

Two Server Queueing System One with Working Vacation and Failure

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Abstract:- In this paper we modeled the system as a Markovian two heterogeneous servers in a queueing system, with Server-1 being completely dependable. Whenever the system, server-1, lacks any users, remains idle and gets ready to serve the arriving new customers. But Server-2 switches over to the working Vacation (WV) mode. Failures occur only when Server-2 is operational. We obtained some performance measures.

Keywords: keyword1, Keyword2, keyword3. working vacation, failure, QBD processes. In service systems, computer systems, and telecommunication systems, there are queueing systems with server failures.

1. Introduction

In service systems, computer systems, and telecommunication systems, there are queueing systems with server failures and money transitions in Banks. The majority of publications discuss trustworthy service stations' queueing methods. However, we frequently encounter situations in practice when service stations may not promptly respond to requests.

Because service station failures and a lack of available repair resources can have a significant impact on system performance, queueing systems with unreliable service stations are worthwhile research issues from the perspective of performance prediction. A parallel queueing system with server failure and repair was examined by Wartenhorst (1995).

First proposed was an M/M/1 queue with balking by Haight (1957). Haight (1959) also suggested an M/M/1 queue with reneging customers. In an M/M/1/N queue, the combined effects of balking and reneging have been studied by Ancker and Gafarian (1963a), (1963b). In 1968, Mitrany and Avi-Itzhak studied an M/M/N queue with plenty of room for server repairs and breakdowns.

In their work, the transformation method was used to determine the moment generating function of the queue size. An M/M/2 queueing system with balking and heterogeneous servers was researched by Singh (1970). In which he discovered the mean queue length and stationary queue length distribution and contrasted the model with homogenous servers with the model with heterogeneous servers. The same model was taken into account by Vinod (1985) utilizing the matrix-geometric solution approach. He put certain limitations on the server downtime for $N = 1$ (either occurring only when the server is active or irrespective of the queue length).

The many servers queueing system M/M/c/N with balking and reneging was examined by Abou-EI-Ata and Hariri in 1992. An optional second channel in a two-phase queueing system was studied by Selvam and Srivasankaran in 1994. A single unstable server system with numerous breakdown modes and two potential stages of repair was researched by Hsien and Andersland in 1995. A total probability decomposition technique was devised by Tang (1997) for obtaining the recursive formulation of the equilibrium system size distribution.

This approach could be used to talk about batch arrival discrete-time or continuous-time queueing systems. An ideal N-policy model for a single server Markovian queue with breakdown, repair, and state-dependent arrival rate was examined by Srivastava and Jain in 1999.

From the perspective of queueing, Wang and Chang (2002) investigated an M/M/R/N queue with balking, reneging, and server breakdowns. This research was expanded upon to examine an M/M/c/N queue with balking, reneging, and server failures. They arrived at the steady-state probabilities in matrix form after solving the steady-state probability equations iteratively. In 2003, Nakdimon and Yechiali examined numerous polling systems that experienced both random failures and fixes. In a two-server queue with Bernoulli schedules and a single vacation policy, where the two servers offer consumers heterogeneous exponential service, Madan et al. (2003) studied the problem. For various states of the servers, they were able to obtain steady-state probability generating functions of the system size. The two phase queueing method with general service periods was taken into consideration by Artalejo and Choudhury in 2004. Wang (2004) talked about server failures and an M/G/1 queue with a second optional service. An M/M/1/N queueing system with balking, reneging, and server vacations was taken into consideration by Zhang et al. (2005). A multi-server queue with bulk arrival and two types of server breakdowns was examined by Sultan et al. (2005). An N policy M/G/1 queueing system with a single detachable, unreliable server, whose arrivals form a Poisson process, is the subject of Wang et al. The distribution of service times, repair times, and startup times is considered to be uniform. The probability-generating function was expressed by Wang et al. (2008) after utilizing LT to study the transient solution of the M/G/1 retrial queueing system with server failure. In other words, they investigated if the model's time-dependent solution existed. Yue et al. (2009) gave the model from Kumar and Madheswari (2005) further thought. They were able to show the conclusions of the conditional stochastic decomposition for the stationary queue length and waiting time. They also obtained the explicit expression of the rate matrix.

Vacation models for several servers are more adaptable and practical than those on a single server. When idle, certain servers in many realistic multiple server systems do auxiliary tasks or take vacations, while other servers are constantly ready to serve incoming clients. "Partial server vacation models" are this class of models.

The remaining chapters are structured as follows. The proposed model description and a QBD model formulation are provided in Section 6.2. The stationary condition is derived in Section 6.3. In 6.4, the explicit matrix form expressions of the steady-state probabilities are obtained. Some performance measures are provided in Section 6.5. There are numerical examples in section 6.6, and there are some observations and thoughts in part 6.7.

2 Proposed Model

A Markovian queueing system with two heterogeneous servers that is totally dependable is how the researcher modeled the system. Server-1 sits idle whenever there are no users in the system and prepares to service any incoming new users. However, Server-2 changes to the WV mode. Failures only happen while Server-2 is operational.

