

A New Approach on Bipolar Second Degree Perfect Fuzzy Matching for Bipolar Fuzzy Graphs Based on Vertices

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Abstract: Graph is a visual representation of data. In mathematics, graph is a structure which creates the pairwise relation between objects. The objects are known as nodes or vertices and the connection between two nodes are known as links or edges. A graph is a symmetric binary relation on a non-empty set V whereas Fuzzy graph is a symmetric binary fuzzy relation on a fuzzy subset. Bipolar Fuzzy graph is a generalization of graph theory by using bipolar Fuzzy set. The bipolar Fuzzy sets are the extension of Fuzzy sets. Our main objective is to introduce Bipolar Second Degree Perfect Fuzzy Matching for Bipolar Fuzzy Graph based on vertices. Secondly, we proved the necessary conditions under which they are equivalent and also proved that for a particular condition a perfect second-degree fuzzy matching is not a $(2, (k_1, k_2))$ regular bipolar fuzzy graph. Next, we discussed about perfect second-degree Fuzzy matching and second degree fuzzy matching number for bipolar fuzzy graph on the cycle.

Keyword: Fuzzy Graph, Bipolar Second-Degree Fuzzy Matching, Bipolar Second Degree Perfect Fuzzy Matching, Bipolar Second-Degree Fuzzy Matching number.

1. Introduction

Leonhard Euler founded the studies of graph theory and topology. He was known as father of graph theory. A graph which has non empty set of vertices connected at most by one edge is called simple graph. In mathematics, a graph can be defined as a pair $G = (V, E)$ where V is the vertices of G and E is the edges of G . Graphs are the effective visual tool store present information quickly and easily. When we represent data in a graph, it can be better understood because the graph can expose the trend or comparison. Graph theory is the study of graphs that deals with the arrangement of certain objects known as vertices and the relationship between them. Graph theory is used to solve problems which have a fairly natural graph structure.

Many of our tools for computing, modeling and reasoning are based on dual logic which means yes or no type or true or false type or more or less type and nothing in between them. This indicates that there is no uncertainty. These assumptions are not justified for all real models. In 1923, Russell wrote that all traditional logic habitually assumes that precise symbols are being employed. It is, therefore, not applicable to this terrestrial life but only to an imagined celestial existence. Zadeh wrote that as the complexity of a system increases, our ability to make precise and yet significant statements about its behavior diminishes until a threshold is reached beyond which precision and significance become almost mutually exclusive characteristics. Probability theory and statistics were used to model uncertainties of reality. Meanwhile more uncertainty theories have been developed. Fuzzy set theory is one of the uncertainty theories which is an extension of dual logic.

Fuzzy means vagueness. True or false is sometimes insufficient when describing human reasoning. Fuzzy logic uses the whole interval between 0 and 1 to describe human reasoning. Fuzzy graph is the generalization of the ordinary graph. A fuzzy set is a class of objects with a continuum of grades of membership. The fuzzy relations between fuzzy sets were first considered by Rosenfeld and he developed the structure of

fuzzy graph obtaining analogs of several graph theoretical concepts. Fuzzy mathematics is the branch of mathematics including fuzzy set theory and fuzzy logic that deals with partial inclusion of elements in a set on a spectrum as opposed to simple binary yes or no (0 or 1) inclusion. In many domains of information processing, bipolarity is a core feature to be considered as positive information represents what is possible or preferred, while negative information represents what is forbidden or surely false. If the information is more over endowed with vagueness and imprecision, then Bipolar fuzzy set constitute an appropriate knowledge representation frame work. The range of membership degree of bipolar fuzzy set is $[-1,1]$. In a bipolar fuzzy set the positive $(0, 1]$ of an element somewhat satisfies the corresponding property the negative membership degree $[-1, 0)$ of an element indicates that the element somewhat satisfies the implicit counter property.

As a generalization of fuzzy set theory Bipolar fuzzy set theory gives more precision, flexibility and compatibility to the system as compared to the classical and fuzzy models. The concept of bipolar fuzzy graphs can be applied in various areas of engineering, computer science, database theory, expert system, neural network, artificial intelligence, signal processing, pattern recognition, robotics, computer networks and medical diagnosis. Recently, bipolar fuzzy graph is a growing research topic as it is the generalization of fuzzy graphs. The notation of matching in a fuzzy graph could be defined using the concepts of effective edges and fractional matching.

