Tuijin Jishu/Journal of Propulsion Technology
ISSN: 1001-4055
Vol. 44 No. 3 (2023)

ℓ-HILBERT MEAN LABELING OF SOME PATH RELATED GRAPHS

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Abstract:
Let G be a graph with p vertices and q edges. The q^{th} hilbert number is denoted by $Ꜧ_q$ and is defined by $Ꜧ_q = 4(q-1)+1$ where $q \geq 1$. A ℓ- hilbert mean labeling is an injective function $f: V(G) \to \{0, 1, 2, \dots, Ꜧ_{\ell+(q-1)}\}$, where $\ell \geq 1$ that induces a bijection $f^*: E(G) \to \{Ꜧ_\ell, Ꜧ_{\ell+1}, Ꜧ_{\ell+2}, \dots, Ꜧ_{\ell+(q-1)}\}$ defined by

$$f^*(uv) = \begin{cases} \dfrac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd} \\ \dfrac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even} \end{cases}$$

for all $uv \in E(G)$. A graph which admits such labeling is called a ℓ- hilbert mean graph. In this paper, a new type of labeling called ℓ- hilbert mean labeling is introduced and the path related graphs is studied.

AMS Subject Classification – 05C78
Keywords: Hilbert numbers, Hilbert mean labeling, Hilbert mean graph.

1. Introduction

Let $G = (V, E)$ be a graph with p vertices and q edges. The graph considered in this paper are simple, finite, undirected and without loops or multiple edges. Terms not defined here are used in the sense of Harary [4]. For number theoretic terminology [1] is followed. A graph labeling is an assignment of integers to the vertices or the edges or both subject to certain conditions. If the domain of the mapping is a set of vertices (edges/both) then the labeling is called a vertex (edge/total) labeling. A dynamic survey of graph labeling is regularly updated by Gallian [2] and it is published by Electronic Journal of Combinatorics. The concept of mean labeling was introduced by S.Somasundaram and R.Ponraj [6]. For Triangular mean labeling and k- Mean labeling refer [5] and [3] and Hilbert mean labeling was introduced in [7].

2. Preliminaries

Definition 2.1: A path P_n is obtained by joining u_i to the consecutive vertices u_{i+1} for $1 \leq i \leq n-1$.
Definition 2.2: The graph obtained by joining a single pendant edge to each vertex of a path P_n is called a comb. It is denoted by $P_n \odot K_1$ (or) P_n^+.
Definition 2.3: Bistar is the graph obtained by joining the apex vertices of two copies of star $K_{1,n}$.
Definition 2.4: The H-graph of path $P_n (n \geq 3)$ is the graph obtained from two copies of P_n with the vertices$u_1, u_2, \cdots, u_n$ and $v_1, v_2, \cdots, v_n$ by joining the vertices $\left(u_{\frac{n+1}{2}}, v_{\frac{n+1}{2}}\right)$ if n is odd and $\left(u_{\frac{n}{2}+1}, v_{\frac{n}{2}}\right)$ if n is even. It is denoted by $H(P_n)$.

Definition 2.5: A F- tree $F(P_n)$ is a graph obtained from path on $n \geq 3$ vertices by appending two pendant edges one to an end vertex and other to vertex adjacent to an end vertex.
Definition 2.6: The ladder graph L_n is a planar, undirected graph with $2n$ vertices and $3n-2$ edges. The ladder graph L_n is a graph obtained as the Cartesian product of P_2 and P_n.
Definition 2.7: The slanting ladder SL_n is a graph that consists of two copies of P_n having vertex set $\{u_i: 1 \leq i \leq n\} \cup \{v_i: 1 \leq i \leq n\}$ and the edge set is formed by adjoining u_{i+1} and v_i for all $1 \leq i \leq n-1$.
Definition 2.8: The graph obtained from a path by attaching exactly two pendant edges to each internal vertex of the path is called a Twig graph and it is denoted by T(n).
Definition 2.9: A graph G with p vertices and q edges is called a mean graph if there is an injective function f from the vertices of G to $\{0,1,2,\ldots,q\}$ in such a way that when each edge $e = uv$ is labeled with $(f(u)+f(v))\,/\,2$ if $f(u)+f(v)$ is even and $(f(u)+f(v)+1)\,/\,2$ if $f(u)+f(v)$ is odd, then the resulting edge labels are distinct. Here f is called a mean labeling of G. If G is a mean graph, then the edges get labels $1,2,\ldots,q$.
Definition 2.10 [3] : A (p,q) graph G is set to have k-mean labeling, if there is an injective function f from the vertices of G to $\{0,1,2,\ldots,k+q-1\}$ such that the induced map f^* defined on E by