2.1 Assumptions Of The Proposed Model

The assumptions of the system model are given as follows:

- (i) The system consists of two servers. While Server-2 is vulnerable to WV and failures, Server-1 is completely reliable. When the system is completely empty, server-2 enters a WV. This WV's duration has a parameter α and follows an exponential distribution. Customers are serviced at a mean rate μ_v that follows an exponential distribution during this WV.
- (ii) Server-2 failures only happen at busy times. Server-2's lifespan and repair times are exponentially distributed with respect to a parameter β .
- (iii) A Poisson process with an arrival rate λ governs consumer arrivals. Depending on how they arrive, clients create a single queue that can hold an endless number of people.

- (iv) A new customer always chooses server-1 if server-1 is idle and server-2 is in a WV mode.
- (v) If a brand-new client arrives and discovers that both servers are occupied, he joins either server in the system.
- (vi) If server-1 is occupied and server-2 is down, customers who are already waiting or who have just arrived must wait in line until server-2's services are resumed or server-1 becomes idle.
- (vii) On an FCFS basis, the two servers offer heterogeneous exponential service to users at service rates of μ_1 and μ_2 , respectively, for Server 1 and Server 2.

2.2 Qbd Process Infinitesimal Generator

Let $X(t)$ represent the status of the servers at time t , and let $N(t)$ be the number of customers in the system at time t .

$$X(t) = \begin{cases} 0, & \text{Server - 1 is the best option, and Server - 2 is on leave.} \\ 1, & \text{Each server is occupied.} \\ 2, & \text{Server - 1 is occupied, and Server - 2 fails.} \end{cases}$$

The SP defines the state space as $\{N(t), X(t)\}$, a CTMC on the state space.

$$E = \{(0, j) \cup (1, j), j = 0, 1, 2\} \cup \{(i, k), i \geq 2, k = 1, 2\}.$$

The levels 0, 1, 2, ... are defined as the sets of the system states $0 = \{(0, 0), (0, 1), (0, 2)\}$, $1 = \{(1, 0), (1, 1), (1, 2)\}$, and $i = \{(i, 1), (i, 2)\}$ if $i \geq 2$. The components of the sets are sorted in lexicographical order. The transition rate matrix Q for the process $\{(N(t), X(t)), t \geq 0\}$ has a block-tridiagonal structure that is given by

$$Q = \begin{bmatrix} B_{00} & B_{01} & & & \\ B_{10} & B_{11} & B_{12} & & \\ & B_{21} & A_1 & A_0 & \\ & & A_2 & A_1 & A_0 \\ & & & A_2 & A_1 & A_0 \\ & & & & \ddots & \ddots & \ddots \end{bmatrix}$$

The matrix Q has the structure of a QBD process and is an infinitesimal generator of the MP $\{(N(t), X(t)), t \geq 0\}$. The square matrices of order 2 that make up the submatrices A_0, A_1 and A_2 are provided by

$$A_0 = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}, A_1 = \begin{bmatrix} -(\lambda + \alpha + \mu_2 + \mu_v) & \alpha \\ \beta & -(\lambda + \beta + \mu_1 + \mu_2) \end{bmatrix},$$

$$A_2 = \begin{bmatrix} \mu_1 + \mu_v & 0 \\ 0 & \mu_1 + \mu_2 \end{bmatrix}$$

The boundary matrices are defined by

$$B_{00} = \begin{bmatrix} -(\lambda + \mu_v) & 0 & \mu_v \\ 0 & -\lambda & 0 \\ 0 & \beta & -(\lambda + \beta) \end{bmatrix}, B_{01} = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$B_{11} = \begin{bmatrix} -(\lambda + \mu_v + \mu_1) & 0 & \mu_v \\ 0 & -(\lambda + \mu_1 + \mu_2) & 0 \\ 0 & 0 & -(\lambda + \beta + \mu_1) \end{bmatrix}$$

$$B_{10} = \begin{bmatrix} \mu_1 & 0 & 0 \\ \mu_1 + \mu_2 & 0 & 0 \\ 0 & \mu_1 & \beta \end{bmatrix}, B_{12} = \begin{bmatrix} \lambda & 0 \\ \lambda & 0 \\ 0 & \lambda \end{bmatrix}, B_{21} = \begin{bmatrix} \mu_2 & \mu_1 & 0 \\ 0 & 0 & \mu_1 \end{bmatrix}$$

3 The System's Stationary State

The stability requirement for the model is stated in the following theorem.

Theorem 6.3.1:

The system's necessary and sufficient condition is $\rho = \frac{\lambda(\alpha+\beta)}{(\alpha+\beta)\mu_1+\beta\mu_2} < 1$

Proof

Define matrix A as $A_0 + A_1 + A_2$. Matrix A can therefore be expressed as

$$A = \begin{bmatrix} -\alpha & \alpha \\ \beta & -\beta \end{bmatrix} \quad (3.1)$$

It is obvious that A is an irreducible MP generator. Let $\pi = (\pi_0, \pi_1)$ be a stationary likelihood vector for this MP. The linear equations $(\pi_0, \pi_1)A = 0$ and $\pi_0 + \pi_1 = 1$ are then satisfied.

