varphi^P(x) \wedge \varphi^P(y)) - \mu^P(\varphi(x), \varphi(y)))$$

$$\bar{\mu}^P(\varphi^P(x), \varphi^P(y)) = (\sigma^P(x) \wedge \sigma^P(y)) - \mu^P(\varphi(x), \varphi(y))$$

$$\bar{\mu}^P(\varphi^P(x), \varphi^P(y)) = \bar{\mu}^P(x, y) \quad \forall (x, y) \in \sigma_{AQSP}^P$$

$$(ii) \bar{\sigma}^N(\varphi^N(x)) = \sigma^N(\varphi^N(x)) = \sigma^N(x) = \bar{\sigma}^N(x)$$

$$\bar{\mu}^N(\varphi^N(x), \varphi^N(y)) = \sigma^N(\varphi^N(x) (\mu_s, \gamma_s) \varphi^N(y)) - \mu^N(\varphi(x), \varphi(y))$$

$$\bar{\mu}^N(\varphi^N(x), \varphi^N(y)) = ((\sigma^N(x) \vee \sigma^N(y)) - \mu^N(\varphi(x), \varphi(y)))$$

$$\bar{\mu}^N(\varphi^N(x), \varphi^N(y)) = \bar{\mu}^N(x, y) \quad \forall (x, y) \in \sigma_{AQSP}^N$$

Similarly, we have the following result for $\varphi^P : \rho^P \rightarrow \rho^P$, $\varphi^N : \rho^N \rightarrow \rho^N$,

$$(iii) \bar{\rho}^P(\varphi^P(x)) = \rho^P(\varphi^P(x)) = \rho^P(x) = \bar{\rho}^P(x)$$

$$\bar{\gamma}^P(\varphi^P(x), \varphi^P(y)) = \rho^P(\varphi^P(x) \wedge \varphi^P(y)) - \gamma^P(\varphi(x), \varphi(y))$$

$$\bar{\gamma}^P(\varphi^P(x), \varphi^P(y)) = ((\rho^P(x) \wedge \rho^P(y)) - \gamma^P(\varphi(x), \varphi(y)))$$

$$\bar{\gamma}^P(\varphi^P(x), \varphi^P(y)) = \bar{\gamma}^P(x, y) \quad \forall (x, y) \in \rho_{AQSP}^P$$

$$(iv) \bar{\rho}^N(\varphi^N(x)) = \rho^N(\varphi^N(x)) = \rho^N(x) = \bar{\rho}^N(x)$$

$$\bar{\gamma}^N(\varphi^N(x), \varphi^N(y)) = \rho^N(\varphi^N(x) \vee \varphi^N(y)) - \gamma^N(\varphi(x), \varphi(y))$$

$$\bar{\gamma}^N(\varphi^N(x), \varphi^N(y)) = ((\rho^N(x) \vee \rho^N(y)) - \gamma^N(\varphi(x), \varphi(y)))$$

$$\bar{\gamma}^N(\varphi^N(x), \varphi^N(y)) = \bar{\gamma}^N(x, y) \quad \forall (x, y) \in \rho_{AQSP}^N, \text{ Hence the theorem is proved.}$$

6. Mathematical Analysis of Mind Influential Conflicts in AQSP Fuzzy Graphs:

The mind influential conflict in AQSP fuzzy graph analysis indicates the membership and non – membership degree value of the high and low self-esteem of a human mind conflict feelings which we denote the node set as (σ, ρ) and the edge set which implies the influential conflict behavior of human mind swing considered as (μ, γ) . Submerging level of conflict state is assumed as frustration, between the interval of confidence $[-0.5, 0.5]$. Using the association of AQSP fuzzy function, we measure the mind influential conflict feelings of contradictory self-esteem level fuzzy degree membership and non – membership value.

6.1 Method of Mind Influential Conflict in AQSP Fuzzy Graphs Case Study:

Infusion of two or more people's combinatoric conflict feelings are taken as $(\sigma, \rho)A_1$, $(\sigma, \rho)A_2$ as vertices in AQSP fuzzy graphs. In this case study two consecutive membership and non – membership values have been taken for Analysis.

1. We denote the two sequential AQSP fuzzy membership and non – membership values measure as (S_1, S_2) of low and high self – esteem, with submerging level of confidence in frustration.
2. Infusion of AQSP fuzzy graphs edge sets is considered as mind influential conflict feelings with their behavioural attitude.
3. Mind influential conflicts in AQSP fuzzy graphs vertices and edges score value gives the result of low and high self-esteem person's dominating Influential behaviours.
4. Each AQSP fuzzy vertex and edge set represents the frustration of submerging membership and non – membership score values which is attained in their conflict and influential feelings by the following equations.

$$(\mu, \gamma)_S^P(x, y) \leq (\sigma, \rho)_S^P(x) \wedge (\sigma, \rho)_S^P(y) \text{ and } (\mu, \gamma)_S^N(x, y) \geq (\sigma, \rho)_S^N(x) \vee (\sigma, \rho)_S^N(y).$$

5. The score values are measured by the AQSP score formula which gives the result of low and high self-esteem influential person, $\frac{1}{n} \left(\frac{1}{l_d^P} \sum \varphi_x^P - \frac{1}{l_d^N} \sum \varphi_x^N \right)$
6. The relationship between conflict and Influential feelings leads to finding the High and low self-esteem and weightage of the influential behaviours.
7. Approach – Approach conflicts = Positive membership quadra value,
Avoidance – Avoidance conflicts = Negative membership quadra value

Frustration conflicts = submerging level of confidence. The case study result

of the mind influential conflict in AQSP Fuzzy Graphs measure is given in the following tables.

