Tuijin Jishu/Journal of Propulsion Technology
ISSN: 1001-4055
Vol. X No. Y (20--)

# A delay-dependent dissipativity and passivity-based synchronization analysis for Markovian jumping complex dynamical network models with distributed coupling delays

R.Sugumar
PG and Research Department of Mathematics, C. Abdul Hakeem College (Autonomous), Melvisharam - 632509, Tamil Nadu, India.Email:loyolasugumar@gmail.com

**Abstract**: This paper studies the issue of delay-dependent dissipativity and passivity-based synchronization analysis for Markovian jumping complex dynamical network (MJCDN) models with distributed coupling delays. The state feedback $H_\infty$ control scheme is designed for the synchronization of the MJCDN closed-loop error model. By constraining the appropriate Lyapunov functional along with the properties of the Kronecker product and using integral inequality techniques,several new delay dependent sufficient conditions are derived in terms of linear matrix inequalities (LMIs), which ensure that the dissipativity and passivity performance of the considered MJCDN model. The results of this analysis are verified using the MATLAB LMI control toolbox. Illustrative examples are provided to demonstrate the feasibility of the results
***Keywords****:* KeyWords: Complex dynamical networks, dissipativity, passivity, Lyapunov functional, Markovian jumping, distributed coupling delay, $H_\infty$ control,2000 AMS Subject Classification: 34D20, 34K20

## 1. Introduction

Complex dynamic networks (CDNs) have attracted attention from many researchers, as they are often used in many real world systems, such as neural networks, secure communication, the world wide web, and others science and engineering domains [1]- [4]. Indeed, CDN models are large-scale networks consisting of a group of coupled nodes with certain topological connections. Each node describes its own specific dynamic behaviors. Furthermore, CDNs models exhibit several complex phenomena, which include dissipativity, passivity, synchronization, and stability. Research on the dynamical behaviors of CDN models has become popular in recent years [5]- [7].

In 1961, Krasovskii and Lidskii introduced a class of Markovian jumping systems. They are modeled by a collection of linear systems that transfer from models corresponding to the Markov chain in a limited mode set a finite set. Markovian jumping systems provides effective optimization models to describe CDNs whose structures is subject to abrupt changes [8]. Obviously, Markovian jumping complex dynamical network (MJCDN) models can be viewed as a class of hybrid models. In this respect, the synchronization analysis of MJCDN models has become an important research focus. Recently, several useful theories have been published [9]- [10]. As an example, exponential synchronization of CDN models with Markovian jumping parameters and stochastic delays was investigated in [10]. Based on the Lyapunov principle and other analytic methods, the sufficient criteria pertaining to finite-time dissipative control of singular discrete-time Markovian jumping systems with actuator saturation and partly unknown transition rates were presented in [11]. In [12], a matrix measure approach was adopted to address the synchronization criteria of CDN models with Markov switching.

Synchronization, as one of the representative dynamical behaviors of CDN models, has received increasing research attention due to its potential application in various fields, such as computer science, cryptography, neuroscience, multimedia, and telecommunication [13]- [15]. Investigations on synchronization pertaining to CDN models are of great significance from both analytical and practical perspectives. However, it is not easy to achieve synchronization without the use of effective controllers. In this regards, researchers have proposed multifarious control methods [16]- [18]. Among these control methods, $H_\infty$ control is an economical and effective solution, which not only reduces channel congestion but also makes full use of the transmission capacity of the network [19]- [20]. It has been confirmed that this method is an efficient and prevalent approach to addressing disturbance attenuation. Fruitful results corresponding to synchronization of CDN models have also been published in the literature [19]- [22].

The dissipativity and passivity issues with respect to neural network models have attracted research consideration [25],[26]. Indeed, the theory of dissipativity introduces the system input and output descriptions and provides new dissipativity concept efficiency by adjustable parameters $(\mathcal{L}, \mathcal{P}, \mathcal{R})$and it is a more general case of passivity and $H_\infty$ performances. In fact, it has been shown that dissipativity is highly capable of disturbance attenuation. As a result, many methods have been developed to address the issues related to dissipativity-based synchronization, and significant developments have been made [25],In [25], under several proposed sufficient conditions, the Markovian jumping neural networks were able to achieve dissipativity and passivity performance by employing integral inequalities and adopting suitable Lyapunov function. In [26]-[33], new passivity criteria for memristor-based neutral-type stochastic bidirectional associative memory neural networks with mixed time-varying delays were obtained. Some of the researchers investigated the passivity and dissipativity analysis of neural networks with various time delay like time varying delay [40], probabilistic time varying delay additive time varying delay ,mixed time delay, time delay in the leakage term[34],

In the current literature, dissipativity and passivity-based synchronization of MJCDN models with time-varying distributed coupling delays is challenging and is still an open research question. Therefore, we aim to fill such gap in this study.

The major contributions of this study are as follows:
(1) A compact MJCDN model is formulated, and the issue corresponding to time-varying distributed coupling delays is taken into account.
(2) A state feedback $H_\infty$ control scheme is designed, which can reduce the control costs, thus it is more economic and practical.
(3) Refined Jensen's inequality and reciprocal convex lemma are utilized to derive the results.
(4) Dissipativity and passivity criteria provides valuable stability results as well as compositional results for the analysis of interconnected systems.

The remaining part of this article is organized as follows. In Section 2, some preliminaries and the CDN model are introduced. In Section 3, novel dissipativity and passivity criteria are presented. In Section 4, simulations are presented to validate the theoretical results. The research findings are summarized in section 5.

**Notation**: Throught this article, $\mathbb{R}^n$ denotes the n-diensional Euclidean space and $\mathbb{R}^{n\times m}$ is the set of $n \times m$ real matrices. In addition,$\mathcal{B}^T$ $and$ $\mathcal{B}^{-1}$ represent the transpose and inverse of matrix$\mathcal{B}$, while I denotes the identity matrix with compatible dimensions.For a real symmetric matices $\mathcal{A}$ $and$ $\mathscr{b}$ $\mathcal{A} \geq \mathscr{b}$ (respectively $\mathcal{A} > \mathscr{b}$) means that matrix $\mathcal{A} - \mathscr{b}$ is positive semi definite (respectively, positive definite). Asterisk "*"in a matrix is used to represent the term iduced by symmetry;$\gamma$ denotes the indes of disturbance attention. While $\|.\|$refers to the Euclidean vector norm. Moreover $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, \mathcal{P})$shows a complete probability space with filtration, $\{\mathcal{F}_t\}t \geq 0$that satisfies the usual condition(i.e the filtration contains all P-null sets and is right continuous). The Kronecker product of matrices $\mathcal{X} \in \mathbb{R}^{n\times m}$ and $\mathcal{Y} \in \mathbb{R}^{p\times q}$ is a matrix in $\mathbb{R}^{mp\times nq}$, and is denoted by $\mathcal{X} \otimes \mathcal{Y}$.Besides that $\mathcal{L}_2[0,\infty)$ is the space of square integrable vector function over $[0,\infty)$, $\mathcal{E}\{.\}$ represent the mathematical operators, while $diag(\ldots.)$denotes the block diagonal matrix.

## 2. Problem description and preliminaries

Given a total probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}t \geq 0, \mathcal{P})$ with characteristic function $\{\mathcal{F}_t\}t \geq 0$ fulfilling the standard conditions(i.e the filtration contain all $\mathcal{P} - null\ sets$ and is right continuous),where $\Omega$ sis the sample space , is the largest algebra of events and $\boldsymbol{\mathcal{P}}$ is the probability measure charterized on $\boldsymbol{\mathcal{F}}$.Let $\{\sigma(t), t \geq o\}$ be right continuous Markovian chain in the Probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}t \geq 0, \mathcal{P})$ taking qualities in the finite space $S = \{1,2, \ldots.m\}$ with generator $\Lambda == \{\lambda_{kq}\}_{m\times n}$ $(k, q \in S)$given by

$$P_r(\rho_{t+\Delta} = q \mid \rho_t = k) = \begin{cases} \lambda_{kq}\Delta + o(\Delta) & if k \neq q \\ 1 + \lambda_{kq}\Delta + o(\Delta) & if k = q \end{cases}$$

where $\lambda > 0, \lim_{t\to 0} \frac{o(\Delta)}{\Delta} = 0$and $\lambda_{kq}$ is the transition rate from node k to mode q satisfying $\lambda_{kq} \geq 0\ for\ k \neq q$ with $\lambda_{kq} = -\sum_{q=1,q\neq k} \lambda_{kq}\ (k, q \in \boldsymbol{S})$

Here we consider the following NJDCM model, with consists of N identical coupled nodes, whereby each node is an n-dimensional dynamical model

$$\dot{x}_z(t) = -\mathscr{b}_1(\rho_t)x_z(t) + \mathscr{f}_1(\rho_t)x_z(t-\tau(t)) + \mathcal{B}_1(f(x_z(t) + \mathcal{A}_1(\rho_t)(f\left(x_z(t-\tau(t))\right) + c\sum_{j=1}^{N}\mathcal{G}_{zj}\mathcal{A}_3(\rho_t)\int_{t-\delta}^{t}x_{j(s)}ds + u_z(t) + \mathcal{D}_1(\rho_t)\omega_z(t), \quad (1)$$

$$y_z(t) = \mathcal{E}_1(\rho_t)x_z(t), \dots\dots z = 1,2\dots N, \quad (2)$$

where $x_z(t) = (x_{z1}(t), x_{z2}(t), \dots\dots x_{zn}(t))^T \in \mathbb{R}^n$ represent the state vector of the z-nodes at time t,respectively.In addition, $f(x_z(t)) = [f(x_{z1}(t)), f(x_{z2}(t)), \dots\dots f(x_{zn}(t))]^T \in \mathbb{R}^n$is a vector valued time varying nonlinear function describing the dynamics nodes; $\mathscr{b}_1(\rho_t) \in \mathbb{R}^{n\times n}$ is a positive diagonal matrix; $\mathscr{f}_1(\rho_t), \mathcal{B}_1(\rho_t), \mathcal{A}_1(\rho_t) \in \mathbb{R}^{n\times n}$are the connection weight matrices and delayed weight matrices,respectively c is coupling strength ; $\mathcal{D}_1(\rho_t) \in \mathbb{R}^{n\times n}$ is a known matrix;$\omega_z(t) \in \mathbb{R}^n$ is the external disturbance which belong to $\mathcal{L}_2[0,\infty), y_z(t) \in \mathbb{R}^n$ is the output node z; $\mathcal{E}_1(\rho_t)$ is the known constant matrix with an appropriate dimension. $\mathcal{A}_2(\rho_t), \mathcal{A}_3(\rho_t)$ describes the constant inter-coupling matrices of the model;$\mathcal{G} = (\mathcal{G}_{zj})_{N\times N}$ is the outer-coupling configuration matrix of the model.If there is coupling between node z and node $(z \neq j)$,then $\mathcal{G}_{zj} = 1$, otherwise $\mathcal{G}_{zj} = 0(z \neq j$.Meanwhile,the diagonal elements of $\boldsymbol{\mathcal{G}}$ are defined as

$$\mathcal{G} = -\sum_{j=1,j\neq z}^{N}\mathcal{G}_{zj} \quad z = 1,2\dots\dots N. \quad (3)$$

Function $\tau(t)$ represent the time varying coupling delay and $\delta$ denotes the distributed coupling delay that satisfies

$$0 \le \tau(t) \le \bar{\tau}, \ \dot{\tau}(t) \le \mu \quad (4)$$

Where $\bar{\tau}$ *and* $\mu$ are known constants

**Assumption 1**: The nonlinear vector-valued function $f(.): \mathbb{R}^n \to \mathbb{R}^n$ satisfies f(0)=0, and the following sector-bounded condition is

$$[f(u) - f(v) - \mathcal{F}_1(u-v)]^T[f(u) - f(v) - \mathcal{F}_2(u-v)] \le 0, for\ all\ u, v \in \mathbb{R}^n \quad (5)$$

where $\mathcal{F}_1$ *and* $\mathcal{F}_2$ are constant matrices with $\mathcal{F}_2 - \mathcal{F}_1 \ge 0$.