That is, $-\pi_0\alpha + \pi_1\alpha = 0$

$\pi_0\beta - \pi_1\beta = 0$ and

$\pi_0 + \pi_1 = 1$ Solving these equations, we have

$$\pi_0 = \frac{\beta}{\alpha+\beta}, \pi_1 = \frac{\alpha}{\alpha+\beta} \quad (3.2)$$

The essential and sufficient factor for the QBD process's stability is

$$\pi A_0 e < \pi A_2 e \quad (3.3)$$

Where e is an ordered 2-column vector whose members are all equal to one. Substituting the value of A_0, A_2 and e in (3.3), we get

$$\lambda(\pi_0 + \pi_1) < \pi_0(\mu_1 + \mu_2) + \pi_1(\mu_2)$$

Using (3.2), we get the stationary condition $\rho < 1$ which becomes

$$\rho = \frac{\lambda(\alpha+\beta)}{(\alpha+\beta)\mu_1+\beta\mu_2} < 1 \quad (3.4)$$

4 Steady State Probabilities

Assume N and X are the stationary random variables that represent the system's customer and server counts, respectively. A symbol for the stationary probability is

$$\begin{aligned} P_{ij} &= \{N = i, X = j\} \\ &= \lim_{t \rightarrow \infty} P\{L(t) = i, J(t) = j\}, (i, j) \in E \end{aligned}$$

Where $j = 0, 1, 2, \dots$ and $i = 1, 2, \dots$. The stationary probability vector P of the generator Q exists under the stationary condition $\rho < 1$. Whereas, and for, this stationary probability vector P is divided as $P = (P_0, P_1, P_2)$, where $P_0 = (P_{00}, P_{01}, P_{02})$, $P_1 = (P_{10}, P_{11}, P_{12})$ and $P_i = (P_{i0}, P_{i2})$ for $i \geq 2$. Using Neuts's (1981) matrix-geometric solution approach the stationary probability vector P is given by

$$P_0 B_{00} + P_1 B_{10} = 0 \quad (4.1)$$

$$P_0 B_{01} + P_1 B_{11} + P_2 B_{21} = 0 \quad (4.2)$$

$$P_1 B_{12} + P_2 (A_1 + R A_2) = 0 \quad (4.3)$$

$$P_i = P_2 R^{i-2}, i = 3, 4, 5, \dots \quad (4.4)$$

and the restoring circumstance

$$P_0 e_1 + P_1 e_1 + P_2 (I - R)^{-1} e = 1 \quad (4.5)$$

Where I is an identity matrix of order two, e_1 is a column vector of order three with a value of one for each element and R , also known as the rate matrix, is the smallest non-negative solution with a spectral radius of less than one to the matrix quadratic equation

$$R^2 A_2 + R A_1 + A_0 = 0 \quad (4.6)$$

Theorem 4.1

If $\rho < 1$, the smallest non-negative answer to matrix equation (4.6) is as follows:

$$R = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix} \quad (4.7)$$

and using the following relation, $RA_2 e = A_0 e$ (4.8)

$$RA_2 e = \begin{bmatrix} r_{11}(\mu_1 + \mu_v) + r_{12}\mu_1 \\ r_{21}(\mu_1 + \mu_v) + r_{22}\mu_1 \end{bmatrix} \quad (4.9)$$

$$A_0 e = \begin{bmatrix} \lambda \\ \lambda \end{bmatrix} \quad (4.10)$$

Using (4.9) and (4.10) in (4.8), we get

$$r_{11}(\mu_1 + \mu_v) + r_{12}(\mu_1 + \mu_2) = \lambda \quad (4.11)$$

$$r_{21}(\mu_1 + \mu_v) + r_{22}(\mu_1 + \mu_2) = \lambda \quad (4.12)$$

From (4.11), we get

$$r_{12} = \left(\frac{1}{\mu_1 + \mu_2} \right) \{ \lambda - (\mu_1 + \mu_v) r_{11} \} \quad (4.13)$$

From (4.12), we get

$$r_{21} = \left(\frac{1}{\mu_1 + \mu_v} \right) \{ \lambda - (\mu_1 + \mu_2) r_{22} \} \quad (4.14)$$

and

$$r_{12} r_{21} = \frac{\lambda^2}{(\mu_1 + \mu_2)(\mu_1 + \mu_v)} - \left(\frac{\lambda}{\mu_1 + \mu_2} \right) r_{11} - \left(\frac{\lambda}{(\mu_1 + \mu_v)} \right) r_{22} + r_{11} r_{22} \quad (4.15)$$

$$R^2 A_2 = \begin{bmatrix} (r_{11}^2 + r_{12} r_{21})(\mu_1 + \mu_v) & r_{12}(r_{11} + r_{22})(\mu_1 + \mu_2) \\ r_{21}(r_{11} + r_{22})(\mu_1 + \mu_v) & (r_{22}^2 + r_{12} r_{21})(\mu_1 + \mu_2) \end{bmatrix} \quad (4.16)$$

$$RA_1 = \begin{bmatrix} -r_{11}(\lambda + \alpha + \mu_1 + \mu_v) + r_{12}\beta & r_{11}\alpha - (\lambda + \beta + \mu_1 + \mu_2)r_{12} \\ -r_{21}(\lambda + \alpha + \mu_1 + \mu_v) + r_{22}\beta & r_{21}\alpha - (\lambda + \beta + \mu_1 + \mu_2)r_{22} \end{bmatrix} \quad (4.17)$$