2. Literature Review

In 1965, Zadeh [32] introduced the notation of fuzzy subset for a set. Then, the theory of fuzzy sets has become a vigorous area of research in different disciplines. In 1975, Rosenfeld [21] discussed the concept of fuzzy graphs. In 1994, Zhang [33] initiated the concept of bipolar fuzzy sets as the generalization of fuzzy sets. A bipolar fuzzy set is an extension of Zadeh's fuzzy set theory, whose membership degree range is $[-1, 1]$. Shakila Banu and Akilandeswari [25] introduced the concept of square perfect fuzzy matching. Seethalakshmi and Gnanajothi [22] derived the necessary conditions of fuzzy graph on a cycle. The concept of $d_2 - degree$ and total $d_2 - degree$ of a vertex in a fuzzy graph was defined by Sekar and Santhimaheswari [24]. Ravi Narayanan and SanthiMaheswari introduced $d_2 - degree$ and $d_m - degree$ of a vertex in bipolar fuzzy graph [18][19]. Anu j Kumar and Pradhan [3] introduced a new concept of fuzzy distance two labeling and discussed some properties related to product fuzzy graph and fuzzy distance two labeling graph. Richa Bansal and Sandhya Rajput [20] proposed new analogous results for fuzzy spanning sub graph, complete, regular and connected fuzzy graph along with proof.

Seethalakshmi and Gnanajothi [23] introduced the notation of bipolar fuzzy matching, bipolar perfect fuzzy matching and proved some results of a bipolar fuzzy graph. H. J. Zimmermann [34] described the basic mathematical framework of fuzzy set theory as well as the most important applications of this theory to other theories and techniques. Arjunan K. and Subramani C. studies some of the properties of fuzzy graph and proved some results [4]. Mythili V.,Kaliyappan M. and Hariharan S. [15] has been reviewed the recent developments in fuzzy graph theory. Sunitha [28] and Animesh Kumar Sharma [5] reviewed the works of fuzzy graphs. Craine [6] analyzed various properties of fuzzy interval graphs. SovanSamanta [30] discussed some result of bipolar fuzzy graph. Sunitha M. S. and Vijaya Kumar A. discussed the concept of characterization of fuzzy trees [29]. Waheed Ahmed Khan, Babir Ali and Abdelghani Taouti [31] introduced and discussed the term bipolar picture fuzzy graph along with some of its fundamental characteristics and applications. Ghorai Ganesh and Pal [11][12] described various properties of m-polar fuzzy graph and they also examined isomorphic properties of m-polar fuzzy graph. Ghorai Ganesh and Pal [10] defined generalized regular bipolar fuzzy graph. Ghorai Ganesh [9] discussed the several characterizations of bipolar fuzzy graph.

Nagoorgani and Basheer Ahmed [15] examined the properties of various types of degree order, size of fuzzy graphs and compared the relationship between them. Muhammad Akram and Wieslaw Dudek [14] introduce the concepts of regular and totally regular bipolar fuzzy graphs. The notation of fuzzy line graph was introduced by Mordeson [13]. Shannon and Atanassov [26] introduced the concept of intuitionistic fuzzy relations & intuitionistic fuzzy graphs and investigated some of their properties in [27]. Parvathi defined operations on intuitionistic fuzzy graphs in [17]. Akram and Dudek has introduced the notations of bipolar fuzzy graphs in [1] and interval valued fuzzy line graphs in [2].

3. Preliminaries

Definition 3.1: A fuzzy graph $G: (\sigma, \mu)$ is a pair of functions (σ, μ) where, $\sigma: V \rightarrow [0, 1]$ is a fuzzy subset of a non-empty set V and $\mu: V \times V \rightarrow [0, 1]$ is a symmetric fuzzy relation on σ such that for all u, v in V , the relation $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$ is satisfied. A fuzzy graph G is called complete fuzzy graph if the relation $\mu(u, v) = \sigma(u) \wedge \sigma(v)$ is satisfied.

Definition 3.2: Let $G: (\sigma, \mu)$ be a fuzzy graph. The d_2 -degree of a vertex u in G is $d_2(u) = \sum \mu^2(u, v)$ where $\mu^2(u, v) = \{ \mu(u, u_1) \wedge \mu(u_1, v) : u, u_1, v \text{ is the shortest path connecting } u \text{ and } v \text{ of length } 2 \}$. Also $\mu(u, v) = 0$ for $(u, v) \notin E$.

The minimum d_2 -degree of G is $\delta_2(G) = \wedge \{d_2(u) : u \in V\}$.

The maximum d_2 -degree of G is $\Delta_2(G) = \vee \{d_2(u) : u \in V\}$.

Definition 3.3: Let $G: (\sigma, \mu)$ be a fuzzy graph on $G^*: (V, E)$. If $d_2(u) = k$ for all $u \in V$ then G is said to be a $(2, k)$ -regular fuzzy graph.