**Remark 1**: If the above assumption considers that the grides $\mathcal{F}_1$ *and* $\mathcal{F}_2$ are diagonal matrices,then the assumption is the common Lispchitz condition.In the event that $\mathcal{F}_1$ *and* $\mathcal{F}_2$ are symmetric matrices as for the origin,it is norm-bounded portrayal.The sector-bounded description on the nonlinear term in Assumptio 1 incorporates the general Lipschitz and norm-bounded conditions,and its description is progressively broad.The sector-bounded nonlinearities are regularly used to display the model in practice.

Let $s(t) \in \mathbb{R}^n$ be the state direction of the unforced isolated node of models (1) and (2) fulfilling

$$\dot{s}(t) == -\mathscr{b}_1(\rho_t)x_z(t) + \mathscr{f}_1(\rho_t)x_z(t-\tau(t)) + \mathcal{B}_1(f(\mathcal{S}(t)) + \mathcal{A}_1(\rho_t)(f\left(\mathcal{S}(t-\tau(t))\right) \quad (6)$$

Then, the synchronization error model is defined as

$$e_z(t) = x_z(t) - \mathcal{S}(t), \quad z = 1,2\dots.N, \quad \mathbf{(7)}$$

So the error dynamic of models (1) and (2) is inferred as follows,

$$\dot{e}_z(t) == -\mathscr{b}_1(\rho_t)e_z(t) + \mathscr{f}_1(\rho_t)e_z(t-\tau(t)) + \mathcal{B}_1(f(e_z(t) + \mathcal{A}_1(\rho_t)(f\left(e_z(t-\tau(t))\right) + c\sum_{j=1}^{N}\mathcal{G}_{zj}\mathcal{A}_2(\rho_t)e_j(t-\tau(t)) + \sum_{j=1}^{N}\mathcal{G}_{zj}\mathcal{A}_3(\rho_t)\int_{t-\delta}^{t}e_{j(s)ds+}u_z(t) + \mathcal{D}_1(\rho_t)\omega_z(t), \quad (8)$$

$y_z(t) = \mathcal{E}_1(\rho_t)x_z(t), \dots \dots . z = 1,2 \dots N,$ (9)

where $f(e_z(t)) = f(x_z(t)) - f(s(t)),$

$f(e_z(t-\tau(t))) = f(x_k(t-\tau(t))) - f(s(t-\tau(t))).$

We use the state feedback control,

$u_z(t) = \mathcal{K}_z(\rho_t)e_z(t), \quad z = 1,2,,,,N,$ (10)

with such error model $H_\infty$ synchronization,where $\mathcal{K}_z$ is the zth node controller gain matrix to be described. Consequently,the closed-loop error model(8) is obtained as follows

$\dot{e}_z(t) ==$
$-\mathcal{b}_1(\rho_t)e_z(t) + \mathcal{f}_1(\rho_t)e_z(t-\tau(t)) + \mathcal{B}_1(f(e_z(t) + \mathcal{A}_1(\rho_t)(f\left(e_z(t-\tau(t))\right) + c\sum_{j=1}^{N}\mathcal{G}_{zj}\mathcal{A}_2(\rho_t)e_j(t-$
$\tau(t)) + \sum_{j=1}^{N}\mathcal{G}_{zj}\mathcal{A}_3(\rho_t)\int_{t-\delta}^{t} e_{j(s)}ds + \mathcal{K}_z(\rho_t)e_z(t) + \mathcal{D}_1(\rho_t)\omega_z(t),$ (11)

$y_z(t) = \mathcal{E}_1(\rho_t)x_z(t), \dots \dots . z = 1,2 \dots N,$ (12)

For convenience each possible value $\rho_t$ is denoted by $k, k \in \mathcal{S}$ in the sequel.As such, we have

$\mathcal{b}_1(\rho_t) = \mathcal{b}_{1k}, \mathcal{f}_1(\rho_t) = \mathcal{f}_{1k}, \mathcal{B}_1(\rho_t) = \mathcal{B}_{1k}, \mathcal{A}_1(\rho_t) = \mathcal{A}_{1k}, \mathcal{A}_2(\rho_t) = \mathcal{A}_{2k}, \mathcal{D}_1(\rho_t) = \mathcal{D}_{1k}, \mathcal{E}_1(\rho_t) = \mathcal{E}_{1k}$ ,

where $\mathcal{b}_{1k}, \mathcal{f}_{1k}\mathcal{B}_{1k,}\mathcal{A}_{1k}\mathcal{A}_{2k}, \mathcal{D}_{1k}, \mathcal{E}_{1k}$ for any $k \in \mathcal{S}$ are known constant matrices of appropriate dimension. Then,the error dynamical models(11) and (12) are defined as follows,

$\dot{e}_z(t) = (-\mathcal{b}_{1k} + \mathcal{K}_{zk})e_z(t) + \mathcal{f}_1(\rho_t)e_z(t-\tau(t)) + \mathcal{B}_{1k}(f(e_z(t) + \mathcal{A}_{1k}(\rho_t)(f\left(e_z(t-\tau(t))\right) +$
$c\sum_{j=1}^{N}\mathcal{G}_{zj}\mathcal{A}_{2k}(\rho_t)e_j(t-\tau(t)) + \sum_{j=1}^{N}\mathcal{G}_{zj}\mathcal{A}_{3k}(\rho_t)\int_{t-\delta}^{t} e_{j(s)ds} + \mathcal{D}_{1k}(\rho_t)\omega_z(t),$ (13)

$y_z(t) = \mathcal{E}_{1k}e_z(t), \dots \dots . z = 1,2 \dots N.$ (14)

By utilizing the Kronecker product,the error dynamical models (13) and (14) can be written in a compact form as

$\dot{e}(t) =$
$(I_N \otimes (-\mathcal{b}_{1k} + \mathcal{K}_{zk}))e(t) + (I_N \otimes \mathcal{f}_1(\rho_t))e(t-\tau(t)) + (I_N \otimes \mathcal{B}_{1k})(f(e(t) + (I_N \otimes \mathcal{A}_{1k})(f\left(e(t-\right.$
$\left.\tau(t))\right) + c(\mathcal{b} \otimes \mathcal{A}_{1k)}e(t-\tau(t)) + (I_N \otimes \mathcal{A}_{3k})\int_{t-\delta}^{t} e_{j(s)ds} + (I_N \otimes \mathcal{D}_{1k})\omega(t)$ (15)

$y(t) = (I_N \otimes \mathcal{E}_{1k})e(t),$ (16)

where

$e(t) = [e_1^T(t)e_2^T(t) \dots \dots . e_N^T(t)]^T,$

$e(t-\tau(t)) = [e_1^T(t-\tau(t))e_2^T(t-\tau(t)t) \dots \dots . e_N^T(t-\tau(t))]^T,$

$f(e(t) = [f^T(e_1(t))f^T(e_2(t)) \dots \dots f^T(e_N(t))]^T,$

$f(e(t-\tau(t)) = [f^T\left(e_1((t-\tau(t)))\right)f^T\left(e_2((t-\tau(t)))\right) \dots \dots f^T(e_N((t-\tau(t)))]^T,$

$\omega(t) = [\omega_1^T(t)\omega_2^T(t) \dots \dots \omega_N^T(t)]^T$,

$\mathcal{K}_k = diag\{\mathcal{K}_{1k}\mathcal{K}_{2k} \dots \dots \mathcal{K}_{Nk}\}$.

The following definitions and lemmas are useful for deriving the main results, which are utilized throught this paper.

**Definition 2.1**[26] Models and (15) and (16) are strictly $(\mathcal{L}, \mathcal{P}, \mathcal{R}) - \gamma$ dissipativity for any $t_f \geq 0$ and some$\gamma \geq 0$, if under zero initial state the following condition meet

$$\mathcal{E}\{\mathcal{G}(\omega, y, t_f)\} \geq \mathcal{E}\{\gamma\tau < \omega, \omega > t_f\}. \tag{17}$$

**Remark 2**: As per definition 2.1 the quadratic energy supply function $\mathcal{G}$ associated with models (15) and (16) is defined by,

$$\mathcal{G}(\omega, y, t_f) =< y, \mathcal{L}y > t_f + 2 < y, \mathcal{P}\omega > t_f +< \omega, \mathcal{R}\omega > t_f \ for \ all \ t_f \geq 0, \tag{18}$$

where $\mathcal{L}, \mathcal{P}, \mathcal{R} \in \mathbb{R}^{n\times m}$ with $\mathcal{L}, \mathcal{R}$ are symmetric matrices.Let $\mathcal{L}_2[0, \infty)$ be the space of square integrable function on $[0, \infty)$.Notations $< y, \mathcal{L}y > t_f, < y, \mathcal{P}\omega > t_f, < \omega, \mathcal{R}\omega > t_f$ represent $\int_0^{t_f} y^T(t)\mathcal{L}y(t)dt, \int_0^{t_f} y^T(t)\mathcal{P}\omega(t)dt$ and $\int_0^{t_f} \omega^T(t)\mathcal{R}\omega(t)dt$ respectively.Similarly,connection (17) can be written in the structure of the following dissipativity execution index models (15) and (16),

$$\mathcal{J}_{\gamma,t_f} = \int_0^{t_f} \mathcal{E}\{\left[\begin{pmatrix} y(t) \\ \omega(t) \end{pmatrix}^T\right] \begin{bmatrix} \mathcal{L} & \mathcal{P} \\ * & \mathcal{R} \end{bmatrix} \begin{pmatrix} y(t) \\ \omega(t) \end{pmatrix} - \gamma\omega^T(t)\omega(t)\}, \tag{19}$$

**Definition 2.2** [27] Complex models (15) and (16) are said to be positive,if there exist a scalar $\gamma > 0$ such that for all $t_f > 0$ and for all solution with zero initial condition,

$$2\int_0^{t_f} \mathcal{E}\{y^T(s)\omega(s)\}ds \geq -\gamma \int_0^{t_f} \mathcal{E}\{\omega^T(s)\omega(s)\}ds \ . \tag{20}$$

**Lemma 2.3** [28] let $\otimes$ denotes the Kronecker product,$x, y, z$ and $w$ are matrices with appropriate dimensions.The following properties hold:

$(\beta x) \otimes y = x \otimes (\beta y)$,

$(x + y) \otimes z = x \otimes z + y \otimes z$,

$(x \otimes y)(z \otimes w) = (xz) \otimes (yw)$,

$(x \otimes y)^T = x^T \otimes y^T$.