Substituting (4.15), (4.16) and A_0 in (4.6), we get

$$(r_{11}^2 + r_{12} r_{21})(\mu_1 + \mu_v) - r_{11}(\lambda + \alpha + \mu_1 + \mu_v) + r_{12}\beta + \lambda = 0 \quad (4.18)$$

$$r_{12}(r_{11} + r_{22})(\mu_1 + \mu_2) + r_{11}\alpha - (\lambda + \beta + \mu_1 + \mu_2)r_{12} = 0 \quad (4.19)$$

$$r_{21}(r_{11} + r_{22})(\mu_1 + \mu_v) - r_{21}(\lambda + \alpha + \mu_1 + \mu_v) + r_{22}\beta = 0 \quad (4.20)$$

$$(r_{22}^2 + r_{12}r_{21})(\mu_1 + \mu_2) + r_{21}\alpha - (\lambda + \beta + \mu_1 + \mu_2)r_{22} + \lambda = 0 \quad (4.21)$$

Using (4.15) the first term of (4.18) is

$$\left(r_{11}^2 + \frac{\lambda^2}{(\mu_1 + \mu_2)(\mu_1 + \mu_v)} - \left(\frac{\lambda}{\mu_1 + \mu_2} \right) r_{11} - \left(\frac{\lambda}{\mu_1 + \mu_v} \right) r_{22} + r_{11}r_{22} \right) (\mu_1 + \mu_v)$$

That is,

$$r_{11}^2(\mu_1 + \mu_v) + \left(\frac{\lambda^2}{\mu_1 + \mu_2} \right) - \left(\frac{\lambda(\mu_1 + \mu_v)}{\mu_1 + \mu_2} \right) r_{11} - \lambda r_{22} + r_{11}r_{22}(\mu_1 + \mu_v) \quad (4.22)$$

Using (4.13) the third term of (4.18) becomes

$$\left(\left(\frac{\lambda}{\mu_1 + \mu_2} \right) - \left(\frac{\mu_1 + \mu_v}{\mu_1 + \mu_2} \right) r_{11} \right) \beta \quad (4.23)$$

Substituting (4.13), (4.14), (4.22) and (4.23) in (4.18), we get

$$\left(r_{11}^2 + \frac{\lambda^2}{(\mu_1 + \mu_2)(\mu_1 + \mu_v)} - \left(\frac{\lambda}{\mu_1 + \mu_2} \right) r_{11} - \left(\frac{\lambda}{\mu_1 + \mu_v} \right) r_{22} + r_{11}r_{22} \right) (\mu_1 + \mu_v)$$

$$- r_{11}(\lambda + \alpha + \mu_1 + \mu_v) + \left(\left(\frac{\lambda}{\mu_1 + \mu_2} \right) - \left(\frac{\mu_1 + \mu_v}{\mu_1 + \mu_2} \right) r_{11} \right) \beta + \lambda = 0$$

$$r_{11}^2 - Ar_{11} - Br_{22} + r_{11}r_{22} + C = 0 \quad (4.24)$$

$$\text{where, } A = \left(\frac{\lambda}{\mu_1 + \mu_2} \right) + \left(\frac{\lambda + \alpha}{\mu_1 + \mu_v} + 1 \right), B = \frac{\lambda}{\mu_1 + \mu_v}, C = \left(\frac{\lambda}{\mu_1 + \mu_v} \right) \left(\frac{\lambda + \beta}{\mu_1 + \mu_2} + 1 \right)$$

Using (4.15) the first term of (4.21) is

$$\left(r_{22}^2 + \frac{\lambda^2}{(\mu_1 + \mu_2)(\mu_1 + \mu_v)} - \left(\frac{\lambda}{\mu_1 + \mu_2} \right) r_{11} - \left(\frac{\lambda}{\mu_1 + \mu_v} \right) r_{22} + r_{11}r_{22} \right) (\mu_1 + \mu_2)$$

That is,

$$\left(r_{22}^2(\mu_1 + \mu_2) + \frac{\lambda^2}{(\mu_1 + \mu_v)} - \lambda r_{11} - \left(\frac{\lambda(\mu_1 + \mu_2)}{\mu_1 + \mu_v} \right) r_{22} + (\mu_1 + \mu_2)r_{11}r_{22} \right) \quad (4.25)$$

Using (4.13) the second term of (4.21) is

$$\left(\left(\frac{\lambda}{\mu_1 + \mu_v} \right) - \left(\frac{\mu_1 + \mu_2}{\mu_1 + \mu_v} \right) r_{22} \right) \alpha \quad (4.26)$$