Definition 3.4: A bipolar fuzzy graph with an underlying set V is defined to be the pair $G: (A, B)$ where $A = (m_1^+, m_1^-)$ is a bipolar fuzzy set on V and $B = (m_2^+, m_2^-)$ is a bipolar fuzzy set on E such that $m_2^+(u, v) \leq \min\{m_1^+(u), m_1^+(v)\}$ and $m_2^-(u, v) \geq \max\{m_1^-(u), m_1^-(v)\}$ for all $(u, v) \in E$. Here A is bipolar fuzzy vertex set of V and B is bipolar fuzzy edge set of E .

Definition 3.5: The positive degree of a vertex $u \in G$ is $d^+(u) = \sum m_2^+(u, v)$. The negative degree of a vertex $u \in G$ is $d^-(u) = \sum m_2^-(u, v)$. The degree of the vertex u is called as $d(u) = (d^+(u), d^-(u))$.

Definition 3.6: Let $G: (A, B)$ be a bipolar fuzzy graph where $A = (m_1^+, m_1^-)$ and $B = (m_2^+, m_2^-)$ are two bipolar fuzzy sets on a non-empty finite set V . Then G is said to be a (c_1, c_2) -regular bipolar fuzzy graph if all the vertices of G has same degree (c_1, c_2) .

Definition 3.7: Let $G: (A, B)$ be a bipolar fuzzy graph. The positive d_2 -degree of a vertex $u \in G$ is defined as $d_2^+(u) = \sum m_2^{(2,+)}(u, v)$ where $m_2^{(2,+)}(u, v) = \{m_2^+(u, u_1) \wedge m_2^+(u_1, v) : u, u_1, v \text{ is the shortest path connecting } u \text{ and } v \text{ of length } 2\}$. The negative d_2 -degree of a vertex $u \in G$ is defined as $d_2^-(u) = \sum m_2^{(2,-)}(u, v)$ where $m_2^{(2,-)}(u, v) = \{m_2^-(u, u_1) \vee m_2^-(u_1, v) : u, u_1, v \text{ is the shortest path connecting } u \text{ and } v \text{ of length } 2\}$. The d_2 -degree of a vertex u is defined as, $d_2(u) = (d_2^+(u), d_2^-(u))$

The minimum d_2 -degree of G is $\delta_2(G) = \wedge \{d_2(u) : u \in V\}$.

The maximum d_2 -degree of G is $\Delta_2(G) = \vee \{d_2(u) : u \in V\}$.

Definition 3.8: Let $G: (A, B)$ be a bipolar fuzzy graph. If $d_2(u) = (c_1, c_2)$ for all $u \in V$ then G is said to be a $(2, (c_1, c_2))$ -regular bipolar fuzzy graph.

4. Bipolar Second Degree Perfect Fuzzy Matching

Definition 4.1: Let $G: (A, B)$ be a bipolar fuzzy graph where $A = (m_1^+, m_1^-)$ and $B = (m_2^+, m_2^-)$ be two bipolar fuzzy sets on a non-empty finite set V and $E \subset V \times V$ respectively. A subset S of V is called a bipolar second-degree fuzzy matching if for each vertex u , we have

$$\sum_{\substack{u \neq v \\ u, v \in V}} m_2^{(2,+)}(u, v) \leq m_1^+(u) \quad \text{and} \quad \sum_{\substack{u \neq v \\ u, v \in V}} m_2^{(2,-)}(u, v) \geq m_1^-(u)$$

Example 4.1: Consider the following bipolar fuzzy graph G on (V, E) .

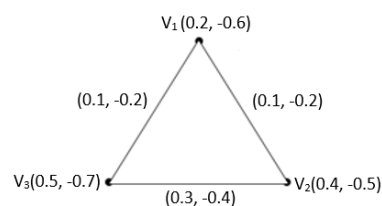


Fig 1: The bipolar fuzzy graph $G: (A, B)$

$$\begin{aligned} \sum_{\substack{v_1 \neq v_3 \\ v_1, v_3 \in V}} m_2^{(2,+)}(v_1, v_3) &= m_2^{(2,+)}(v_1, v_3) + m_2^{(2,+)}(v_1, v_2) \\ &= \{m_2^+(v_1, v_2) \wedge m_2^+(v_2, v_3)\} + \{m_2^+(v_1, v_3) \wedge m_2^+(v_3, v_2)\} \\ &= \{0.1 \bigwedge 0.3 + 0.1 \bigwedge 0.3\} = 0.1 + 0.1 = 0.2 \leq m_1^+(v_1) \end{aligned}$$

$$\begin{aligned} \sum_{\substack{v_2 \neq v_1 \\ v_2, v_1 \in V}} m_2^{(2,+)}(v_2, v_1) &= m_2^{(2,+)}(v_2, v_1) + m_2^{(2,+)}(v_2, v_3) \\ &= \{0.3 \bigwedge 0.1 + 0.1 \bigwedge 0.1\} = 0.1 + 0.1 = 0.2 \leq m_2^+(v_2) \end{aligned}$$

$$\begin{aligned} \sum_{\substack{v_3 \neq v_2 \\ v_3, v_2 \in V}} m_2^{(2,+)}(v_3, v_2) &= m_2^{(2,+)}(v_3, v_2) + m_2^{(2,+)}(v_3, v_1) \\ &= \{0.1 \bigwedge 0.1 + 0.3 \bigwedge 0.1\} = 0.2 \leq m_3^+(v_3) \end{aligned}$$