$$f^*(uv) = \begin{cases} \frac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd} \\ \frac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even} \end{cases}$$

is a bijection from E to $\{k,k+1,k+2,\ldots,k+q-1\}$. A graph that admits a k-mean labeling is called a k-mean graph.
Definition 2.11: The n^{th} hilbert number $Ӈ_n$ is given by the formula $4(n-1)+1$ for $n \geq 1$. The first few hilbert numbers are $1,5,9,13,17,21,25,29,33,37,41,45,49,53,57$, etc.
Definition 2.12: Let G be a graph with p vertices and q edges. The n^{th} hilbert number is denoted $Ӈ_n$ and is defined by $Ӈ_n = 4(n-1)+1$ where $n \geq 1$. A hilbert mean labeling is an injective function $f: V(G) \to \{0,1,2,\ldots,Ӈ_q\}$, where $Ӈ_q$ is the q^{th} hilbert number that induces a bijection $f^*: E(G) \to \{Ӈ_1, Ӈ_2, \ldots, Ӈ_q\}$ defined by

$$f^*(uv) = \begin{cases} \dfrac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd} \\ \dfrac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even} \end{cases}$$

for all $uv \in E(G)$. A graph which admits such labeling is called a hilbert mean graph.

3. Main Results

Definition 3.1: Let G be a graph with p vertices and q edges. The q^{th} hilbert number is denoted by $Ӈ_q$ and is defined by $Ӈ_q = 4(q-1)+1$ where $q \geq 1$. A ℓ- hilbert mean labeling is an injective function $f: V(G) \to \{0,1,2,\ldots,Ӈ_{\ell+(q-1)}\}$ where $\ell \geq 1$, that induces a bijection $f^*: E(G) \to \{Ӈ_\ell, Ӈ_{\ell+1}, Ӈ_{\ell+2}, \ldots, Ӈ_{\ell+(q-1)}\}$ defined by

$$f^*(uv) = \begin{cases} \dfrac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd} \\ \dfrac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even} \end{cases}$$

for all $uv \in E(G)$. A graph which admits such labeling is called a ℓ- hilbert mean graph.
Theorem 3.2: P_m is a ℓ- hilbert mean graph, where $m \geq 2$.
Proof: Let $G = P_m$. Let $V(G) = \{x_i : 1 \leq i \leq m\}$ and $E(G) = \{x_i\, x_{i+1} : 1 \leq i \leq m-1\}$.
We observe that G has m vertices and $m-1$ edges.
Define $f : V(G) \to \{0,1,2,\ldots,Ӈ_{\ell+m-2}\}$ as follows.
Case 1: $m \text{ is odd and } \ell \geq 2, f(x_1) = 4\ell - 7$
For $2 \leq i \leq m$,

$$f(x_i) = \begin{cases} 4(\ell+i-3)+1, & i \text{ is odd} \\ 4(\ell+i-2), & i \text{ is even} \end{cases}$$

Case 2: $m \text{ is even and } l \geq 1,$

For $1 \leq i \leq m$,

$$f(x_i) = \begin{cases} 4(\ell + i - 2), & i \text{ is odd} \\ 4(\ell + i - 3) + 1, & i \text{ is even} \end{cases}$$

Clearly f is injective and the induced edge labeling $f^* : E(G) \to \{Ꜧ_\ell, Ꜧ_{\ell+1} \dots, Ꜧ_{\ell+m-2}\}$ is defined as follows.

$f^*(x_i\, x_{i+1}) = Ꜧ_{\ell+(i-1)}$, where $1 \leq i \leq m-1$

Thus, we get the induced edge labels as $Ꜧ_\ell, Ꜧ_{\ell+1} \dots, Ꜧ_{\ell+m-2}$.

Hence P_m is a ℓ- hilbert mean graph, where $m \geq 2$.

Example 3.3: The ℓ- hilbert mean labeling of P_4 is shown in figure 1.

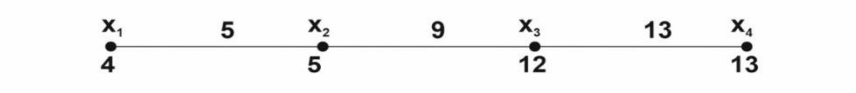

Figure - 1

Theorem 3.4: P_m^+ is a ℓ- hilbert mean graph, where $m \geq 3$.