Substituting (4.25) and (4.26) in (4.21), we get

$$\left(r_{22}^2 (\mu_1 + \mu_2) + \frac{\lambda^2}{(\mu_1 + \mu_v)} - \lambda r_{11} - \left(\frac{\lambda (\mu_1 + \mu_2)}{\mu_1 + \mu_v} \right) r_{22} + (\mu_1 + \mu_2) r_{11} r_{22} \right)$$

$$\left(\left(\frac{\lambda}{\mu_1 + \mu_v} \right) - \left(\frac{\mu_1 + \mu_2}{\mu_1 + \mu_v} \right) r_{22} \right) \alpha - (\lambda + \beta + \mu_1 + \mu_2) r_{22} + \lambda = 0$$

$$r_{22}^2 - A_1 r_{22} - B_1 r_{11} + r_{11} r_{22} + C_1 = 0 \quad (4.27)$$

$$\text{where, } A_1 = \left(\frac{\lambda + \alpha}{\mu_1 + \mu_v} \right) + \left(\frac{\lambda + \beta}{\mu_1 + \mu_2} + 1 \right), B_1 = \left(\frac{\lambda}{\mu_1 + \mu_2} \right) C_1 = \left(\frac{\lambda}{\mu_1 + \mu_2} \right) \left[\frac{\lambda}{\mu_1 + \mu_v} \{1 + \alpha\} + 1 \right]$$

From equation (4.27), we get

$$r_{11} = \left(\frac{r_{22}^2 - A_1 r_{22} + C_1}{B_1 - r_{22}} \right) \quad (4.28)$$

Equation (4.24), when equation (4.28) is substituted, yields the cubic equation with one variable, r_{22} , as follows:

$$Q r_{22}^3 + R r_{22}^2 + S r_{22} + T = 0 \quad (4.29)$$

Where

$$Q = A + B - A_1,$$

$$R = A_1^2 - A^2 + 3C_1 + C - B_1 \{A + A_1 - C_1 - 2B\},$$

$$S = AC_1 - 2A_1 C_1 + B_1 \{A^2 - 2C - BB_1\}$$

$$T = CB_1^2 - C_1 \{AB_1 - C_1\}.$$

$$\text{To solve (4.29) let } r_{22} = y - \left(\frac{R}{3Q} \right),$$

$$\text{where, } y = z - \frac{U}{3z}, z = \sqrt[3]{-\frac{V}{2} \pm \sqrt{D}}, U = -\frac{R^2}{3Q^2} + \frac{S}{Q}, V = \frac{2R^3}{27Q^3} - \frac{RS}{3Q^2} + \frac{T}{Q}$$

$$\text{and } D = \frac{V^2}{4} + \frac{U^3}{27}$$

Here, $D > 0$, thus, the equation (4.29) has two complex roots in addition to one real root. Let this true root be r_{22} .

The third degree equation's root formula can be used to generate the explicit expression of r_{22} . Equation (4.28) yields r_{11} , equation (4.13) yields r_{12} , and equation (4.14) yields r_{21} .

From equation (4.1), we get

$$P_0 = -P_1 B_{10} B_{00}^{-1} \quad (4.30)$$

Substituting (4.30) in equation (4.2), we get

$$P_1 = P_2 \left[B_{10} B_{00}^{-1} B_{01} - B_{11} \right]^{-1} B_{21} \quad (4.31)$$

Using the equation (4.31) in equation (4.30), we get

$$P_0 = -P_2 \left[B_{10} B_{00}^{-1} B_{01} - B_{11} \right]^{-1} B_{21} B_{10} B_{00}^{-1} \quad (4.32)$$

Using the equations (4.31) and (4.30) in equation (4.5), we get

$$P_2 = \left[B_{21} \left[e_1 - B_{10} B_{00}^{-1} e \right] \left[B_{10} B_{00}^{-1} B_{01} - B_{11} \right]^{-1} + \left[(I - R)^{-1} e \right] \right]^{-1} \quad (4.33)$$

5 Performance Indicators

We can calculate certain system performance metrics using the steady state probability vector shown above.

Theorem5.1:

(a) The system's anticipated customer base is indicated by

$$E(N) = P_1 e_1 + P_2 e \left\{ 2(I - R)^{-1} + R(I - R)^{-2} \right\}$$

(b) The anticipated number of patrons in line is indicated by $E(N_q) = P_2 e (I - R)^{-2}$

Proof:

The system's anticipated customer base is indicated by

$$\begin{aligned} E(N) &= \sum_{n=1}^{\infty} n P(N = n) \\ &= P(N = 1) + 2P(N = 2) + 3P(N = 3) + \cdots \\ &= P_1 e_1 + 2P_2 e + 3P_2 e R + 4P_2 e R^2 + \cdots \\ E(N) &= P_1 e_1 + P_2 e \left[2 \{ 1 + R + R^2 + \cdots \} + \{ 1 + 2R + 3R^2 + \cdots \} \right] \\ E(N) &= P_1 e_1 + P_2 e \left[2(I - R)^{-1} + R(I - R)^{-2} \right] \end{aligned}$$

The anticipated number of consumers standing in line is indicated by

$$E(N_q) = \sum_{n=1}^{\infty} (n-1) P(N = n)$$

$$= P(N = 2) + 2P(N = 3) + 3P(N = 4) + \cdots$$

$$= P_2 e + 2RP_2 e + 3R^2 P_2 e + \cdots$$

$$E(N_q) = P_2 e (I - R)^{-2}$$

6 Numerical Illustrations

Table 6.1 and Table 6.2 explain the effect of the arrival rate λ and the service rate μ_1 in system length and queue length respectively. Table 6.3 and Table 6.4 explain the effect of the service rates μ_1 and μ_2 in system length and queue length respectively. Table 6.5 and Table 6.6 explain the effect of α and β in system length and queue length respectively.