Similarly,

$$\begin{aligned} \sum_{\substack{v_1 \neq v_3 \\ v_1, v_3 \in V}} m_2^{(2,-)}(v_1, v_3) &= \{-0.2 \vee -0.4 + -0.2 \vee -0.4\} \\ &= \{-0.2 + -0.2\} = -0.4 \geq m_1^-(v_1) \end{aligned}$$

$$\begin{aligned} \sum_{\substack{v_2 \neq v_1 \\ v_2, v_1 \in V}} m_2^{(2,-)}(v_2, v_1) &= \{-0.4 \vee -0.2 + -0.2 \vee -0.2\} \\ &= \{-0.2 + -0.2\} = -0.4 \geq m_1^-(v_2) \end{aligned}$$

$$\begin{aligned} \sum_{\substack{v_3 \neq v_2 \\ v_3, v_2 \in V}} m_2^{(2,-)}(v_3, v_2) &= \{-0.2 \vee -0.2 + -0.4 \vee -0.2\} \\ &= \{-0.2 + -0.2\} = -0.4 \geq m_1^-(v_3) \end{aligned}$$

Thus $\{v_1, v_2, v_3\}$ is a bipolar second-degree fuzzy matching on G .

Definition 4.2: A bipolar second-degree fuzzy matching S is called a bipolar second degree perfect fuzzy matching if,

$$\sum_{\substack{u \neq v \\ u, v \in V}} m_2^{(2,+)}(u, v) = m_1^+(u) \text{ and } \sum_{\substack{u \neq v \\ u, v \in V}} m_2^{(2,-)}(u, v) = m_1^-(u)$$

Definition 4.3: Let G be a bipolar fuzzy graph on the underlying graph (V, E) . Let S be a bipolar second-degree fuzzy matching for G . The bipolar second-degree fuzzy matching number $\Gamma(G)$ is defined as:

$$\Gamma(G) = \left(\sum_{u \in S} m_2^{(2,+)}(u, v), \sum_{u \in S} m_2^{(2,-)}(u, v) \right)$$

Example 4.2: In this example we have consider Figure 1 and Example 3.1

$$\Gamma(G) = \left(\sum_{u \in S} m_2^{(2,+)}(u, v), \sum_{u \in S} m_2^{(2,-)}(u, v) \right) \text{ where } S = \{v_1, v_2, v_3\}$$

$$\sum_{u \in S} m_2^{(2,+)}(u, v) = 0.2 + 0.2 + 0.2 = 0.6$$

$$\sum_{u \in S} m_2^{(2,-)}(u, v) = (-0.4) + (-0.4) + (-0.4) = -1.2$$

Therefore $\Gamma(G) = (0.6, -1.2)$

Theorem 4. 1: Let $G : (A, B)$ be a bipolar fuzzy graph on the cycle $G^* : (V, E)$. Then edges of the bipolar fuzzy graph of G is half of their vertices iff all the vertices of G are bipolar second degree perfect fuzzy matching and is equivalent to $(2, (k_1, k_2))$ is regular bipolar fuzzy graph.

Proof

Suppose that $m_1^+(u)$ and $m_1^-(u)$ are constant functions. Let $m_1^+(u) = k_1$ and $m_1^-(u) = k_2$ for all $u \in V$, $m_2^+(u, v) = k_1/2$ and $m_2^-(u, v) = k_2/2$ for all $(u, v) \in E$. Assume that G is a $(2, (k_1, k_2))$ regular bipolar fuzzy graph on the cycle $G^* : (V, E)$.

Then $d_2^+(u) = k_1$ and $d_2^-(u) = k_2$ (i.e.) $d_2(u) = (k_1, k_2)$.