**Lemma 2.4** [29] For any constant model,

$w \in \mathbb{R}^n, w = w^T > 0$ Scalars a and b with $a < b$ and a vector function $x: [a, b] \to \mathbb{R}^n$ such that the integrations concerned are well defined.As such the following inequality holds:

$$\left[\int_a^b x(s)ds\right]^T w \left[\int_a^b x(s)ds \leq (b - a)\left[\int_a^b x^T(s)wx(s)ds\right]\right], \tag{21}$$

**Lemma 2.5**[30] (Reciprocal connectivity lemma) For any vectors $x_1, x_2$ constant matrices $\mathcal{P}_i \quad i = 1,2, ..4 \ \mathcal{U}$ and real scalar $\alpha \geq 0. \beta \geq 0$ satisfying $\alpha + \beta = 1$ the following inequality holds:

$$-\frac{1}{\alpha}x_1^T\mathcal{P}_1x_1-\frac{1}{\beta}x_2^T\mathcal{P}_2x_2-\frac{\beta}{\alpha}x_1^T\mathcal{P}_3x_1-\frac{\alpha}{\beta}x_2^T\mathcal{P}_4x_1\leq-\begin{bmatrix}x_1\\x_2\end{bmatrix}\begin{bmatrix}\mathcal{P}_1 & \mathcal{U}\\ * & \mathcal{P}_2\end{bmatrix}^T\begin{bmatrix}x_1\\x_2\end{bmatrix}, \quad (22)$$

subject to

$$0\leq\begin{bmatrix}\mathcal{P}_1+\mathcal{P}_3 & \mathcal{U}\\ * & \mathcal{P}_2+\mathcal{P}_4\end{bmatrix}.$$

**Lemma 2.6** [31] (Refined Jensen-based inequality) For a given matrix $\mathcal{B}\in\mathbb{S}^+$ and a function $\varphi:[a,b]\to\mathbb{R}^n$whose derivatives $\varphi\in(C[a,b],\mathbb{R}^n)$the following inequality holds;

$$\int_a^b\varphi^T(s)\mathcal{B}\varphi(s)ds\geq\frac{1}{b-a}\hat{\varphi}^T\widehat{\mathcal{B}}\hat{\varphi}, \quad (23)$$

where

$$\widehat{\mathcal{B}}=diag\{\mathcal{B},3\mathcal{B},5\mathcal{B}\},\hat{\varphi}=[\varphi_1^T\varphi_2^T\varphi_3^T]^T,\varphi_1=\varphi(b)-\varphi(a),\ \varphi_2=\varphi(b)+\varphi(a)-\frac{2}{b-a}\int_a^b\varphi(s)ds,$$

$$\varphi_3=\varphi(b)-\varphi(a)-\frac{6}{b-a}\int_a^b\varphi(s)ds-\frac{12}{(b-a)^2}\int_a^b\int_s^b\varphi(u)duds.$$

## 3.Main Results

We present delay dependent dissipativity condition synchronization of MJCDM models (15) and (16).For simplicity denote

$\mathcal{F}_1=I_N\otimes(\mathcal{U}_1^T\mathcal{U}_2+\mathcal{U}_2^T\mathcal{U}_1),\mathcal{F}_2=I_N\otimes(\mathcal{U}_1^T+\mathcal{U}_2^T)$

### 3.1 Dissipativity Analysis

By utilizing Lyapunov-Karasovskii functional (LKF) and LMI methods,we amnalyze dissipativity of MJCDM models (15) and (16) with time varying distributed coupling delays

**Theorem 3.1** Based on Assumption 1.given scalars $\bar{\tau}>0,\delta>0\ and\ \mu>0$ error models(15) and (16) are strictly $(\mathcal{L},\mathcal{P},\mathcal{R})-\gamma dissipative$ if there exist a positive definite matrices $\mathcal{P}_k>0,\mathcal{L}_1>0,\mathcal{L}_2>0,\mathcal{M}_1>0,\mathcal{Z}>0,\mathcal{Z}_1>0,\mathcal{Z}_2>0$, for any consistant matrix $\mathcal{T}_1$ ,appropriate dimensional matrices $\Lambda_1,\Lambda_2$ and positive scalars $\gamma$>0,$\alpha_{1,}\alpha_2>0$ such that the following condition holds:

$$\begin{bmatrix}I_N\otimes\mathcal{M}_1 & I_N\otimes\mathcal{T}_1\\ * & I_N\otimes\mathcal{M}_1\end{bmatrix}>0\ , \quad (24)$$

$$\varphi_k=\left[\varphi_{(l,m,k)}\right]_{11\times11}<0, \quad (25)$$

where

$\varphi_{1,1,k}=(I_N\otimes\mathcal{L}_1)+(I_N\otimes\mathcal{L}_2)+\sum_{q=1}^N\lambda_{kq}\left(I_N\otimes\mathcal{P}_q\right)+\bar{\tau}(I_N\otimes\mathcal{N})(I_N\otimes\mathcal{E}_{1k})-\left(I_N\otimes\mathcal{M}_1\right)-9(I_N\otimes\mathcal{Z})-(\alpha_1\mathcal{F}_1)-(I_N\otimes\Lambda_1)\left(I_N\otimes\mathcal{b}_{1,k}\right)-\left(I_N\otimes\mathcal{b}_{1k})^T(I_N\otimes\Lambda_1)^T\right)+(I_N\otimes\Lambda_1)(I_N\otimes\mathcal{K}_k)+(I_N\otimes\mathcal{K}_k)^T(I_N\otimes\Lambda_1)^T+\delta^2(I_N\otimes\mathcal{Z}_1)(I_N\otimes\mathcal{Z}_2)-(I_N\otimes\mathcal{E}_{1k})^T\boldsymbol{\mathcal{L}}(I_N\otimes\mathcal{E}_{1k})$,

$\varphi_{12,k}=(I_N\otimes\mathcal{P}_k)+3(I_N\otimes\mathcal{Z})-(I_N\otimes\Lambda_1)-(I_N\otimes\mathcal{b}_{1k})^T(I_N\otimes\Lambda_2)^T+(I_N\otimes\mathcal{K}_k)^T(I_N\otimes\Lambda_2)^T$,

$\varphi_{13,k}=(I_N\otimes\mathcal{M}_1)-(I_N\otimes\mathcal{T}_1)+3(I_N\otimes\mathcal{Z})+(I_N\otimes\Lambda_1)c(\mathcal{G}\otimes\mathcal{A}_{2k})+(I_N\otimes\Lambda_1)(I_N\otimes\mathcal{f}_{1k})$,

$\varphi_{14,k}=(I_N\otimes\mathcal{T}_1),\varphi_{1,6,k}=-\frac{52}{\bar{\tau}}(I_N\otimes\mathcal{Z}),\varphi_{1,7,k=\frac{120}{\bar{\tau}^2}}(I_N\otimes\mathcal{Z})$,

$\varphi_{18,k}=\alpha_1\mathcal{F}_2+(I_N\otimes\Lambda_1)(I_N\otimes\mathcal{B}_{1k}),\varphi_{1,9,k}=(I_N\otimes\Lambda_1)(I_N\otimes\mathcal{A}_{1k}),\ \varphi_{1,10,k}=(I_N\otimes\Lambda_1)(I_N\otimes\mathcal{A}_{3k})$,

$\varphi_{1,11,k}=(I_N\otimes\Lambda_1)(I_N\otimes\mathcal{D}_{1k}-(I_N\otimes\mathcal{E}_{1k})^T\mathcal{f},\varphi_{2,2,k}=\bar{\tau}^2(I_N\otimes\mathcal{Z})-(I_N\otimes\Lambda_2)-(I_N\otimes\Lambda_2)^T$,

Tuijin Jishu/Journal of Propulsion Technology
ISSN: 1001-4055
Vol. X No. Y (20--)

$\varphi_{2,3,k} = (I_N \otimes \Lambda_2) c (\mathcal{G} \otimes \mathcal{A}_{2k}) + (I_N \otimes \Lambda_2)(I_N \otimes \mathcal{f}_{1k}),\ \varphi_{2,8,k} = (I_N \otimes \Lambda_2)(I_N \otimes \mathcal{B}_{1k}),$

$\varphi_{2,9,k} = (I_N \otimes \Lambda_2)(I_N \otimes \mathcal{A}_{1k}),\ \varphi_{2,10,k} = (I_N \otimes \Lambda_2)(\mathcal{G} \otimes \mathcal{A}_{3k}),\ \varphi_{2,11,k} = (I_N \otimes \Lambda_2)(I_N \otimes \mathcal{D}_{1k}),$

$\varphi_{3,3,k} = -(1-\mu)(I_N \otimes \mathcal{L}_1) + (I_N \otimes \mathcal{T}_1)(I_N \otimes \mathcal{T}_1)^T - 9(I_N \otimes \mathcal{Z}) - \alpha_2 \mathcal{F}_1,\ \varphi_{3,4,k} = -2(I_N \otimes \mathcal{T}_1) + (I_N \otimes \mathcal{M}_1),$

$\varphi_{3,6,k} = \frac{36}{\bar{\tau}}(I_N \otimes \mathcal{Z}),\ \varphi_{3,7,k} = -\frac{180}{\bar{\tau}^2}(I_N \otimes \mathcal{Z}),\ \varphi_{3,9,k} = \alpha_2 \mathcal{F}_1,\ \varphi_{4,4,k} = -(I_N \otimes \mathcal{M}_1)\text{-}(I_N \otimes \mathcal{L}_2),$

$\varphi_{5,5,k} = -(I_N \otimes \mathcal{L}_2),\ \varphi_{6,6,k} = -\frac{192}{\bar{\tau}^2}(I_N \otimes \mathcal{Z}),\ \varphi_{6,7,k} = \frac{720}{\bar{\tau}^3}(I_N \otimes \mathcal{Z}),\ \varphi_{7,7,k} = -\frac{720}{\bar{\tau}^4}(I_N \otimes \mathcal{Z}), \varphi_{8,8,k} = -2\alpha_1 \mathrm{I}$

$\varphi_{9,9,k} = -2\alpha_2 \mathrm{I} \quad \varphi_{10,10,k} = -(I_N \otimes \mathcal{Z}_1),$

$\varphi_{11,11,k} = -(\mathcal{R} - \gamma I)$, and the remaining coefficient are zero