$\lambda \setminus \mu_1$	5	6	7	8	9	10	15
10	4.4532	3.2899	2.5682	2.1134	1.8355	1.6854	2.2438
15	9.5870	6.9541	5.2544	4.1057	3.3082	2.7490	1.8689
20	16.7910	12.1695	9.1545	7.0835	5.6090	4.5332	2.1597
25	26.0597	18.9292	14.2576	11.0286	8.7088	6.9941	2.8898
30	37.3935	27.2333	20.5625	15.9371	12.5997	10.1186	3.9889

Table 6.1

$\lambda \setminus \mu_1$	5	6	7	8	9	10	15
10	0.0575	0.0829	0.1133	0.1490	0.1902	0.2374	0.5907
15	0.0511	0.0761	0.1068	0.1433	0.1857	0.2343	0.5826
20	0.0432	0.0660	0.0945	0.1291	0.1698	0.2169	0.5565
25	0.0363	0.0565	0.0823	0.1140	0.1519	0.1962	0.5211
30	0.0306	0.0484	0.0714	0.1002	0.1350	0.1760	0.4831

Table 6.2

μ_1 / μ_2	5	6	7	8	9	10	15
5	19.6660	18.8465	18.2846	17.9080	17.6685	17.5329	17.7836
6	14.7920	14.1790	13.7347	13.4168	13.1950	13.0478	12.9925
7	11.5863	11.1270	10.7785	10.5172	10.3245	10.1860	10.0058
8	9.3443	9.0004	8.7278	8.5154	8.3522	8.2289	8.0036
9	7.7057	7.4489	7.2363	7.0646	6.9280	6.8210	6.5874

10	6.4097	6.2778	6.1124	5.9739	5.8604	5.7688	5.5438
15	3.2544	3.2210	3.1760	3.1290	3.0842	3.0434	2.9049

Table 6.3

μ_1/μ_2	5	6	7	8	9	10	15
5	0.0119	0.0101	0.0089	0.0080	0.0074	0.0069	0.0055
6	0.0164	0.0135	0.0117	0.0103	0.0094	0.0086	0.0066
7	0.0217	0.0176	0.0149	0.0130	0.0116	0.0106	0.0077
8	0.0279	0.0222	0.0185	0.0159	0.0141	0.0127	0.0089
9	0.0350	0.0274	0.0225	0.0192	0.0168	0.0150	0.0102
10	0.0555	0.0332	0.0270	0.0228	0.0198	0.0175	0.0116
15	0.0955	0.0710	0.0557	0.0455	0.0383	0.0330	0.0196

Table 6.4

α/β	0	1	2	3	4	5	6
0	18.2243	17.8490	17.4883	17.1435	16.8152	16.5034	16.2081
1	17.9067	17.5329	17.1748	16.8332	16.5087	16.2012	15.9106
2	17.6510	17.2864	16.9370	16.6039	16.2873	15.9875	15.7041
3	17.4753	17.1173	16.7744	16.4473	16.1366	15.8422	15.5640
4	17.3501	16.9969	16.6585	16.3358	16.0292	15.7387	15.4642
5	17.2681	16.9074	16.5723	16.2528	15.9492	15.6617	15.3899

Table 6.5

α/β	0	1	2	3	4	5	6
0	0.0209	0.0188	0.0170	0.0153	0.0138	0.0125	0.0113
1	0.0075	0.0069	0.0064	0.0058	0.0054	0.0049	0.0045
2	0.0039	0.0036	0.0033	0.0031	0.0029	0.0026	0.0025
3	0.0024	0.0022	0.0021	0.0019	0.0018	0.0017	0.0015
4	0.0016	0.0015	0.0014	0.0013	0.0012	0.0011	0.0011