By the definition of d_2 – degree of vertex in bipolar fuzzy graph

$$d_2^+(u) = \sum_{\substack{u \neq v \\ u, v \in V}} m_2^{(2,+)}(u, v) \text{ and } d_2^-(u) = \sum_{\substack{u \neq v \\ u, v \in V}} m_2^{(2,-)}(u, v)$$

(i.e.)

$$\sum_{\substack{u \neq v \\ u, v \in V}} m_2^{(2,+)}(u, v) = k_1 \text{ and } \sum_{\substack{u \neq v \\ u, v \in V}} m_2^{(2,-)}(u, v) = k_2$$

Since G is $(2, (k_1, k_2))$ – regular bipolar fuzzy graph.

$$\Rightarrow \sum_{\substack{u \neq v \\ u, v \in V}} m_2^{(2,+)}(u, v) = m_1^+(u) \text{ and } \sum_{\substack{u \neq v \\ u, v \in V}} m_2^{(2,-)}(u, v) = m_1^-(u)$$

Each vertex of u satisfies the bipolar second degree perfect fuzzy matching in G . Now suppose that G is a bipolar second degree perfect fuzzy matching on G . Since G is a bipolar fuzzy graph on the cycle and only two edges are incident with each vertex for cycles.

For any vertex $u \in V$,

$$\sum_{\substack{u \neq v \\ u, v \in V}} m_2^{(2,+)}(u, v) = m_1^+(u) \text{ and } \sum_{\substack{u \neq v \\ u, v \in V}} m_2^{(2,-)}(u, v) = m_1^-(u)$$

By definition,

$$d_2^+(u) = \sum_{\substack{u \neq v \\ u, v \in V}} m_2^{(2,+)}(u, v) \text{ and } d_2^-(u) = \sum_{\substack{u \neq v \\ u, v \in V}} m_2^{(2,-)}(u, v)$$

$$\Rightarrow d_2^+(u) = k_1 \text{ and } d_2^-(u) = k_2$$

Hence G is a $(2, (k_1, k_2))$ regular bipolar fuzzy graph. The converse of the theorem is trivially true.

Example 4.3: Consider the bipolar fuzzy graph $G : (A, B)$ on $G^* : (V, E)$ in Figure 2.

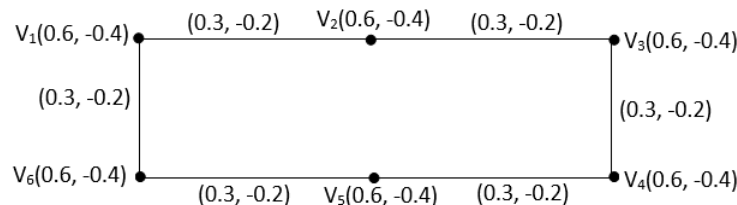


Fig 2: $(2, (0.6, -0.4))$ – regular bipolar fuzzy graph.

$$\begin{aligned} \sum_{\substack{v_1 \neq v_3 \\ v_1, v_3 \in V}} m_2^{(2,+)}(v_1, v_3) &= m_2^{(2,+)}(v_1, v_3) + m_2^{(2,+)}(v_1, v_5) \\ &= \{m_2^+(v_1, v_2) \wedge m_2^+(v_2, v_3)\} + \{m_2^+(v_1, v_6) \wedge m_2^+(v_6, v_5)\} \\ &= \{0.3 \wedge 0.3\} + \{0.3 \wedge 0.3\} = 0.6 = m_1^+(v_1) \end{aligned}$$

Similarly,

$$\sum_{\substack{v_2 \neq v_4 \\ v_2, v_4 \in V}} m_2^{(2,+)}(v_2, v_4) = 0.6 = m_1^+(v_2)$$

$$\sum_{\substack{v_3 \neq v_5 \\ v_3, v_5 \in V}} m_2^{(2,+)}(v_3, v_5) = 0.6 = m_1^+(v_3)$$

$$\sum_{\substack{v_4 \neq v_6 \\ v_4, v_6 \in V}} m_2^{(2,+)}(v_4, v_6) = 0.6 = m_1^+(v_4)$$

$$\sum_{\substack{v_5 \neq v_1 \\ v_5, v_1 \in V}} m_2^{(2,+)}(v_5, v_1) = 0.6 = m_1^+(v_5)$$

$$\sum_{\substack{v_6 \neq v_2 \\ v_6, v_2 \in V}} m_2^{(2,+)}(v_6, v_2) = 0.6 = m_1^+(v_6)$$

$$\begin{aligned} \sum_{\substack{v_1 \neq v_3 \\ v_1, v_3 \in V}} m_2^{(2,-)}(v_1, v_3) &= m_2^{(2,-)}(v_1, v_3) + m_2^{(2,-)}(v_1, v_5) \\ &= \{m_2^-(v_1, v_2) \vee m_2^-(v_2, v_3)\} + \{m_2^-(v_1, v_6) \vee m_2^-(v_6, v_5)\} \\ &= \{-0.2 \vee -0.2\} + \{-0.2 \vee -0.2\} = (-0.2) + (-0.2) = -0.4 = m_1^-(v_1) \end{aligned}$$