Proof: Let $G = P_m^+$. Let $V(G) = \{x_i, y_i : 1 \leq i \leq m\}$ and $E(G) = \{y_i\, y_{i+1} : 1 \leq i \leq m-1\} \cup \{x_i\, y_i : 1 \leq i \leq m\}$.

We observe that G has $2m$ vertices and $2m-1$ edges.

Define $f : V(G) \to \{0, 1, 2, \dots, Ꜧ_{\ell+2m-2}\}$ as follows.

$f(x_i) = 4(\ell + 2i - 3), \quad 1 \leq i \leq m$

$f(y_i) = 4(\ell + 2i - 3) + 1, \quad 1 \leq i \leq m$

Clearly f is injective and the induced edge labeling $f^*: E(G) \to \{Ꜧ_\ell, Ꜧ_{\ell+1}, \dots, Ꜧ_{\ell+2m-2}\}$ is defined as follows.

$f^*(y_i\, y_{i+1}) = Ꜧ_{\ell+i-1}$, where $1 \leq i \leq m-1$

$f^*(x_i\, y_i) = Ꜧ_{\ell+2(i-1)}$, where $1 \leq i \leq m$

Thus, we get the induced edge labels as $Ꜧ_\ell, Ꜧ_{\ell+1} \dots, Ꜧ_{\ell+2m-2}$.

Hence P_m^+ is a ℓ- hilbert mean graph, where $m \geq 3$.

Example 3.5: The ℓ- hilbert mean labeling of P_4^+ is shown in figure 2.

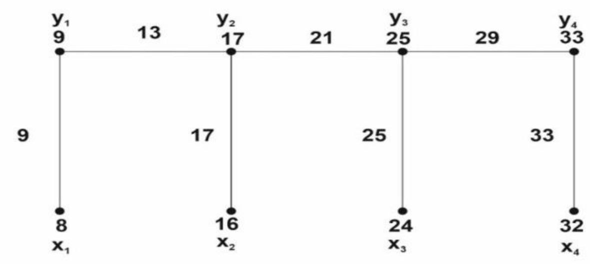

Figure - 2

Theorem 3.6: $B_{n,n}$ is a ℓ- hilbert mean graph, where $n \geq 2$.

Proof: Let $G = B_{n,n}$, where $n \geq 2$.

Let $V(G) = \{x, y, x_i, y_i : 1 \leq i \leq n\}$ and $E(G) = \{xy, xx_i, yy_i : 1 \leq i \leq n\}$

We observe that G has $2n+2$ vertices and $2n+1$ edges.

Define $f: V(G) \to \{0, 1, 2, \dots, Ꜧ_{\ell+2n}\}$ is defined as follows.

$f(x) = 4(\ell - 1)$, $f(x_i) = 4(\ell + 2i - 3) + 1, \quad 1 \leq i \leq n$

$f(y_i) = 4(\ell + 2i - 1), \quad 1 \leq i \leq n$, $f(y) = 4(\ell + 2n - 1) + 1$.

Clearly f is injective and the induced edge labeling f^* is defined as follows.

$f^*(xx_i) = Ꜧ_{\ell+(i-1)}$ where $1 \leq i \leq n$, $f^*(xy) = H_{\ell+n}$,

$f^*(yy_i) = Ꜧ_{\ell+(n+i)}$ where $1 \leq i \leq n$

Thus, we get the induced edge labels as $Ꜧ_\ell, Ꜧ_{\ell+1}, \dots, Ꜧ_{\ell+2n}$.

Hence $B_{n,n}$ is a ℓ- hilbert mean graph, where $n \geq 3$.

Example 3.7: ℓ- hilbert mean labeling of $B_{4,4}$ is shown in figure 3.