Table 6.6

Figure 6.1

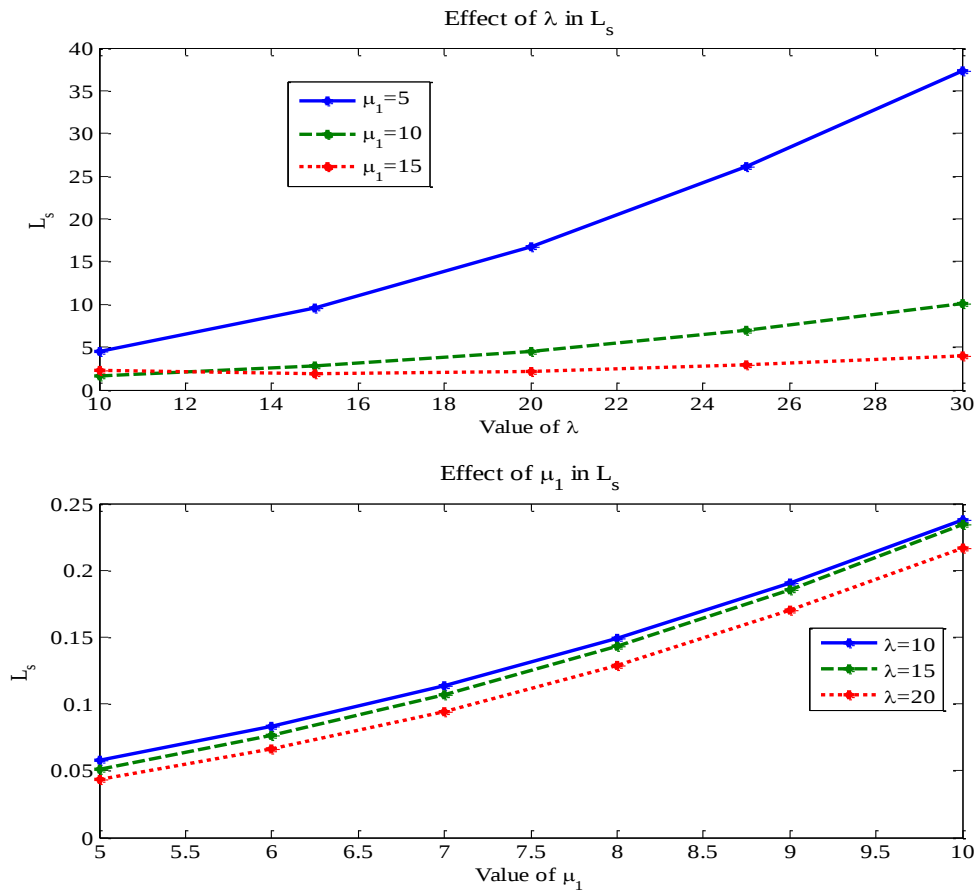


Figure 6.2

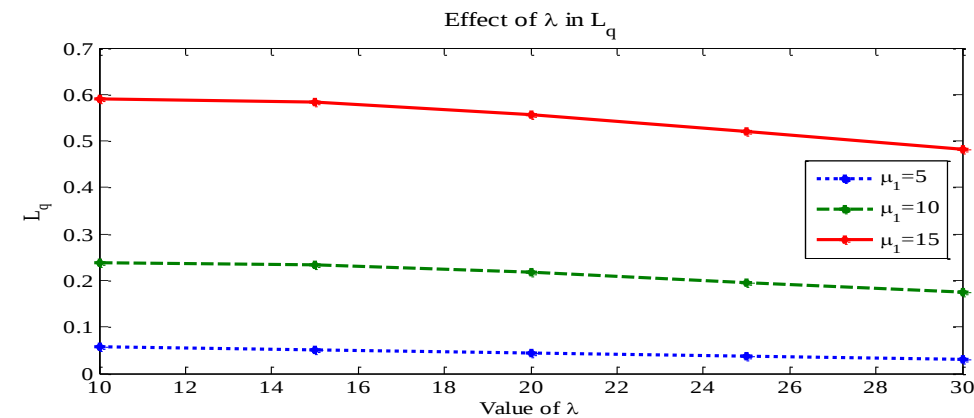


Figure 6.3

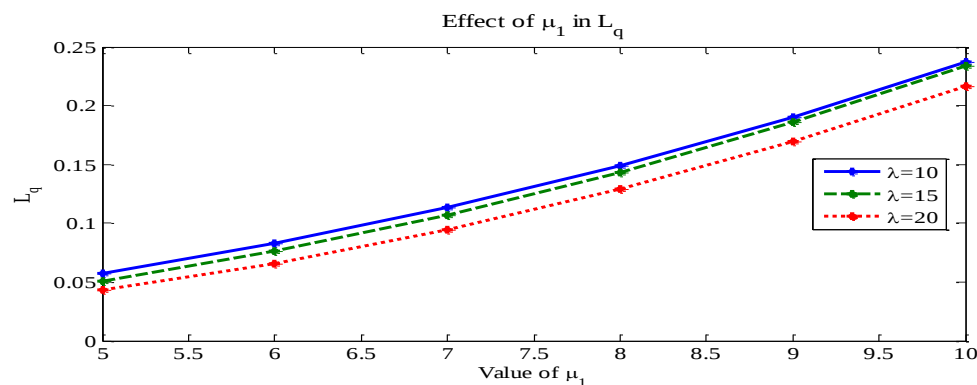


Figure 6.4

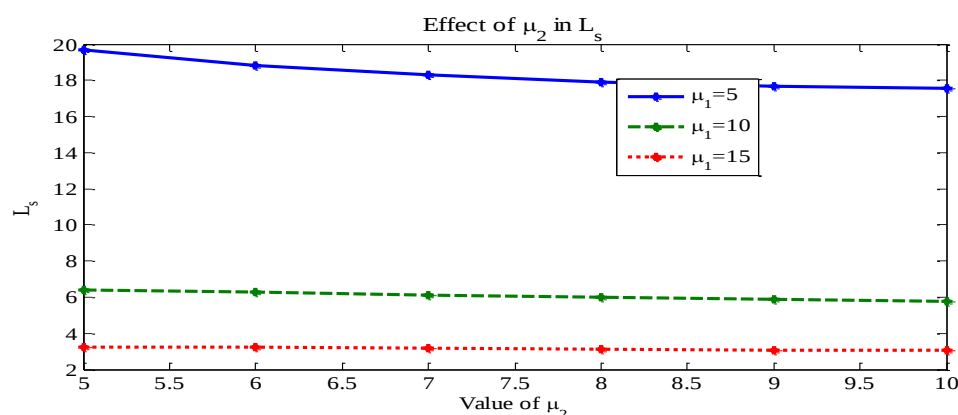


Figure 6.5

7 Some Observations And Remarks

Fix the values of μ_2 and β and increasing the values of μ_1 and α sufficiently large one can observe that both the system and queue length is very small.

8. References

- [1] Wartenhorst, P.N. (1995): Parallel queueing systems with server breakdown and repair, *European Journal of Operational Research*, 82, pp. 302–322.
- [2] Haight, F.(1957): Queueing with balking. *Biometrika*, 44, pp. 360–369.
- [3] Haight, F.(1959): Queueing with reneging. *Metrika*, 2, pp. 186–197.
- [4] Ancker, Jr, C., and Gafarian, A. (1963a): Some queueing problems with balking and reneging: I. *Operations Research*, 11, pp. 88–100.
- [5] Ancker, Jr.C., and Gafarian, A. (1963b): Some queueing problems with balking and reneging: II. *Operations Research*, 11, pp. 928–937.
- [6] Mitrany, I. L., and Avi-Itzhak, B. (1968): A many-server queue with service interruptions, *Operations Research*, 16, pp. 628-638.
- [7] Singh, V.P. (1970): Two-server Markovian queues with balking: heterogeneous vs. homogeneous servers, *Operations Research*, 18, pp. 145-159.
- [8] Vinod, B. (1985): Unreliable queueing systems, *Computers and Operations Research*, 12, pp. 322- 340.
- [9] Abou El-Ata, M., and Hariri, A. (1992): The M/M/C/N queue with balking and reneging. *Computers and Operations Research*, 19, pp. 713–716.

-
- [10] **Selvam, D., and Sivasankaran, V. (1994):** A two-phase queueing system with server vacation's, **Operations Research Letters**, 15 (3), pp. 163-168.
 - [11] **Hsien, Y.C., and Andersland, M.S.(1995):** Repairable single server systems with multiple breakdown modes, **Microelectron. Reliab.** 35 (2), pp. 309–318.
 - [12] **Tang, Y.(1997):** A single server M/G/1 queueing system subject to breakdowns-Some reliability and queueing problems, **Microelectronics Reliability**. 37, pp. 315–321.