Similarly,

$$\sum_{\substack{v_2 \neq v_4 \\ v_2, v_4 \in V}} m_2^{(2,-)}(v_2, v_4) = -0.4 = m_2^-(v_2)$$

$$\sum_{\substack{v_3 \neq v_5 \\ v_3, v_5 \in V}} m_2^{(2,-)}(v_3, v_5) = -0.4 = m_2^-(v_3)$$

$$\sum_{\substack{v_4 \neq v_6 \\ v_4, v_6 \in V}} m_2^{(2,-)}(v_4, v_6) = -0.4 = m_2^-(v_4)$$

$$\sum_{\substack{v_5 \neq v_1 \\ v_5, v_1 \in V}} m_2^{(2,-)}(v_5, v_1) = -0.4 = m_2^-(v_5)$$

$$\sum_{\substack{v_6 \neq v_2 \\ v_6, v_2 \in V}} m_2^{(2,-)}(v_6, v_2) = -0.4 = m_2^-(v_6)$$

Hence G is a bipolar second degree perfect fuzzy matching and also $(2, (0.6, -0.4))$ regular bipolar fuzzy graph.

Theorem 4.2: Let $G : (A, B)$ be a $(2, (k_1, k_2))$ - regular bipolar fuzzy graph on the cycle $G^* : (V, E)$. $m_1^+(u)$, $m_1^-(u)$, $m_2^+(u, v)$ and $m_2^-(u, v)$ are constant functions where, $m_2^+(u, v) \leq m_1^+(u)$, $m_2^-(u, v) \geq m_1^-(u)$, $m_2^+(u, v) \neq \frac{1}{2}m_1^+(u)$ and $m_2^-(u, v) \neq \frac{1}{2}m_1^-(u)$, for all $u \in V, (u, v) \in E$ then V is not a bipolar second degree perfect fuzzy matching of G .

Proof:

Let $m_1^+(u) = k_1$, $m_1^-(u) = k_2$, $m_2^+(u, v) = c_1$ and $m_2^-(u, v) = c_2$ are constant functions for all $u \in V$ and $(u, v) \in E$ where $c_1 \leq k_1$, $c_2 \geq k_2$, $c_1 \neq \frac{k_1}{2}$ and $c_2 \neq \frac{k_2}{2}$. To prove V is not a bipolar second degree perfect fuzzy matching of G . Suppose that G is bipolar fuzzy graph on the cycle and only two edges are incident with each vertex for the cycles.

For any vertex $u \in V$,

$$\begin{aligned} \sum_{\substack{u \neq w \\ u, w \in V}} m_2^{(2,+)}(u, w) &= m_2^{(2,+)}(u, w) + m_2^{(2,+)}(u, v) \\ &= c_1 + c_1 \\ &= 2c_1 \leq k_1 \Rightarrow c_1 \leq \frac{k_1}{2} \text{ but } c_1 \neq \frac{k_1}{2} \\ &\Rightarrow c_1 < k_1 \\ &\Rightarrow \sum_{\substack{u \neq w \\ u, w \in V}} m_2^{(2,+)}(u, w) \neq k_1 \Rightarrow \sum_{\substack{u \neq w \\ u, w \in V}} m_2^{(2,+)}(u, w) \neq m_1^+(u) \text{ for all } u \in V \end{aligned}$$

Similarly,

$$\sum_{\substack{u \neq w \\ u, w \in V}} m_2^{(2,-)}(u, w) \neq m_1^-(u) \text{ for all } u \in V$$

Hence V is not a bipolar second degree perfect fuzzy matching of G .

Example 3.4: Consider the following bipolar fuzzy graph G on (V, E)

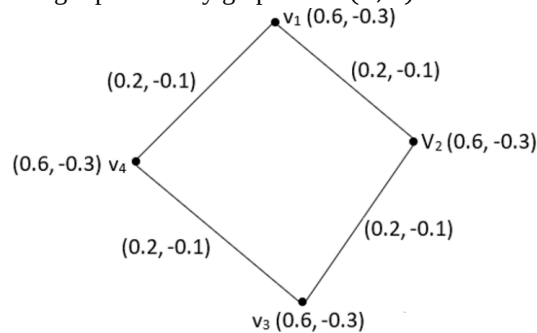


Fig 3: $(2, (0.4, -0.2))$ – regular bipolar fuzzy graph

$$\sum_{\substack{v_1 \neq v_3 \\ v_1, v_3 \in V}} m_2^{(2,+)}(v_1, v_3) = 0.4 \neq 0.6 = m_1^+(v_1)$$

$$\sum_{\substack{v_2 \neq v_4 \\ v_2, v_4 \in V}} m_2^{(2,+)}(v_2, v_4) = 0.4 \neq 0.6 = m_1^+(v_2)$$

$$\sum_{\substack{v_3 \neq v_1 \\ v_3, v_1 \in V}} m_2^{(2,+)}(v_3, v_1) = 0.4 \neq 0.6 = m_1^+(v_3)$$

$$\sum_{\substack{v_4 \neq v_2 \\ v_4, v_2 \in V}} m_2^{(2,+)}(v_4, v_2) = 0.4 \neq 0.6 = m_1^+(v_4)$$