**Proof**: To establish the dissipativity conditions,the following LKF is introduced for the closed-loop error models(15) and (16)

$$\mathcal{v}(e(t),k,t) = \sum_{l=1}^{7} \mathcal{v}_1(e(t),k,t) \tag{26}$$

where

$$\mathcal{v}_1(e(t),k,t) = e^T(t)(I_N \otimes \mathcal{P}_k)e(t), \tag{27}$$

$$\mathcal{v}_2(e(t),k,t) = \int_{t-\tau(t)}^{t} e^T(s)(I_N \otimes \mathcal{L}_1)e(s)ds + \int_{t-\tau(t)}^{t} e^T(s)(I_N \otimes \mathcal{L}_2)e(s)ds, \tag{28}$$

$$\mathcal{v}_3(e(t),k,t) = \bar{\tau} \int_{-\bar{\tau}}^{0} \int_{t+\theta}^{t} \dot{e}^T(s)(I_N \otimes \mathcal{M}_1)\dot{e}(s)dsd\theta, \tag{29}$$

$$\mathcal{v}_4(e(t),k,t) = \int_{-\bar{\tau}}^{0} \int_{t+\theta}^{t} e^T(s)(I_N \otimes \mathcal{N})e(s)dsd\theta, \tag{30}$$

$$\mathcal{v}_5(e(t),k,t) = \int_{-\bar{\tau}}^{0} \int_{t+\theta}^{t} \dot{e}^T(s)(I_N \otimes \mathcal{Z})\dot{e}(s)dsd\theta, \tag{31}$$

$$\mathcal{v}_6(e(t),k,t) = \delta \int_{-\bar{\delta}}^{0} \int_{t+\theta}^{t} e^T(s)(I_N \otimes \mathcal{Z}_1)e(s)dsd\theta, \tag{32}$$

$$\mathcal{v}_7(e(t),k,t) = \int_{t-\delta}^{t} e^T(s)(I_N \otimes \mathcal{L}_2)e(s)ds, \tag{33}$$

Let $\boldsymbol{\mathcal{L}}$ be a weak infinitesimal operator[36] of the random process $\{e(t),k,t \geq 0\}$along with models (15) and (16) be defined as

$$\mathcal{L}\mathcal{v}(e(t),k,t \geq 0) = \lim_{\Delta \to 0^+} \frac{1}{\mathcal{v}}\{\mathcal{E}[\mathcal{v}(e(t+\Delta),k,t+\Delta)], e(t),k\} - \mathcal{v}(e(t),k,) \tag{34}$$

By calculating the infinitesimal generator of $\mathcal{v}(e(t),k,t)$ along the rajector of the closed loop error models (15) and (16) be defined as ,

$$\mathcal{L}\mathcal{v}_1(e(t),k,t) = 2e^T(t)(I_N \otimes \mathcal{P}_k)\dot{e}(t) + e^T(t)\sum_{q=1}^{N} \lambda_{kq}\left(I_N \otimes \mathcal{P}_q\right)e(t), \tag{35}$$

$$\mathcal{L}\mathcal{v}_2(e(t),k,t) \leq e^T(t)\big((I_N \otimes \mathcal{L}_1) + (I_N \otimes \mathcal{L}_2)\big)e(t) - (1-\mu)e^T(t-\tau(t))(I_N \otimes \mathcal{L}_1)(t-\tau(t)) - e^T(t-\bar{\tau})(I_N \otimes \mathcal{L}_2)(t-\bar{\tau}), \tag{36}$$

$$\mathcal{L}\mathcal{v}_3(e(t),k,t) = \bar{\tau}^2 \dot{e}^T(t)(I_N \otimes \mathcal{M}_1)\dot{e}(t) - \bar{\tau}\int_{t-\tau(t)}^{t} \dot{e}^T(I_N \otimes \mathcal{M}_1)\dot{e}(s)ds \quad , \tag{37}$$

$$\mathcal{L}\mathcal{v}_4(e(t),k,t) = \bar{\tau}e^T(t)((I_N \otimes \mathcal{N})e(t) - \int_{t-\tau(t)}^{t} e^T(s)((I_N \otimes \mathcal{N})e(s))ds, \tag{38}$$

$$\mathcal{L}\mathcal{v}_5(e(t),k,t) = \bar{\tau}\dot{e}^T(t)((I_N \otimes \mathcal{Z})\dot{e}(t) - \int_{t-\tau(t)}^{t} \dot{e}^T(s)((I_N \otimes \mathcal{Z})\dot{e}(s))ds, \tag{39}$$

$$\mathcal{L}\mathcal{v}_6(e(t),k,t) = \delta^2 e^T(t)(I_N \otimes \mathcal{Z}_1)e(t) - \delta\int_{t-\delta}^{t} e^T(s)(I_N \otimes \mathcal{Z}_1)e(s), \tag{40}$$

$$\mathcal{L}v_7(e(t),k,t) = e^T(t)(I_N \otimes \mathcal{Z}_2)e(t) - e^T(t-\delta)(I_N \otimes \mathcal{Z}_2)e(t-\delta), \quad (41)$$

Based on (37) by using lemmas 2.4 and 2.5 we obtain ,

$$-\bar{\tau}\int_{t-\tau(t)}^{t} e^T(I_N \dot{\otimes} \mathcal{M}_1)\dot{e}(s)ds \ \le \bar{\tau}\int_{t-\tau(t)}^{t} e^T(I_N \dot{\otimes} \mathcal{M}_1)\dot{e}(s)ds \ - \bar{\tau}\int_{t-\bar{\tau}}^{t-\tau(t)} e^T(I_N \dot{\otimes} \mathcal{M}_1)\dot{e}(s)ds \ ,$$

$$\le -\left(\frac{\bar{\tau}}{\tau(t)}\right)\left(\int_{t-\tau(t)}^{t}\dot{e}(s)ds\right)^T (I_N \otimes \mathcal{M}_1)\left(\int_{t-\tau(t)}^{t}\dot{e}(s)ds\right) - \frac{\bar{\tau}}{\bar{\tau}-\tau(t)}\left(\int_{t-\tau}^{t-\tau(t)}\dot{e}(s)ds\right)^T (I_N \otimes \mathcal{M}_1)\left(\int_{t-\tau}^{t-\tau(t)}\dot{e}(s)ds\right)$$

$$\le \begin{bmatrix} e(t) - e(t-\tau(t) \\ e\big(t-\tau(t)\big) - e(t-\bar{\tau}) \end{bmatrix}^T \begin{bmatrix} (I_N \otimes \mathcal{M}_1) & (I_N \otimes \mathcal{T}_1) \\ * & (I_N \otimes \mathcal{M}_1) \end{bmatrix} \quad (42)$$

$$\le \begin{bmatrix} e(t) - e(t-(t) \\ e\big(t-\tau(t)\big) - e(t-\bar{\tau}) \end{bmatrix}, \quad (43)$$

By using Lemma 2.4 we obtain

$$-\int_{t-\bar{\tau}}^{t} e^T(s)((I_N \otimes \mathcal{N})e(s))ds \le -\frac{1}{\bar{\tau}}\left(\int_{t-\tau(t)}^{t} e(s)ds\right)^T (I_N \otimes \mathcal{N})\left(\int_{t-\tau(t)}^{t} e(s)ds\right), \quad (44)$$

$$-\delta\int_{t-\delta}^{t} e^T(s)(I_N \otimes \mathcal{Z}_1)e(s) \le \left(\int_{t-\delta}^{t} e(s)ds\right)^T (I_N \otimes \mathcal{Z}_1)\left(\int_{t-\delta}^{t} e(s)ds\right). \quad (45)$$

Using lemma 2.6 we obtain

$$\int_{t-\bar{\tau}}^{t} \dot{e}^T(s)((I_N \otimes \mathcal{Z})\dot{e}(s))ds \le$$
$$\begin{bmatrix} e(t) \\ e(t-\tau(t)) \\ \frac{1}{\tau}\int_{t-\bar{\tau}}^{t} e(s)ds \\ \frac{2}{\bar{\tau}^2}\int_{t-\bar{\tau}}^{t} e(s)dsd\theta \end{bmatrix}^T \begin{bmatrix} -9(I_N \otimes \mathcal{Z}) & 3(I_N \otimes \mathcal{Z}) & -28(I_N \otimes \mathcal{Z}) & 30(I_N \otimes \mathcal{Z}) \\ 3(I_N \otimes \mathcal{Z}) & -9(I_N \otimes \mathcal{Z}) & 36(I_N \otimes \mathcal{Z}) & -30(I_N \otimes \mathcal{Z}) \\ -24(I_N \otimes \mathcal{Z}) & 36(I_N \otimes \mathcal{Z}) & -192(I_N \otimes \mathcal{Z}) & 180(I_N \otimes \mathcal{Z}) \\ 30(I_N \otimes \mathcal{Z}) & -30(I_N \otimes \mathcal{Z}) & 180(I_N \otimes \mathcal{Z}) & -180(I_N \otimes \mathcal{Z}) \end{bmatrix} \begin{bmatrix} e(t \\ e(t-\tau(t)) \\ \frac{1}{\tau}\int_{t-\bar{\tau}}^{t} e(s)ds \\ \frac{2}{\bar{\tau}^2}\int_{t-\bar{\tau}}^{t} e(s)dsd\theta \end{bmatrix} \quad (46)$$

On the other hand we can obtain from the assumption 1 that,

$2[f^T e(t) - \mathcal{U}_1 e^T(t)][fe(t) - \mathcal{U}_2 e(t)] \le 0.$

There exist $\alpha_1 > 0, \alpha_2 > 0$ such that ,

$$-\alpha_1 \begin{bmatrix} e(t) \\ fe(t) \end{bmatrix}^T \begin{bmatrix} \mathcal{F}_1 & -\mathcal{F}_2 \\ * & 2I \end{bmatrix} \begin{bmatrix} e(t) \\ fe(t) \end{bmatrix} \ge 0, \quad (47)$$

$$-\alpha_2 \begin{bmatrix} e(t-\tau(t)) \\ fe(t-\tau(t)) \end{bmatrix}^T \begin{bmatrix} \mathcal{F}_1 & -\mathcal{F}_2 \\ * & 2I \end{bmatrix} \begin{bmatrix} e(t-\tau(t)) \\ fe(t-\tau(t)) \end{bmatrix} \ge 0. \quad (48)$$

More over (35) and (36), it is obvious that there exist appropriate dimensional matrices $\Lambda_1$ and $\Lambda_2$ and the following holds

$2[e^T(t)(I_N \otimes \Lambda_1) + \dot{e}^T(t)(I_N \otimes \Lambda_2)[-\dot{e}(t) + \big(I_N \otimes (-\mathcal{b}_{1k} + \mathcal{K}_k) e(t)\big) + (I_N \otimes \mathcal{f}_{1k}) e(t-\tau(t)] + (I_N \otimes \mathcal{B}_{1k}) f e(t)) + (I_N \otimes \mathcal{A}_{1k}) f\big(e(t-\tau(t)\big) + c(\mathcal{G} \otimes \mathcal{A}_{1k}) e\big(t - \tau\tau(t)\big) + c(\mathcal{G} \otimes \mathcal{A}_{3k}) \int_{t-\delta}^{t} e(s) ds + (I_N \otimes \mathcal{D}_{1k}) \omega(t)]$ (49)