 - [13] **Wang, K. H., and Chang, Y. C. (2002):** Cost analysis of a finite M/M/R queueing system with balking, reneging and server breakdowns, **Mathematical Methods of Operations Research**, 56, pp.169-180.
 - [14] **Madan, K. C., Abu-Dayyeh, W. and Gharaibeh, M. (2003):** On two parallel servers with random breakdowns, **Soochow Journal of Mathematics**, 29(4), pp. 413-423.
 - [15] **Nakdimon, O., and Yechiali, U. (2003):** Polling systems with breakdowns and repairs, **European Journal of Operational Research**, 149, pp. 588-613.
 - [16] **Artalejo, J.R., and Choudhury, G. (2004):** Steady state analysis of an M/G/1 queue with repeated attempts and two-phase service, **Quality Technology and Quantitative Management**, 1 (2), pp. 189–199.
 - [17] **Wang, J. (2004):** An M/G/1 queue with second optional service and server breakdowns, **Computer and Mathematical Applications**, 47, pp. 1713–1723.
 - [18] **Zhang, Y., Yue, D., and Yue. W.(2005):** Analysis of an M/M/1/N queue with balking, reneging and server vacations. **Proceeding of the Fifth International Symposium**, pp. 37- 47.
 - [19] **Sultan, A.M., Hassan, N.A., and Elhamy, N.M. (2005):** Computational analysis of a multi-server bulk arrival with two modes server breakdown, **Mathematical Computational Applications**, 10 (2), pp. 249–259.
 - [20] **Wang, K., Wang, T., and Pearn, W. (2007):** Optimal control of the N policy M/G/1 queueing system with server breakdowns and general startup times, **Applied Mathematical Modeling**, 31, pp. 2199-2212.
 - [21] **Wang, J., Liu, B., and Li, J. (2008):** Transient analysis of an M/G/1 retrial queue subject to disasters and server failures, **European Journal of Operational Research**, 189, pp. 1118-1132.
 - [22] **Yue, D., Yu, J., and Yue, W. (2009):** A Markovian queue with two heterogeneous servers and multiple vacations, **Journal of Industrial and Management Optimization** 5, pp. 453-465.
 - [23] **Kumar, B. K., and Madheswari, S. P.(2005):** An M/M/2 queueing system with heterogeneous servers and multiple vacations, **Mathematical and Computer Modelling**, 41, pp. 1415-1429.