Tuijin Jishu/Journal of Propulsion Technology
ISSN: 1001-4055
Vol. 44 No. 3 (2023)

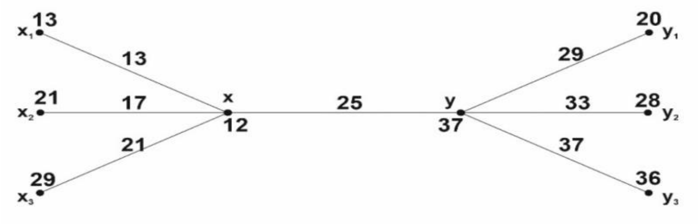

Figure - 3

Theorem 3.8: $H(P_n)$ is a ℓ- hilbert mean graph, where $n \geq 3$.
Proof: Let $G = H(P_n)$, Let $V(G) = \{x_i, y_i : 1 \leq i \leq n\}$and
$E(G) = \{x_i y_i : 1 \leq i \leq n\} \cup \{x_i x_{i+1}, y_i\, y_{i+1} : 1 \leq i \leq n-1\}$
Let G has $2n$ vertices and $2n-1$ edges.
We define a labeling $f : V(G) \to \{0, 1, 2, \dots, Ӈ_{\ell+2n-2}\}$ as follows.
Case 1: n is odd, For $1 \leq i \leq n$,

$$f(x_i) = \begin{cases} 4(\ell + i - 2), & i \text{ is odd} \\ 4(\ell + i - 3) + 1, & i \text{ is even} \end{cases}$$

$$f(y_i) = \begin{cases} 4(\ell + n + i - 3) + 1, & i \text{ is odd} \\ 4(\ell + n + i - 2), & i \text{ is even} \end{cases}$$

Case 2: n is even, For $1 \leq i \leq n$,

$$f(x_i) = \begin{cases} 4(\ell + i - 2), & i \text{ is odd} \\ 4(\ell + i - 3) + 1, & i \text{ is even} \end{cases}$$

$$f(y_i) = \begin{cases} 4(\ell + n + i - 2), & i \text{ is odd} \\ 4(\ell + n + i - 3) + 1, & i \text{ is even} \end{cases}$$

Clearly f is injective and the induced edge labeling f^* is defined as follows.

$f^*\left(x_{\frac{n+1}{2}}, y_{\frac{n+1}{2}}\right) = H_{\ell+(n-1)}$, where n *is odd*

$f^*\left(x_{\frac{n+2}{2}}, y_{\frac{n}{2}}\right) = H_{\ell+(n-1)}$, where n *is even*

$f^*(x_i\, x_{i+1}) = H_{\ell+(i-1)}$, where $1 \leq i \leq n-1$

$f^*(y_i\, y_{i+1}) = H_{\ell+(n+i-1)}$, where $1 \leq i \leq n-1$

Thus, we get the induced edge labels as $Ӈ_\ell, Ӈ_{\ell+1} \dots, Ӈ_{\ell+2n-2}$.
Hence $H(P_n)$ is a ℓ- hilbert mean graph, where $n \geq 3$.
Example 3.9: ℓ- hilbert mean labeling of $H(P_3)$ is shown in figure 4.

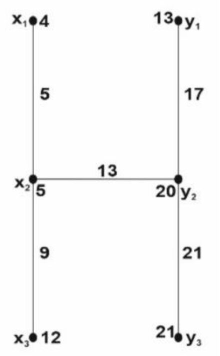

Figure – 4

Theorem 3.10: $F(P_n)$ is a ℓ- hilbert mean graph, where n is even and $n \geq 4$.
Proof: Let $G = F(P_n)$. Let $V(G) = \{x_i : i = 1, 2\} \cup \{y_i : 1 \leq i \leq n\}$ and
$E(G) = \{x_i y_i : i = 1, 2\} \cup \{y_i y_{i+1} : 1 \leq i \leq n-1\}$.
We observe that G has $n+2$ vertices and $n+1$ edges.
We define a labeling $f : V(G) \to \{0, 1, 2, \dots, Ӈ_{\ell+n}\}$ as follows.

$f(y_1) = 4\ell - 3,\ f(x_i) = 4(\ell + 2i - 3), \quad i = 1, 2$

For $1 \le i \le n - 1$,

$$f(y_{i+1}) = \begin{cases} 4(\ell + i) + 1, & i \text{ is odd} \\ 4(\ell + i + 1), & i \text{ is even} \end{cases}$$

Clearly f is injective and the induced edge labeling f^* is defined as follows.

$f^*(x_i y_i) = H_{\ell+2(i-1)}$, where $i = 1, 2$, $f^*(y_1 y_2) = Ӈ_{\ell+1}$,

$f^*(y_i y_{i+1}) = Ӈ_{\ell+(i+1)}$, where $2 \le i \le n - 1$

Thus, we get the induced edge labels as $Ӈ_{\ell}, Ӈ_{\ell+1} \dots, Ӈ_{\ell+n}$.