From (35)-(46) and adding (47)-(49) we have ,

$$\mathcal{E}\{\mathcal{L}\mathcal{v}e(t), k, t\} \leq$$

$$\mathcal{E}\{2e^T(t)(I_N \otimes \mathcal{P}_k)\dot{e}(t) + e^T(t) \sum_{q=1}^{N} \lambda_{kq} \left(I_N \otimes \mathcal{P}_q\right) e(t) + e^T(t)\big((I_N \otimes \mathcal{L}_1) + (I_N \otimes \mathcal{L}_2)\big) e(t) - (1-\mu) e^T(t - \tau(t))(I_N \otimes \mathcal{L}_1)(t-\tau(t)) - e^T(t-\bar{\tau})(I_N \otimes \mathcal{L}_2)(t-\bar{\tau})\} + \bar{\tau}^2 \dot{e}^T(t)(I_N \otimes \mathcal{M}_1)\dot{e}(t) - \begin{bmatrix} e(t) - e(t-\tau(t)) \\ e\big(t-\tau(t)\big) - e(t-\bar{\tau}) \end{bmatrix}^T \begin{bmatrix} (I_N \otimes \mathcal{M}_1) & (I_N \otimes \mathcal{T}_1) \\ * & (I_N \otimes \mathcal{M}_1) \end{bmatrix} \begin{bmatrix} e(t) - e(t-\tau(t)) \\ e\big(t-\tau(t)\big) - e(t-\bar{\tau}) \end{bmatrix} + \bar{\tau} e^T(t)((I_N \otimes \mathcal{N}) e(t) - \frac{1}{\bar{\tau}} \left(\int_{t-\tau(t)}^{t} e(s) ds\right)^T (I_N \otimes \mathcal{N}) \left(\int_{t-\tau(t)}^{t} e(s) ds\right) + \bar{\tau} \dot{e}^T(t)((I_N \otimes \mathcal{Z})\dot{e}(t) +$$

$$\begin{bmatrix} e(t) \\ e\big(t-\tau(t)\big) \\ \frac{1}{\tau}\int_{t-\bar{\tau}}^{t} e(s) ds \\ \frac{2}{\bar{\tau}^2}\int_{t-\bar{\tau}}^{t} e(s) ds d\theta \end{bmatrix}^T \begin{bmatrix} -9(I_N \otimes \mathcal{Z}) & 3(I_N \otimes \mathcal{Z}) & -28(I_N \otimes \mathcal{Z}) & 30(I_N \otimes \mathcal{Z}) \\ 3(I_N \otimes \mathcal{Z}) & -9(I_N \otimes \mathcal{Z}) & 36(I_N \otimes \mathcal{Z}) & -30(I_N \otimes \mathcal{Z}) \\ -24(I_N \otimes \mathcal{Z}) & 36(I_N \otimes \mathcal{Z}) & -192(I_N \otimes \mathcal{Z}) & 180(I_N \otimes \mathcal{Z}) \\ 30(I_N \otimes \mathcal{Z}) & -30(I_N \otimes \mathcal{Z}) & 180(I_N \otimes \mathcal{Z}) & -180(I_N \otimes \mathcal{Z}) \end{bmatrix} \begin{bmatrix} e(t \\ e\big(t-\tau(t)\big) \\ \frac{1}{\tau}\int_{t-\bar{\tau}}^{t} e(s) ds \\ \frac{2}{\bar{\tau}^2}\int_{t-\bar{\tau}}^{t} e(s) ds d\theta \end{bmatrix} +$$

$$\delta^2 e^T(t)(I_N \otimes \mathcal{Z}_1) e(t) - \left(\int_{t-\delta}^{t} e(s) ds\right)^T (I_N \otimes \mathcal{Z}_1) \left(\int_{t-\delta}^{t} e(s) ds\right) e^T(t)(I_N \otimes \mathcal{Z}_2) e(t) - e^T(t-\delta)(I_N \otimes \mathcal{Z}_2) e(t - \delta) - \alpha_1 \begin{bmatrix} e(t) \\ fe(t) \end{bmatrix}^T \begin{bmatrix} \mathcal{F}_1 & -\mathcal{F}_2 \\ * & 2I \end{bmatrix} \begin{bmatrix} e(t) \\ fe(t) \end{bmatrix} - \alpha_2 \begin{bmatrix} e\big(t-\tau(t)\big) \\ fe\big(t-\tau(t)\big) \end{bmatrix}^T \begin{bmatrix} \mathcal{F}_1 & -\mathcal{F}_2 \\ * & 2I \end{bmatrix} \begin{bmatrix} e\big(t-\tau(t)\big) \\ fe\big(t-\tau(t)\big) \end{bmatrix} + 2[e^T(t)(I_N \otimes \Lambda_1) + \dot{e}^T(t)(I_N \otimes \Lambda_2)[-\dot{e}(t) + \big(I_N \otimes (-\mathcal{b}_{1k} + \mathcal{K}_k) e(t)\big) + (I_N \otimes \mathcal{f}_{1k}) e(t-\tau(t)] + (I_N \otimes \mathcal{B}_{1k}) f e(t) + (I_N \otimes \mathcal{A}_{1k}) f\big(e\big(t-\tau(t)\big) + c(\mathcal{G} \otimes \mathcal{A}_{1k}) e\big(t-\tau\tau(t)\big) + c(\mathcal{G} \otimes \mathcal{A}_{3k}) \int_{t-\delta}^{t} e(s) ds + (I_N \otimes \mathcal{D}_{1k}) \omega(t)]. \quad (50)$$

Define the following dissipativity condition for models (15) and (16)

$$\mathcal{J}_{\gamma,t_f} = \int_0^{t_f} \mathcal{E}\{\left[\begin{pmatrix} y(t) \\ \omega(t) \end{pmatrix}^T\right] \begin{bmatrix} \mathcal{L} & \mathcal{P} \\ * & \mathcal{R} \end{bmatrix} \begin{pmatrix} y(t) \\ \omega(t) \end{pmatrix} - \gamma \omega^T(t)\omega(t)\}\, dt, \quad (51)$$

Thus ,we have,

$$\mathcal{E}\{\mathcal{L}\mathcal{v}\big(e(t)\big), k.t\} - y^T(t)\mathcal{L}y(t) - 2y^T(t)\mathcal{P}\omega(t) - \omega^T(t)(\mathcal{R} - \gamma I)\omega(t) \leq \mathcal{E}\{\zeta^T(t)\varphi_k\zeta(t)\}, \quad (52)$$

where $\zeta^T(t) = [e^T(t)\dot{e}^T(t)e^T\big(t-\tau(t)\big)e^T(t-\bar{\tau})e^T(1-\delta) \int_{t-\bar{\tau}}^{t} e(s) ds \int_{t-\bar{\tau}}^{t}\int_{\theta}^{t} e(s) ds d\theta\, f^T\big(e(t)\big) f^T\big(e(t - \tau(t))\big] \int_{t-\delta}^{t} e(s) ds \omega^T(t)]$

In addition we can confirm that $\varphi_k < 0$ holds.it is obvious that for $\varphi_k < 0$ (52) implies that,

$$\mathcal{E}\{y^T(t)\mathcal{L}y(t) + 2y^T(t)\mathcal{P}\omega(t) + \omega^T(t)\mathcal{R}\omega(t)\} \geq \mathcal{E}\{\mathcal{L}\mathcal{v}\big(e(t)\big), k.t + \gamma\omega^T(t))\omega(t)\}. \quad (53)$$

Integrating both sides of (53) from 0 $to\ t_f$ we have

$$\mathcal{E}\{\int_0^{t_f}[y^T(t)\mathcal{L}y(t) + 2y^T(t)\mathcal{P}\omega(t) + \omega^T(t)\mathcal{R}\omega(t)]dt\} \geq \mathcal{E}\int_0^{t_f}\{\mathcal{L}\mathcal{v}\big(e(t)\big), k.t + \gamma\omega^T(t))\omega(t)\}\, dt, \quad (54)$$

$$\mathcal{E}\{\mathcal{G}\big(y, \omega, t_f\big)\} \geq \mathcal{E}\{\gamma < \omega, \omega > t_f + (t_f, t, k - \mathcal{v}(e(0,t,k)) \geq \mathcal{E}\{\gamma < \omega, \omega >\}t_f\}. \quad (55)$$

For all $t_f \geq 0$.Therefore, error models (15) and (16) are strictly $(\mathcal{L}, \mathcal{P}, \mathcal{R}) - \gamma\ dissipative$ according to Definition 2.1 This complete the proof.

**Theorem 3.2** .Based on Assumption 1.given scalars $\bar{\tau} > 0, \delta > 0$ $and$ $\mu > 0$ $\beta_1, \beta_2 > 0$ error models (15) and (16) are strictly $(\mathcal{L}, \mathcal{P}, \mathcal{R}) - \gamma dissipative$ if there exist a positive definite matrices $\mathcal{P}_k > 0, \mathcal{L}_1 > 0, \mathcal{L}_2 > 0, \mathcal{M}_1 > 0, \mathcal{Z} > 0, \mathcal{Z}_1 > 0, \mathcal{Z}_2 > 0$, for any conssistant matrix $\mathcal{T}_1$ , appropriate dimensional matrices $\wedge, K_k > 0$ and positive scalars $\gamma$>0,$\alpha_{1,}\alpha_2 > 0$ such that the following condition holds:

$$\begin{bmatrix} I_N \otimes \mathcal{M}_1 & I_N \otimes \mathcal{T}_1 \\ * & I_N \otimes \mathcal{M}_1 \end{bmatrix} > 0 \quad (56)$$

$$\hat{\varphi}_k = [\varphi_{(l,m,k)}]_{11\times 11} < 0, \quad (57)$$

where

$\hat{\varphi}_{1,1,k} = (I_N \otimes \mathcal{L}_1) + (I_N \otimes \mathcal{L}_2) + \sum_{q=1}^{N} \lambda_{kq} \left(I_N \otimes \mathcal{P}_q\right) + \bar{\tau}(I_N \otimes \mathcal{N})(I_N \otimes \mathcal{E}_{1k}) - \left(I_N \otimes \mathcal{M}_1\right) - 9(I_N \otimes \mathcal{Z}) - (\alpha_1 \mathcal{F}_1) - (I_N \otimes \wedge_1)\left(I_N \otimes \mathscr{b}_{1,k}\right) - (I_N \otimes \mathscr{b}_{1k})^T \left(I_N \otimes \wedge_1)^T\right) + (I_N \otimes \wedge_1)(I_N \otimes \mathcal{K}_k) + (I_N \otimes \mathcal{K}_k)^T (I_N \otimes \wedge_1)^T + \delta^2 (I_N \otimes \mathcal{Z}_1)(I_N \otimes \mathcal{Z}_2) - (I_N \otimes \mathcal{E}_{1k})^T \mathcal{L}(I_N \otimes \mathcal{E}_{1k})$,

$\hat{\varphi}_{12,k} = (I_N \otimes \mathcal{P}_k) + 3(I_N \otimes \mathcal{Z}) - \beta_1 (I_N \otimes \wedge) - \beta_2 (I_N \otimes \mathscr{b}_{1k})^T (I_N \otimes \wedge)^T + \beta_2 (I_N \otimes \mathcal{K}_k)^T$,