Similarly,

$$\sum_{\substack{v_1 \neq v_3 \\ v_1, v_3 \in V}} m_2^{(2,-)}(v_1, v_3) = -0.2 \neq -0.3 = m_1^-(v_1)$$

$$\sum_{\substack{v_2 \neq v_4 \\ v_2, v_4 \in V}} m_2^{(2,-)}(v_2, v_4) = -0.2 \neq -0.3 = m_1^-(v_2)$$

$$\sum_{\substack{v_3 \neq v_1 \\ v_3, v_1 \in V}} m_2^{(2,-)}(v_3, v_1) = -0.2 \neq -0.3 = m_1^-(v_3)$$

$$\sum_{\substack{v_4 \neq v_2 \\ v_4, v_2 \in V}} m_2^{(2,-)}(v_4, v_2) = -0.2 \neq -0.3 = m_1^-(v_4)$$

Hence G is not a bipolar second degree perfect fuzzy matching.

Theorem 4.3: Let G be a bipolar second degree perfect fuzzy matching on the cycle $G^*: (V, E)$ of length $n \geq 5$.

$$\text{If } m_1^+(u_i) = \begin{cases} \frac{3k_1}{2}, i = 1, 2, n-1, n \\ 2k_1, i = 3, 4, \dots, n-2 \end{cases}$$

$$m_1^-(u_i) = \begin{cases} \frac{-3k_2}{2}, i = 1, 2, n-1, n \\ -2k_2, i = 3, 4, \dots, n-2 \end{cases}$$

for all $u \in V$

$$m_2^+(e_i) = \begin{cases} k_1, i = 1, 2, \dots, n-1 \\ \frac{k_1}{2}, i = n \text{ and} \end{cases}$$

$$m_2^-(e_i) = \begin{cases} -k_2, i = 1, 2, \dots, n-1 \\ -\frac{k_2}{2}, i = n \text{ for all} \end{cases}$$

$(u, v) \in E$. Then G is not a $(2, (k_1, k_2))$ – regular bipolar fuzzy graph.

Proof:

Let $G : (A, B)$ be a bipolar second degree perfect fuzzy matching on the cycle $G^*: (V, E)$ is any length ≥ 5 . To prove that G is not a $(2, (k_1, k_2))$ – regular bipolar fuzzy graph. Let $v_1, v_2, \dots, v_n$ be the vertices and $e_1, e_2, \dots, e_n$ be the edges of the cycle of G^* in that order.

By definition,

$$d_2^+(u) = \sum_{\substack{u \neq v \\ u, v \in V}} m_2^{(2,+)}(u, v) \text{ and } d_2^-(u) = \sum_{\substack{u \neq v \\ u, v \in V}} m_2^{(2,-)}(u, v)$$

$$\begin{aligned} d_2^+(v_1) &= \sum_{\substack{v_1 \neq v_3 \\ v_1, v_3 \in V}} m_2^{(2,+)}(v_1, v_3) \\ &= m_2^{(2,+)}(v_1, v_3) + m_2^{(2,+)}(v_1, v_{n-1}) \\ &= \{m_2^+(v_1, v_2) \wedge m_2^+(v_2, v_3)\} + \{m_2^+(v_1, v_n) \wedge m_2^+(v_n, v_{n-1})\} \\ &= \{m_2^+(e_1) \wedge m_2^+(e_2)\} + \{m_2^+(e_n) \wedge m_2^+(e_{n-1})\} \\ &= \{k_1 \wedge k_1\} + \left\{\frac{k_1}{2} \wedge k_1\right\} \\ &= k_1 + \frac{k_1}{2} = \frac{3k_1}{2} \\ &= m_1^+(v_1) \\ d_2^+(v_2) &= \{m_2^+(e_2) \wedge m_2^+(e_3)\} + \{m_2^+(e_1) \wedge m_2^+(e_n)\} \\ &= \{k_1 \wedge k_1\} + \left\{k_1 \wedge \frac{k_1}{2}\right\} \\ &= k_1 + \frac{k_1}{2} = \frac{3k_1}{2} \\ &= m_1^+(v_2) \end{aligned}$$

$$\begin{aligned} d_2^+(v_i) &= \{m_2^+(e_i) \wedge m_2^+(e_{i+1})\} + \{m_2^+(e_{i-1}) \wedge m_2^+(e_{i-2})\} \\ &= \{k_1 \wedge k_1\} + \{k_1 \wedge k_1\} = 2k_1 \\ &= m_1^+(v_i) \end{aligned}$$