$F(P_n)$ is a ℓ- hilbert mean graph, where n is even and $n \ge 4$.

Example 3.11: ℓ- hilbert mean labeling of $F(P_4)$ is shown in figure 5.

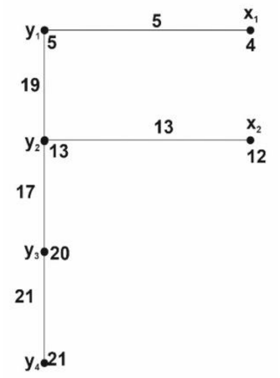

Figure - 5

Theorem 3.12: L_n is a ℓ- hilbert mean graph, where $n \ge 3$.

Proof: Let $G = L_n$. Let $V(G) = \{x_i, y_i : 1 \le i \le n\}$ and

$E(G) = \{x_i y_i : 1 \le i \le n\} \cup \{x_i x_{i+1}, y_i y_{i+1} : 1 \le i \le n - 1\}$

Let G has $2n$ vertices $3n - 2$ edges. Define $f : V(G) \to \{0, 1, 2, \dots, Ӈ_{\ell+3n-3}\}$ as follows.

Case 1: n *is odd* and $\ell \ge 1$,

For $1 \le i \le n$,

$$f(x_i) = \begin{cases} 4(\ell + i - 2), & i \text{ is odd} \\ 4(\ell + i - 3), & i \text{ is even} \end{cases}$$

$$f(y_i) = \begin{cases} 4(\ell + 2n + i - 4) + 1, & i \text{ is odd} \\ 4(\ell + 2n + i - 3), & i \text{ is even} \end{cases}$$

Case 2: n *is even* and $\ell \ge 2$, $f(x_1) = 4\ell - 7$,

For $2 \le i \le n$,

$$f(x_i) = \begin{cases} 4(\ell + i - 3) + 1, & i \text{ is odd} \\ 4(\ell + i - 2), & i \text{ is even} \end{cases}$$

$$f(y_i) = \begin{cases} 4(\ell + 2n + i - 3), & i \text{ is odd} \\ 4(\ell + 2n + i - 4) + 1, & i \text{ is even} \end{cases}$$

Clearly f is injective and the induced edge labeling f^* is defined as follows.

$f^*(x_i x_{i+1}) = H_{\ell+(i-1)}$, where $1 \le i \le n - 1$,

$f^*(y_i y_{i+1}) = H_{\ell+(2n+i-2)}$, where $1 \le i \le n - 1$,

$f^*(x_i y_i) = H_{\ell+(n+i-2)}$, where $1 \le i \le n$

Thus, we get the induced edge labels as $Ӈ_{\ell}, Ӈ_{\ell+1} \dots, Ӈ_{\ell+3n-3}$.

Hence L_n is a ℓ- hilbert mean graph, where $n \ge 3$.

Example 3.13: ℓ- hilbert mean labeling of L_3 is shown in figure 6.

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ISSN: 1001-4055
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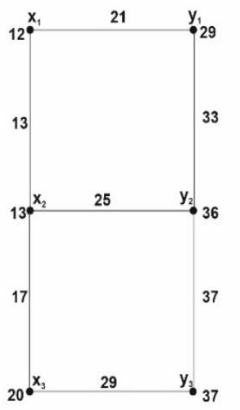

Figure - 6

Theorem 3.14: SL_n is a ℓ- hilbert mean graph, where n is even and $n \geq 3$.
Proof: Let $G = SL_n$. Let $V(G) = \{x_i, y_i : 1 \leq i \leq n\}$and
$E(G) = \{x_i x_{i+1}, y_i y_{i+1}, x_{i+1} y_i : 1 \leq i \leq n-1\}$
Let G has $2n$ vertices $3n-3$ edges. Define $f : V(G) \to \{0, 1, 2, \ldots, Ꜧ_{\ell+3n-4}\}$as follows.
Case 1: n *is odd* and $\ell \geq 2$, $f(x_1) = 4\ell - 7$,
For $2 \leq i \leq n$,

$$f(x_i) = \begin{cases} 4(\ell + i - 3) + 1, & i \text{ is odd} \\ 4(\ell + i - 2), & i \text{ is even} \end{cases}$$

$$f(y_i) = \begin{cases} 4(\ell + 2n + i - 5), & i \text{ is odd} \\ 4(\ell + 2n + i - 4) + 1, & i \text{ is even} \end{cases}$$