$\hat{\varphi}_{13,k} = (I_N \otimes \mathcal{M}_1) - (I_N \otimes \mathcal{T}_1) + 3(I_N \otimes \mathcal{Z}) + \beta_1 (I_N \otimes \wedge) c(\mathcal{G} \otimes \mathcal{A}_{2k}) + \beta_2 (I_N \otimes \wedge)(I_N \otimes \mathscr{f}_{1k})$,

$\hat{\varphi}_{1,4,k} = \varphi_{1,4,k}, \hat{\varphi}_{1,6,k} = \varphi_{1,6,k}, \hat{\varphi}_{1,7,k} = \varphi_{1,7,k}$,

$\hat{\varphi}_{18,k} = \alpha_1 \mathcal{F}_2 + \beta_1 (I_N \otimes \wedge)(I_N \otimes \mathcal{B}_{1k})$,$\hat{\varphi}_{1,9,k} = \beta_1 (I_N \otimes \wedge)(I_N \otimes \mathcal{A}_{1k})$, $\hat{\varphi}_{1,10,k} = \beta_1 (I_N \otimes \wedge)(I_N \otimes \mathcal{A}_{3k})$,

$\hat{\varphi}_{1,11,k} = \beta_1 (I_N \otimes \wedge_1)(I_N \otimes \mathcal{D}_{1k} - (I_N \otimes \mathcal{E}_{1k})^T \mathscr{f}$, $\hat{\varphi}_{2,2,k} = \bar{\tau}^2 (I_N \otimes \mathcal{Z}) - \beta_2 (I_N \otimes \wedge) - \beta_2 (I_N \otimes \wedge)^T$,

$\hat{\varphi}_{2,3,k} = \beta_2 (I_N \otimes \wedge) c(\mathcal{G} \otimes \mathcal{A}_{2k}) + \beta_2 (I_N \otimes \wedge)(I_N \otimes \mathscr{f}_{1k})$, $\hat{\varphi}_{2,8,k} = \beta_2 (I_N \otimes \wedge)(I_N \otimes \mathcal{B}_{1k})$,

$\hat{\varphi}_{2,9,k} = \beta_2 (I_N \otimes \wedge)(I_N \otimes \mathcal{A}_{1k})$, $\hat{\varphi}_{2,10,k} = \beta_2 (I_N \otimes \wedge)(\mathcal{G} \otimes \mathcal{A}_{3k})$,

$\hat{\varphi}_{2,11,k} = \beta_2 (I_N \otimes \wedge)(I_N \otimes \mathcal{D}_{1k})$,

$\hat{\varphi}_{3,3,k} = \varphi_{3,3,k}, \hat{\varphi}_{3,4,k} = \varphi_{3,4,k}, \hat{\varphi}_{3,6,k} = \varphi_{3,6,k}, \hat{\varphi}_{3,7,k} = \varphi_{3,7,k}$

$\hat{\varphi}_{3,9,k} = \varphi_{3,9,k}, \hat{\varphi}_{4,4,k} = \varphi_{4,4,k}, \hat{\varphi}_{5,5,k} = \varphi_{5,5,k}, \hat{\varphi}_{6,6,k} = \varphi_{6,6,k}$

$\hat{\varphi}_{6,7,k} = \varphi_{6,7,k}, \hat{\varphi}_{7,7,k} = \varphi_{7,7,k}, \hat{\varphi}_{8,8,k} = \varphi_{8,8,k}, \hat{\varphi}_{9,9,k} = \varphi_{9,9,k}$

$\hat{\varphi}_{10,10,k} = \varphi_{10,10,k}, \hat{\varphi}_{11,11,k} = \varphi_{11,11,k}$,

and $\varphi_{(r\times s)}$ is defined in Theorem 3.1 Furtheremore,the desired controller gains are given as

$$\mathcal{K}_k = \wedge^{-1} \mathcal{K}_k, \quad (58)$$

**Proof**: Denote $\wedge_1 = \beta_1 \wedge, \wedge_2 = \beta_2 \wedge$.Then we can obtain inequality (57).According to theorem 3.1 ,we can conclude that error models (15) and (16) are strictly $(\mathcal{L}, \mathcal{P}, \mathcal{R}) - \gamma$ $dissipative$ according to Definition 2.1. This completes the proof.

**Remark 3**: Passivity and dissipativity theory has been used as a powerful analysis and design tool in many applications,such as large space structures, chemical process, multi-agent systems, haptic teleoperation systems, and cyber-physical systems.

### 3.2 Passivity Analysis

Tuijin Jishu/Journal of Propulsion Technology
ISSN: 1001-4055
Vol. X No. Y (20--)

We consider the passivity analysis of synchronization of MJCDM models with time-varying coupling delays

**Remark:4** Theorem 3.2 Provides MJDCMs with $H_\infty$ control dissipativity modes (15) and (16).Research on dissipativity analysis for various dynamical networks have recived increasing attention [33].As a special case of dissipativity ,the passivity analysis has been widely studied [34] . By applying $\mathcal{L} = 0, \mathcal{P} = I$ and $\mathcal{R}$=2$\gamma I$,the dissipativity performance(17) is reduced to the passivity condition (20)

Based on Theorem 3.2, we have corollary 3.3 for the passivity analysis of models (15) and (16) in the absence of $H_\infty$ control term $(u(t) = 0)$.In this case, models (15) and (16) reduces to

$$\dot{e}(t) = (I_N \otimes (-\mathscr{b}_{1k} + \mathcal{K}_{zk}))e(t) + (I_N \otimes \mathscr{f}_1(\rho_t))\big(t - \tau(t)\big) + e(I_N \otimes \mathcal{B}_{1k})(f(e(t) + (I_N \otimes \mathcal{A}_{1k})(f\Big(e\big(t - \tau(t)\big)\Big) + c(\mathscr{b} \otimes \mathcal{A}_{1k)}e\big(t - \tau(t)\big) + (I_N \otimes \mathcal{A}_{3k})\int_{t-\delta}^{t} e(s)ds + (I_N \otimes \mathcal{D}_{1k})\mathscr{w}(t), \qquad (59)$$

$$y(t) = (I_N \otimes \mathcal{E}_{1k})e(t). \qquad (60)$$

**Corollary 3.3**. Based on Assumption 1.given scalars $\bar{\tau} > 0, \delta > 0$ $and$ $\mu > 0$ error models(59) and (60) are positive in the sense of Definition 2.2 if there exist positive definite matrices $\mathcal{P}_k > 0, \mathcal{L}_1 > 0, \mathcal{L}_2 > 0, \mathcal{M}_1 > 0, \mathcal{Z} > 0, \mathcal{Z}_1 > 0, \mathcal{Z}_2 > 0$, for any conssistant matrix $\mathcal{T}_1$ ,appropriate dimensional matrices $\Lambda_1$ $and$ $\Lambda_2$and superb scalars $\gamma$>0,$\alpha_{1,}\alpha_2 > 0$ such that the following condition holds:

$$\begin{bmatrix} I_N \otimes \mathcal{M}_1 & I_N \otimes \mathcal{T}_1 \\ * & I_N \otimes \mathcal{M}_1 \end{bmatrix} > 0 \qquad (61)$$

$$\Psi_k = \left[\Psi_{(l,m,k)}\right]_{11\times 11} < 0, \qquad (62)$$

Where

$$\Psi_{1,1,k} = (I_N \otimes \mathcal{L}_1) + (I_N \otimes \mathcal{L}_2) + \sum_{q=1}^{N} \lambda_{kq}\left(I_N \otimes \mathcal{P}_q\right) + \bar{\tau}(I_N \otimes \mathcal{N})(I_N \otimes \mathcal{E}_{1k}) - \left(I_N \otimes \mathcal{M}_1\right) - 9(I_N \otimes \mathcal{Z}) - (\alpha_1 \mathcal{F}_1) - (I_N \otimes \Lambda_1)\left(I_N \otimes \mathscr{b}_{1,k}\right) - (I_N \otimes \mathscr{b}_{1k})^T (I_N \otimes \Lambda_1)^T\big) + (I_N \otimes \Lambda_1)(I_N \otimes \mathcal{K}_k) + (I_N \otimes \mathcal{K}_k)^T (I_N \otimes \Lambda_1)^T + \delta^2 (I_N \otimes \mathcal{Z}_1)(I_N \otimes \mathcal{Z}_2) - (I_N \otimes \mathcal{E}_{1k})^T \mathcal{L}(I_N \otimes \mathcal{E}_{1k}),$$

$$\Psi_{12,k} = (I_N \otimes \mathcal{P}_k) + 3(I_N \otimes \mathcal{Z}) - (I_N \otimes \Lambda_1) - (I_N \otimes \mathscr{b}_{1k})^T (I_N \otimes \Lambda_2)^T + (I_N \otimes \mathcal{K}_k)^T (I_N \otimes \Lambda_2)^T,$$

$$\Psi_{13,k} = (I_N \otimes \mathcal{M}_1) - (I_N \otimes \mathcal{T}_1) + 3(I_N \otimes \mathcal{Z}) + (I_N \otimes \Lambda_1)c(\mathcal{G} \otimes \mathcal{A}_{2k}) + (I_N \otimes \Lambda_1)(I_N \otimes \mathscr{f}_{1k}),$$

$$\Psi_{14,k} = (I_N \otimes \mathcal{T}_1), \Psi_{1,6,k} = -\frac{52}{\bar{\tau}}(I_N \otimes \mathcal{Z}), \Psi_{1,7,k = \frac{120}{\bar{\tau}^2}}(I_N \otimes \mathcal{Z}),$$

$$\Psi_{18,k} = \alpha_1 \mathcal{F}_2 + (I_N \otimes \Lambda_1)(I_N \otimes \mathcal{B}_{1k}), \Psi_{1,9,k} = (I_N \otimes \Lambda_1)(I_N \otimes \mathcal{A}_{1k}),$$

$$\Psi_{1,10,k} = (I_N \otimes \Lambda_1)(I_N \otimes \mathcal{A}_{3k}),\ \Psi_{1,11,k} = (I_N \otimes \Lambda_1)(I_N \otimes \mathcal{D}_{1k} - (I_N \otimes \mathcal{E}_{1k})^T \mathscr{f},$$

$$\Psi_{2,2,k} = \bar{\tau}^2 (I_N \otimes \mathcal{Z}) - (I_N \otimes \Lambda_2) - (I_N \otimes \Lambda_2)^T,\ \Psi_{2,3,k} = (I_N \otimes \Lambda_2)c(\mathcal{G} \otimes \mathcal{A}_{2k}) + (I_N \otimes \Lambda_2)(I_N \otimes \mathscr{f}_{1k}),$$

$$\Psi_{2,8,k} = (I_N \otimes \Lambda_2)(I_N \otimes \mathcal{B}_{1k}),\ \Psi_{2,9,k} = (I_N \otimes \Lambda_2)(I_N \otimes \mathcal{A}_{1k}),$$

$$\Psi_{2,10,k} = (I_N \otimes \Lambda_2)(\mathcal{G} \otimes \mathcal{A}_{3k}),\ \Psi_{2,11,k} = (I_N \otimes \Lambda_2)(I_N \otimes \mathcal{D}_{1k}),$$

$$\Psi_{3,3,k} = -(1-\mu)(I_N \otimes \mathcal{L}_1) + (I_N \otimes \mathcal{T}_1)(I_N \otimes \mathcal{T}_1)^T - 9(I_N \otimes \mathcal{Z}) - \alpha_2 \mathcal{F}_1,$$

$$\Psi_{3,4,k} = -2(I_N \otimes \mathcal{T}_1) + (I_N \otimes \mathcal{M}_1),$$

$$\Psi_{3,6,k} = \frac{36}{\bar{\tau}}(I_N \otimes \mathcal{Z}),\ \Psi_{3,7,k} = -\frac{180}{\bar{\tau}^2}(I_N \otimes \mathcal{Z}),$$