Table .1: Combinatorics Conflict Feelings of Human Mind in Vertices

Person - 1 $Node(\sigma, \rho)A_1$	Person - 2 $Node(\sigma, \rho)A_2$	Mind Conflicts	Conflict feelings
u_1	v_1	Approach – Approach conflicts	High self - esteem
u_2	v_2	Avoidance – Avoidance conflicts	Low self - esteem
u_3	v_3	Frustration conflicts	Stressful feelings

¹Table. 1. represents the Mind conflicts and Conflict feelings of two persons'

Table. 2: Combinatorics Influential Conflict Feelings of Human Mind in edges

Person - 1 $edge(\mu, \gamma)B_1$	Person - 2 $edge(\mu, \gamma)B_2$	Mind Influential Conflicts Behaviours	Influential Conflict feelings
u_1u_2	v_1v_2	Membership Conflicts Behaviours	Positive Influential Traumatic feelings
u_1u_3	v_1v_3	Non – membership Conflicts Behaviour	Negative Influential Traumatic feelings

²Table. 2. exemplify the AQSP fuzzy graph Combinatorics Influential Conflict Feelings of Human influential behaviours.

Table .3: Combinatorics Conflict Feelings of Human Mind in AQSP fuzzy graph membership and non – membership nodes

Person - 1 $Node(\sigma, \rho)A_1$	Membership and Non – Membership values	Person - 2 $Node(\sigma, \rho)A_1$	Membership and Non – Membership values
u_1	$S_1(0.9, -0.8, 0.4, -0.3)$ $S_2(0.8, -0.7, 0.3, -0.2)$	v_1	$S_1(0.7, -0.7, 0.2, -0.2)$ $S_2(0.6, -0.5, 0.1, 0.0)$
u_2	$S_1(1.0, -1.0, 0.5, -0.5)$ $S_2(1.0, -0.9, 0.5, -0.4)$	v_2	$S_1(1.0, -0.9, 0.5, -0.5)$ $S_2(0.9, -1.0, 0.4, -0.5)$
u_3	$S_1(0.6, -0.5, 0.1, 0.0)$ $S_2(0.6, -0.6, 0.1, -0.1)$	v_3	$S_1(0.6, -0.7, 0.1, 0.2)$ $S_2(0.8, -0.8, 0.3, -0.3)$

³Table.3. express the AQSP fuzzy graph membership and non - membership values withCombinatorics Influential Conflict Feelings of Human Mind.

Table. 4: Combinatorics Conflict Feelings of Human Mind in AQSP fuzzy graph edges

Person - 1 $edge(\mu, \gamma)B_1$	Membership and Non – Membership values	Person - 2 $edge(\mu, \gamma)B_1$	Membership and Non – Membership values
u_1u_2	$S_1(0.6, -0.6, 0.1, -0.1)$ $S_2(0.6, -0.6, 0.1, -0.1)$	u_1u_2	$S_1(0.7, -0.6, 0.2, -0.1)$ $S_2(0.6, -0.5, 0.1, 0.0)$
u_1u_3	$S_1(0.7, -0.7, 0.2, -0.2)$ $S_2(0.8, -0.8, 0.3, -0.3)$	u_1u_3	$S_1(0.6, -0.6, 0.1, -0.1)$ $S_2(0.8, -0.8, 0.3, -0.3)$

⁴Table. 4. shows the AQSP fuzzy graph Combinatorics Conflict Feelings of Human Mind with membership and non - membership values.

Table. 5: Combinatorics Conflict Feelings of Human Mind in AQSPFuzzy Graph nodes Scores.

Person - 1 $Node(\sigma, \rho)A_1$	Person 1 $(\sigma, \rho)A_1$ Score Values	Person - 2 $Node(\sigma, \rho)A_1$	Person 2 $(\sigma, \rho)A_2$ Score Values
u_1	0.456	v_1	0.533
u_2	1.0	v_2	0.990
u_3	0.733	v_3	0.630

⁵Table.5. gives the vertex set score value of Combinatorics ConflictFeelings of Human Mind in AQSP Fuzzy nodes Score values.

Table. 6: Combinatorics Conflict Feelings of Human Mind in AQSPFuzzy graph edge set Scores.

	Person 1 $(\mu, \gamma)B_1$ Score Values	$edge(\mu, \gamma)B_2$	Person 2 $(\mu, \gamma)B_2$ Score Values
u_1u_2	0.466	u_1u_2	0.500
u_1u_3	0.646	u_1u_3	0.633

⁶Table.6. explains the Combinatorics Conflict Feelings of HumanMind in AQSP Fuzzy edge set Score values.