Case 2: n *is even* and $\ell \geq 2$,
For $1 \leq i \leq n$,

$$f(x_i) = \begin{cases} 4(\ell + i - 2), & i \text{ is odd} \\ 4(\ell + i - 3) + 1, & i \text{ is even} \end{cases}$$

$$f(y_i) = \begin{cases} 4(\ell + 2n + i - 4), & i \text{ is odd} \\ 4(\ell + 2n + i - 5) + 1, & i \text{ is even} \end{cases}$$

Clearly f is injective and the induced edge labeling f^* is defined as follows.
$f^*(x_i x_{i+1}) = H_{\ell+(i-1)}$, where $1 \leq i \leq n-1$
$f^*(y_i y_{i+1}) = H_{\ell+(2n+i-3)}$, where $1 \leq i \leq n-1$
$f^*(x_{i+1} y_i) = H_{\ell+(n+i-2)}$, where $1 \leq i \leq n-1$
Thus, we get the induced edge labels as $Ꜧ_\ell, Ꜧ_{\ell+1}, \ldots, Ꜧ_{\ell+3n-4}$.
Hence SL_n is a ℓ- hilbert mean graph, where n is even and $n \geq 3$.
Example 3.15: ℓ- hilbert mean labeling of SL_4 is shown in figure 7.

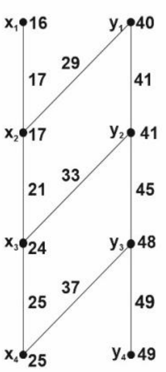

Figure - 7

Theorem 3.16: $T(n)$ is a ℓ- hilbert mean graph, where n is $even$ and $n \geq 4$.

Proof: Let $G = T(n)$. Let $V(G) = \{x_i : 1 \leq i \leq n\} \cup \{ y_i, z_i : 1 \leq i \leq n-2\}$

$E(G) = \{x_i\ x_{i+1} : 1 \leq i \leq n-1\} \cup \{y_i z_i : 1 \leq i \leq n-2\} \cup \{x_i z_i : 1 \leq i \leq n-2\}$

Let G has $3n-4$ vertices and $3n-5$ edges.

Define a function $f : V(G) \rightarrow \{0, 1, 2, \ldots, Ꜧ_{\ell+2n-2}\}$ as follows.

For $1 \leq i \leq n$,

$$f(x_i) = \begin{cases} 4(\ell + 3i - 4), & i \text{ is odd} \\ 4(\ell + 3i - 7) + 1, & i \text{ is even} \end{cases}$$

$$f(y_i) = \begin{cases} 4(\ell + 3i - 2), & i \text{ is odd} \\ 4(\ell + 3i - 5) + 1, & i \text{ is even} \end{cases}$$

$$f(z_i) = \begin{cases} 4(\ell + 3i), & i \text{ is odd} \\ 4(\ell + 3i - 3) + 1, & i \text{ is even} \end{cases}$$

Clearly f is injective and the induced edge labeling f^* is defined as follows.

$f^*(x_i\, x_{i+1}) = H_{\ell+(i-1)}$, where $1 \leq i \leq n-1$

$f^*(x_{i+1} y_i) = H_{\ell+(3i-2)}$, where $1 \leq i \leq n-2$

$f^*(x_{i+1}\, z_i) = H_{\ell+(3i-1)}$, where $1 \leq i \leq n-2$

Thus, we get the induced edge labels as $Ꜧ_\ell, Ꜧ_{\ell+1}, \ldots, Ꜧ_{\ell+2n-2}$.

Hence $T(n)$ is a ℓ- hilbert mean graph, where n is $even$ and $n \geq 4$.

Example 3.17: ℓ- hilbert mean labeling of $T(4)$ is shown in figure 8.

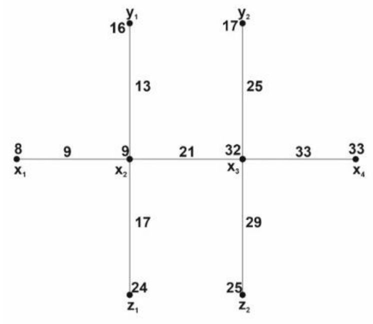

Figure - 8

4. Conclusion

In this paper, we have introduced ℓ- hilbert mean labeling and studied ℓ- hilbert mean labeling of some path related graphs. This work contributes several new results to the theory of graph labeling.

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