$\Psi_{3,9,k} = \alpha_2\mathcal{F}_1$, $\Psi_{4,4,k} = -(I_N \otimes \mathcal{M}_1)$-$(I_N \otimes \mathcal{L}_2)$, $\Psi_{5,5,k} = -(I_N \otimes \mathcal{L}_2)$, $\Psi_{6,6,k} = -\frac{192}{\bar{\tau}^2}(I_N \otimes \mathcal{Z})$,

$\Psi_{6,7,k} = \frac{720}{\bar{\tau}^3}(I_N \otimes \mathcal{Z})$, $\Psi_{7,7,k} = -\frac{720}{\bar{\tau}^4}(I_N \otimes \mathcal{Z})$, $\Psi_{8,8,k} = -2\alpha_1 \mathrm{I}$

$\Psi_{9,9,k} = -2\alpha_2 \mathrm{I}$ $\Psi_{10,10,k} = -(I_N \otimes \mathcal{Z}_1)$

$\Psi_{11,11,k} = -\gamma I$, and the remaining terms are zero

**Proof** :Consider the LKF candidate as in Theorem 3.1 by excluding the control term.Following the same argument of Theorem 3.1 we have

$$\mathcal{E}\{\mathcal{L}v(e(t)),k.t\} - y^T(t)\mathcal{L}y(t) - 2y^T(t)\mathcal{P}\omega(t) - \omega^T(t)(\mathcal{R} - \gamma I)\omega(t) \le \mathcal{E}\{\zeta^T(t)\varphi_k\zeta(t)\}, \tag{63}$$

where $\zeta^T(t) = [e^T(t)\dot{e}^T(t)e^T(t-\tau(t))e^T(t-\bar{\tau})e^T(1-\delta)\int_{t-\bar{\tau}}^{t} e(s)ds \int_{t-\bar{\tau}}^{t}\int_{\theta}^{t} e(s)dsd\theta\, f^T(e(t))f^T(e(t-\tau(t))] \int_{t-\delta}^{t} e(s)ds\omega^T(t)]$.

If $\Psi_k < 0$ then (63) implies that

$$\mathcal{E}\{\mathcal{L}v(e(t)),k,t) - 2y^T(t)\omega(t) - \omega^T(t)\omega(t)\} \le 0. \tag{64}$$

By integrating (64) from $0$ $to$ $t_f$ we have

$$2\int_0^{t_f} \mathcal{E}\{y^T\omega(s)\}ds \ge \mathcal{E}\left\{v\left(e(t_f)\right),k,t\right) - v(e(0),k,t) - \gamma\int_0^{t_f}\omega^T(s)\omega(s)ds\},$$

$$\ge \gamma\int_0^{t_f}\omega^T(s)\omega(s)ds. \tag{65}$$

For all $t_f \ge 0$.Therefore models (59) and (60) are passive according to definition 2.2 .This complets the proof of corollary 3.3.

**Remark:5** Models (59) and (60) without Markovian jumping reduce the following form,

$$\dot{e}(t) = (I_N \otimes \mathscr{b}_1)e(t) + (I_N \otimes \mathscr{f}_1)(t-\tau(t)) + (I_N \otimes \mathcal{B}_{1k})(f(e(t) + (I_N \otimes \mathcal{A}_1)(f\left(e(t-\tau(t))\right) + c(\mathcal{G} \otimes \mathcal{A}_1)e(t-\tau(t)) + (\mathcal{G} \otimes \mathcal{A}_3)\int_{t-\delta}^{t} e(s)ds + (I_N \otimes \mathcal{D}_1)w(t), \tag{66}$$

$$y(t) = (I_N \otimes \mathcal{E}_1)e(t), \tag{67}$$

**Corollary 3.4**.Based on Assumption 1.given scalars $\bar{\tau} > 0, \delta > 0$ $and$ $\mu > 0$ models(66) and (67) are passive in the definition 2.2 strictly if there exist a positive definite matrices $\mathcal{P} > 0, \mathcal{L}_1 > 0, \mathcal{L}_2 > 0, \mathcal{M}_1 > 0, \mathcal{Z} > 0$, $\mathcal{Z}_1 > 0, \mathcal{Z}_2 > 0$, for any consistant matrix $\mathcal{T}_1$ , appropriate dimensional matrices $\wedge_1, \wedge_2$ and high-quality scalars scalars $\gamma$>0,$\alpha_1,\alpha_2 > 0$ such that the following condition holds:

$$\begin{bmatrix} I_N \otimes \mathcal{M}_1 & I_N \otimes \mathcal{T}_1 \\ * & I_N \otimes \mathcal{M}_1 \end{bmatrix} > 0\,, \tag{68}$$

$$\Sigma_k = [\Sigma_{(l,m,k)}]_{11\times 11} < 0, \tag{69}$$

where

$\Sigma_{1,1} =$
$(I_N \otimes \mathcal{L}_1) + (I_N \otimes \mathcal{L}_2) + \bar{\tau}(I_N \otimes \mathcal{N})(I_N \otimes \mathcal{E}_{1k}) - (I_N \otimes \mathcal{M}_1) - 9(I_N \otimes \mathcal{Z}) - (\alpha_1\mathcal{F}_1) - (I_N \otimes \wedge_1)(I_N \otimes \mathscr{b}_{1,k}) - (I_N \otimes \mathscr{b}_{1k})^T(I_N \otimes \wedge_1)^T) + (I_N \otimes \wedge_1)(I_N \otimes \mathcal{K}_k) + (I_N \otimes \mathcal{K}_k)^T(I_N \otimes \wedge_1)^T + \delta^2(I_N \otimes \mathcal{Z}_1)(I_N \otimes \mathcal{Z}_2)$,$\Sigma_{12} = (I_N \otimes \mathcal{P}) + 3(I_N \otimes \mathcal{Z}) - (I_N \otimes \wedge_1) - (I_N \otimes \mathscr{b}_1)^T(I_N \otimes \wedge_2)^T$,$\Sigma_{13} = (I_N \otimes \mathcal{M}_1) - (I_N \otimes \mathcal{T}_1) + 3(I_N \otimes \mathcal{Z}) + (I_N \otimes \wedge_1)c(\mathcal{G} \otimes \mathcal{A}_2) + (I_N \otimes \wedge_1)(I_N \otimes \mathscr{f}_1)\Sigma_{14} = (I_N \otimes \mathcal{T}_1)$,$\Sigma_{1,6} = -\frac{52}{\bar{\tau}}(I_N \otimes \mathcal{Z})$,$\Sigma_{1,7=\frac{120}{\bar{\tau}^2}}(I_N \otimes \mathcal{Z})$,

$\Sigma_{18} = \alpha_1\mathcal{F}_2 + (I_N \otimes \wedge_1)(I_N \otimes \mathcal{B}_1)$,$\Sigma_{1,9} = (I_N \otimes \wedge_1)(I_N \otimes \mathcal{A}_1)$,

$\sum_{1,10} = (I_N \otimes \Lambda_1)(I_N \otimes \mathcal{A}_3)$, $\sum_{1,11} = (I_N \otimes \Lambda_1)(I_N \otimes \mathcal{D}_1 - (I_N \otimes \mathcal{E}_1)^T$,

$\sum_{2,2} = \bar{\tau}^2(I_N \otimes \mathcal{M}_1) + \overline{\tau(I_N \otimes \mathcal{Z})} - (I_N \otimes \Lambda_2) - (I_N \otimes \Lambda_2)^T$,$\sum_{2,3} = (I_N \otimes \Lambda_2)(I_N \otimes \mathscr{f}_1) + (I_N \otimes \Lambda_2)c(\mathcal{G} \otimes \mathcal{A}_2)$,

$\sum_{2,8,k} = (I_N \otimes \Lambda_2)(I_N \otimes \mathcal{B}_1)$,$\varphi_{2,9,k} = (I_N \otimes \Lambda_2)(I_N \otimes \mathcal{A}_{1k})$,$\varphi_{2,10,k} = (I_N \otimes \Lambda_2)(\mathcal{G} \otimes \mathcal{A}_{3k})$,$\varphi_{2,11,k} =$ $(I_N \otimes \Lambda_2)(I_N \otimes \mathcal{D}_{1k})$ ,$\varphi_{3,3,k} = -(1-\mu)(I_N \otimes \mathcal{L}_1) + (I_N \otimes \mathcal{T}_1)(I_N \otimes \mathcal{T}_1)^T - 9(I_N \otimes \mathcal{Z}) - \alpha_2 \mathcal{F}_1$,

$\sum_{3,4} = -2(I_N \otimes \mathcal{T}_1) + (I_N \otimes \mathcal{M}_1)$, $\sum_{3,6} = \frac{36}{\bar{\tau}}(I_N \otimes \mathcal{Z})$, $\sum_{3,7} = -\frac{180}{\bar{\tau}^2}(I_N \otimes \mathcal{Z})$,

$\sum_{3,9} = \alpha_2 \mathcal{F}_1$, $\sum_{4,4} = -(I_N \otimes \mathcal{M}_1)$-$(I_N \otimes \mathcal{L}_2)$,$\sum_{5,5} = -(I_N \otimes \mathcal{L}_2)$, $\sum_{6,6} = -\frac{192}{\bar{\tau}^2}(I_N \otimes \mathcal{Z})$,$\sum_{6,7} = \frac{720}{\bar{\tau}^3}(I_N \otimes \mathcal{Z})$,
$\sum_{7,7} = -\frac{720}{\bar{\tau}^4}(I_N \otimes \mathcal{Z})$, $\sum_{8,8} = -2\alpha_1 \mathrm{I}$ $\sum_{9,9} = -2\alpha_2 \mathrm{I}$ $\sum_{10,10} = -(I_N \otimes \mathcal{Z}_1)$

$\sum_{11,11} = -\gamma I$, and the remaining terms are zero

**Proof**: The proof is the same as that of Corollary 3.3, hence, it is omitted.

**Remark 6**: It is mentioned that employing an augmented LKF and zero integral inequalities can lead to less conservative results when compare to the LKF in the papers [35,-36]. The proposed model consists of a distributed coupling delays with newly augmented form of LKF.Most of the papers[37-39]used the Kronecker product method tostudy the complex dynamical networks. In this paper, we employed the transformation method. In order to reduce the computational burden there are various methods are approached but in this work, the refined Jensen's inequality and reciprocal convex lemma are used in the proof of Theorems, which helps to reduces the computational burden. As a result, proposed passivity and dissipativity analysis gives better results while maintaining lower computational burden.