6.2. AQSP Fuzzy Graph Mathematical Combinatorics Method:

$$x \rightarrow y = \begin{cases} 1, & x \leq y \\ x, & x = y \\ y, & x > y \end{cases} \quad \text{and} \quad (G_1 * G_2): G_{min} = {}^T M$$

$$A_1 = [0.4 \quad 1.0 \quad 0.7] \quad A_2 = [0.5 \quad 1.0 \quad 0.6]$$

$$B_1 = [0.4 \quad 0.6] \quad B_2 = [0.5 \quad 0.6]$$

Relationship between conflict feelings and influential behaviours of persons A_1 and B_1 :

$$R_1(A_1, B_1) = \begin{bmatrix} 0.4 \\ 1.0 \\ 0.7 \end{bmatrix} {}^T M [0.4 \quad 0.6] = \begin{bmatrix} 0.4 & 1.0 \\ 0.4 & 0.6 \\ 0.7 & 0.6 \end{bmatrix}$$

$$R_2(A_2, B_2) = \begin{bmatrix} 0.5 \\ 1.0 \\ 0.6 \end{bmatrix} {}^T M [0.5 \quad 0.6] = \begin{bmatrix} 0.5 & 1.0 \\ 0.5 & 0.6 \\ 0.5 & 0.6 \end{bmatrix}$$

Combinatoric convolution of mind conflict and influential behaviour $R^* = R_1 \cap R_2$

$$R^* = \begin{bmatrix} 0.4 & 1.0 \\ 0.4 & 0.6 \\ 0.7 & 0.6 \end{bmatrix} {}^T M \begin{bmatrix} 0.5 & 1.0 \\ 0.5 & 0.6 \\ 0.5 & 0.6 \end{bmatrix} = \begin{bmatrix} 0.4 & 1.0 \\ 0.4 & 0.6 \\ 0.4 & 0.6 \end{bmatrix}$$

Max – min convolution of low and high influential fuzzy value $R^* = [0.4 \quad 1.0]$

Let A' be common influential and conflict feelings of $G_1(B_1)$ and $G_2(B_2)$ where B_1 and B_2 are intersecting influential conflict feelings. i.e. $A' = [0.4, 0.7]$ of persons A_1 and A_2 .

6.3. Intersecting influential conflict feelings of $[G_1(B_1), G_2(B_2)]$ Augment relation in AQSP Fuzzy Graph:

We denote the common conflict and influential behaviour of $[G_1(B_1), G_2(B_2)]$ A as $B' = A' \cdot R$

$$[0.4, 0.7] {}^T M \begin{bmatrix} 0.4 & 1.0 \\ 0.4 & 0.6 \\ 0.4 & 0.6 \end{bmatrix} = \begin{bmatrix} 0.4 & 0.7 \\ 0.4 & 0.6 \\ 0.4 & 0.6 \end{bmatrix} = [0.4, 0.7]$$

common conflict and influential behaviours of G_1 and $G_2 = [0.4, 0.7]$

We find the common conflict feelings and influential behaviours of G_1 and G_2 by using AQSP Augment relation in R_1 is given as,

$$B'_1 = A' * R_1 = [0.4, 0.7] \text{ then,}$$

$$B'_2 = A' * R_2 \text{ is measured as } [0.4, 0.7] {}^T M \begin{bmatrix} 0.5 & 1.0 \\ 0.5 & 0.6 \\ 0.5 & 0.6 \end{bmatrix} = \begin{bmatrix} 0.4 & 0.7 \\ 0.4 & 0.6 \\ 0.4 & 0.6 \end{bmatrix} = [0.4, 0.7]$$

$$B'_2: A' * R_2 = [0.4, 0.7].$$

Therefore, the weightage of common level mind influential conflict feelings and behaviours of two persons' low and high self-esteem is measured as $(G_1 * G_2)(B'_1 {}^T M B'_2) = [0.4, 0.7]$.

The dominating conflict and influential behaviours person is $G_1(A_1)$

The low self-esteem of $G_1(A_1) = 0.4$ and the high self-esteem of $G_1(A_1) = 0.7$

7. Conclusion:

Mind Influential Conflicts in AQSP fuzzy graph is a subjective valuation assigned by one or more human persons mind conflict feelings and influential behaviours. In this monograph we have analysed the mind conflict feelings and influential conflict behaviours with submerging level of frustrations in AQSP fuzzy graph with Alternate Quadra level. The partition of AQSP is considered in to two.

- (i) Approach – Approach conflict feelings (High self-esteem) and the alternate of it is given as
- (ii) Avoidance – Avoidance conflict feelings (Low self-esteem).

The submerging level of confidence is taken as frustration and stressful state between the range of $[-0.5, 0.5]$. The forces of conflict influential feelings behavior are in fact responsible for mental conflict feelings. Making use of the mathematical method of AQSP fuzzy graph we can find the weightage of the conflict feelings and influential behavioural feelings of the human mind. Because conflict feelings are a state of uncertain affairs in which two or more influential behaviours trends are evoked by AQSP fuzzy graphs.

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