$$\begin{aligned} d_2^+(v_{n-1}) &= \{m_2^+(e_{n-1}) \wedge m_2^+(e_n)\} + \{m_2^+(e_{n-2}) \wedge m_2^+(e_{n-3})\} \\ &= \left\{k_1 \wedge \frac{k_1}{2}\right\} + \{k_1 \wedge k_1\} \\ &= \frac{k_1}{2} + k_1 = \frac{3k_1}{2} \\ &= m_1^+(v_{n-1}) \\ d_2^+(v_n) &= \{m_2^+(e_n) \wedge m_2^+(e_1)\} + \{m_2^+(e_{n-1}) \wedge m_2^+(e_{n-2})\} \\ &= \left\{\frac{k_1}{2} \wedge k_1\right\} + \{k_1 \wedge k_1\} \\ &= \frac{k_1}{2} + k_1 = \frac{3k_1}{2} \\ &= m_1^+(v_n) \end{aligned}$$

Similarly,

$$\begin{aligned} d_2^-(v_1) &= \frac{-3k_2}{2} = m_1^-(v_1) \\ d_2^-(v_2) &= \frac{-3k_2}{2} = m_1^-(v_2) \end{aligned}$$

$$d_2^-(v_i) = -2k_2 = m_1^-(v_i)$$

$$\begin{aligned} d_2^-(v_{n-1}) &= \frac{-3k_2}{2} = m_1^-(v_{n-1}) \\ d_2^-(v_n) &= \frac{-3k_2}{2} = m_1^-(v_n) \end{aligned}$$

Hence, G is not a $(2, (k_1, k_2))$ regular bipolar fuzzy graph.

Example 4.5: Consider the following bipolar fuzzy graph G on (V, E)

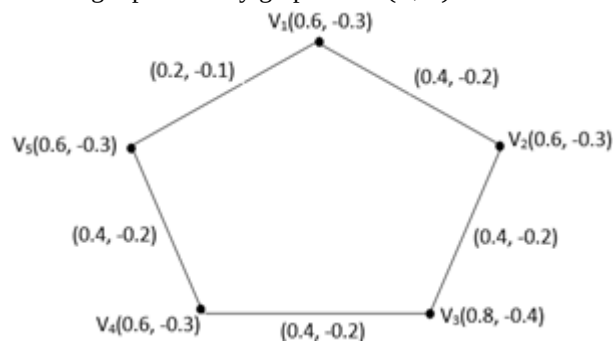


Fig 4: Bipolar Second degree perfect fuzzy matching

$$\begin{aligned} d_2^+(v_1) &= \sum_{\substack{v_1 \neq v_3 \\ v_1, v_3 \in V}} m_2^{(2,+)}(v_1, v_3) \\ &= m_2^{(2,+)}(v_1, v_3) + m_2^{(2,+)}(v_1, v_4) \\ &= \{0.4 \wedge 0.4\} + \{0.2 \wedge 0.4\} \end{aligned}$$

$$= 0.4 + 0.2$$

$$= 0.6 = m_1^+(v_1)$$

$$\text{Similarly, } d_2^+(v_2) = 0.4 + 0.2 = 0.6 = m_1^+(v_2)$$

$$d_2^+(v_3) = 0.4 + 0.4 = 0.8 = m_1^+(v_3)$$

$$d_2^+(v_4) = 0.2 + 0.4 = 0.6 = m_1^+(v_4)$$

$$d_2^+(v_5) = 0.2 + 0.4 = 0.6 = m_1^+(v_5)$$

$$d_2^-(v_1) = (-0.2) + (-0.1) = -0.3 = m_1^-(v_1)$$

$$d_2^-(v_2) = (-0.2) + (-0.1) = -0.3 = m_1^-(v_2)$$

$$d_2^-(v_3) = (-0.2) + (-0.2) = -0.4 = m_1^-(v_3)$$

$$d_2^-(v_4) = (-0.1) + (-0.2) = -0.3 = m_1^-(v_4)$$

$$d_2^-(v_5) = (-0.1) + (-0.2) = -0.3 = m_1^-(v_5)$$

Hence, G is not a $(2, (0.8, -0.4))$ regular bipolar fuzzy graph.

5. Conclusion:

In this paper, we have introduced bipolar second degree perfect fuzzy matching for bipolar fuzzygraph based on vertices on the cycle. We proved the necessary conditions under which they are equivalent and also proved that, for a particular condition, a bipolar second degree perfect fuzzy matching is not a $(2, (k_1, k_2))$ – regular bipolar fuzzy graph. We also discussed bipolar second degree perfect fuzzy matching and bipolar second degree fuzzy matching number for cycle graph. In future, we will extend this work for complete graph, regular graph and finally for any connected graph.

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