**Remark 7**: When compare to the results in [48], this paper deals with the distributed coupling delay. Thus, it is necessary to investigate the problem of MJCDN model with distributed coupling delay in order to get some less conservative results,which is the motivation for this work,as it is theoretical and practical significance. In[40]and[41]onlydissipative analysis of systems are investigated but this paper deals with both passivity and dissipativity of MJCDN model, which is the main advantages of the paper.

**4 NumericalExample**

A numerical example to illustrate the feasibility and effectiveness of the obtained results is presented, as follows. Example 1: Consider models (13) and (14) that consist of 3 identical nodes and κ = 2 with time-varying distributed coupling delays in which each node is a two-dimensional MJCDN model described by

$$\dot{e}_z(t) = (I_N \otimes (-\mathscr{b}_{1k} + \mathcal{K}_{zk}))e_z(t) + (I_N \otimes \mathscr{f}_1(\rho_t))e_z(t-\tau(t)) + (I_N \otimes \mathcal{B}_{1k})(f(e_z(t) + (I_N \otimes \mathcal{A}_{1k})(f\left(e(t-\tau(t))\right) + c(\mathscr{b} \otimes \mathcal{A}_{1k})e(t-\tau(t)) + (I_N \otimes \mathcal{A}_{3k})\int_{t-\delta}^{t} e_{j(s)ds} + (I_N \otimes \mathcal{D}_{1k})\omega_z(t) \quad (70)$$

$$y_z(t) = (I_N \otimes \mathcal{E}_{1k})e(t) z = 1,2,3 \ k = 1,2. \quad (71)$$

Ther Parameters are given as follows,

$\mathscr{b}_{11} = \begin{bmatrix} -8.1 & 0 \\ 0 & -8.1 \end{bmatrix}$, $\mathscr{f}_{11} = \begin{bmatrix} -0.1 & 0.1 \\ 0 & -0.51 \end{bmatrix}$, $\mathcal{B}_{11} = \begin{bmatrix} 0.2 & 1.2 \\ -0.5 & 1.2 \end{bmatrix}$, $\mathcal{A}_{11} = \begin{bmatrix} -0.31 & 0.2 \\ 0.3 & -0.33 \end{bmatrix}$,

$\mathcal{A}_{21} = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}$, $\mathcal{A}_{31} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$, $\mathcal{D}_{11} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$, $\mathcal{E}_{11} = \begin{bmatrix} 0.2 & 0.2 \\ -0.3 & -0.3 \end{bmatrix}$,

$\mathscr{b}_{12} = \begin{bmatrix} -1.4 & 0 \\ 0 & -1.4 \end{bmatrix}$, $\mathscr{f}_{12} = \begin{bmatrix} -0.1 & 0.1 \\ -0.1 & -0.2 \end{bmatrix}$, $\mathcal{B}_{12} = \begin{bmatrix} 0.4 & 0.7 \\ 0.1 & 0 \end{bmatrix}$, $\mathcal{A}_{12} = \begin{bmatrix} 0.2 & 0.6 \\ 0.5 & 0.1 \end{bmatrix}$,

$\mathcal{A}_{22} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\mathcal{A}_{32} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\mathcal{D}_{12} = \begin{bmatrix} 0.31 & 1 \\ 0.4 & -0.21 \end{bmatrix}$, $\mathcal{E}_{12} = \begin{bmatrix} 0.1 & 0.1 \\ 0.3 & -0.3 \end{bmatrix}$.

The nonlinear function is chosen as

$$f(x_z(t)) = \begin{bmatrix} -0.3x_{z1} + \tanh(0.1x_{z1}) + 0.1x_{z2} \\ 0.5x_{z1} - \tanh(0.2x_{z1}) \end{bmatrix}.$$

It is clear that $f(x_z(t))$ satisfies Asuumption 1 with

$$\mathcal{U}_1 = \begin{bmatrix} -0.31 & 0.1 \\ 0 & 0.5 \end{bmatrix}, \qquad \mathcal{U}_2 = \begin{bmatrix} -0.2 & 0.1 \\ 0 & 0.31 \end{bmatrix}.$$

The outer coupling matrix is

$$\mathcal{G} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 1 & 1 & -2 \end{bmatrix}.$$

The coupling strength c is chosen as c=0.5 Markov process$\{\rho_t, t \geq 0\}$ takes value in $\mathcal{S} = \{1,2\}$ with generator $\Delta = \begin{bmatrix} -7 & 7 \\ 6 & -6 \end{bmatrix}$.The objective here is to find the dissipativity performance $\gamma$ such that models (70) and (71) are $(\mathcal{L}, \mathcal{P}, \mathcal{R}) - \gamma\ dissipative$. In this regard,we choose

$$\mathcal{L} = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}, \mathcal{P} = \begin{bmatrix} 0.1 & -0.1 \\ -0.1 & 0.5 \end{bmatrix} \mathcal{R} = \begin{bmatrix} 12 & 0 \\ 0 & 12 \end{bmatrix}. \quad (72)$$

Let us consider $\beta_1 = 2.99, \beta_2 = 1, \delta = 0.6\delta, \bar{\tau} = 0.25, \mu = 0.5$ .Then use the MATLAB LMI control toolbox to solve LMI (58) in theorem 3.2. The corresponding feasible solution as follows

$$\mathcal{P}_1 = \begin{bmatrix} 521.1461 & 249.5286 \\ 249.5286 & 512.7735 \end{bmatrix}, \quad \mathcal{P}_2 = \begin{bmatrix} 526.8866 & 264.1885 \\ 264.1885 & 531.1361 \end{bmatrix}, \mathcal{L}_1 = 10^3 \begin{bmatrix} 1.6423 & -0.0384 \\ -0.0384 & 1.4812 \end{bmatrix},$$

$$\mathcal{L}_2 = 10^3 \begin{bmatrix} 0.8965 & 0.0329 \\ 0.0329 & 1.0343 \end{bmatrix}, \mathcal{M}_1 = \begin{bmatrix} -17.3881 & -22.4879 \\ -22.4878 & -162.0464 \end{bmatrix}, \mathcal{N} = 10^3 \begin{bmatrix} -4.4606 & -0.4874 \\ -0.4875 & -4.3168 \end{bmatrix},$$

$$\mathcal{L} = 10^{-8} \begin{bmatrix} 0.6085 & 0.0000 \\ 0.0000 & 0.6085 \end{bmatrix}, \mathcal{L}_1 = \begin{bmatrix} 891.2228 & 1.7535 \\ 1.7534 & 919.6291 \end{bmatrix}, \mathcal{L}_2 = \begin{bmatrix} 898.2967 & -0.0000 \\ -0.0000 & 898.2965 \end{bmatrix},$$

$$\mathcal{X}_1 = 10^3 \begin{bmatrix} -1.1048 & -0.3208 \\ -0.3208 & 0.1223 \end{bmatrix}, \mathcal{X}_2 = \begin{bmatrix} -463.0821 & -285.8192 \\ -285.8192 & -471.7092 \end{bmatrix}, \Delta = \begin{bmatrix} 100.8260 & -0.2354 \\ -0.2354 & 68.1842 \end{bmatrix},$$

$$\gamma = 920.3727, \alpha_1 = 413.7550, \alpha_2 = 385.1395.$$

The corresponding gain matrices in (58) are given by,

$$\mathcal{K}_1 = \Delta^{-1}\mathcal{X}_1 = \begin{bmatrix} -10.9691 & -3.1775 \\ -4.7469 & 1.7829 \end{bmatrix}, \mathcal{K}_2 = \Delta^{-1}\mathcal{X}_2 = \begin{bmatrix} -4.6026 & -2.8511 \\ -4.2079 & -6.9281 \end{bmatrix}.$$

**5 Conclusion**

We have examined dissipativity and passivity-based synchronization of MJCDN models with time-varying distributed coupling delays in this study. In order to derive the closed-loop error model of the considered MJCDNs, a state feedback $H_\infty$ control scheme has been designed. By constraining the appropriate Lyapunov functional along with the properties of the Kronecker product and by using integral inequality techniques, several new delay-dependent dissipativity and passivity conditions have been derived in terms of LMIs. The results of this study have been verified with the MATLAB LMI toolbox. Numerical example have been presented to show the feasibility of the results. The method developed in this study can be utilized to examine other types of neural network models. As such, we will study coupled CDN models with complex-valued inputs in our further work.

**References**

[1] D. Watts, S. Strogatz, Collective dynamics of small-world networks, Nature, 393(6684) (1998) 440 – 442.
[2] X. Li, G. Chen, Synchronization and desynchronization of complex dynamical networks: an engineering viewpoint, IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications, 50(11) (2003) 1381 – 1390.

[3] Y. Liu, Z. Wang, J. Liang, X. Liu, Synchronization and state estimation for discrete-time complex networks with distributed delays, IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics, 38(5) (2008) 1314 –1325.
[4] X. Wang, G. Chen, Complex networks: small world, scale free and beyond, IEEE Circuits and Systems Magazine, 3 (2003) 6 – 20.
[5] M. E. J. Newman, The structure and function of complex networks, SIAM Review, 45 (2003) 167 – 256.
[6] S. Boccaletti, V. Latora, Y. Marenu, M. Chavez, D. U. Hwang, Complex networks: structure and dynamics, Physics Reports, 424 (2006) 175 – 308.
[7] P. L. Lu, Y. Yang, Synchronization of a class of complex networks, Chinese control conference, (2012) 1136 – 1141.
[8] N. N. Krasovskii, E. A. Lidskii, Analysis design of controller in systems with random attributes, Part I. Automation and Remote Control, 22 (1961) 1021 – 1025.
[9] Y. Liu, Z.Wang, J. Liang, X. Liu, Stability and synchronization of discrete-time Markovian jumping neural networks with mixed mode dependent time delays, IEEE Transactions on Neural Networks and Learning Systems, 20 (2009) 1102 – 1116.
[10] J. W. Yi, Y. W. Wang, J. W. Xiao, Y. Huang, Exponential synchronization of complex dynamical networks with Markovian jumping parameters and stochastic delays and its application to multi-agent systems, Communications in Nonlinear Science and Numerical Simulation, 18 (2013) 1175 – 1192..
[11] Y. Ma, X. Jia, D. Liu, Finite-time dissipative control for singular discrete-time Markovian jump systems with actuator saturation and partly unknown transition rates, Applied Mathematical Modelling, 53 (2018) 49 – 70.
[12] L. J. Pan, J. D. Cao, J. Q. Hu, Synchronization for complex networks with Markov switching via matrix measure approach, Applied Mathematical Modelling, 39 (2015) 5636 – 5649.
[13] L. Lu, B. He, C. Man, Passive synchronization for Markov jump genetic oscillator networks with time-varying delays, Mathematical Biosciences, 262 (2015) 80 – 87.
[14] J. Lu, G. Chen, A time-varying complex dynamical network model and its controlled synchronization criteria, IEEE Transactions on Automatic Control, 50 (2005) 841 – 846.
[15] D. H. wang, M. Chavez, A. Amann, S. Boccaletti, Synchronization in complex networks with age ordering, Physical Review Letters, 94 (2005) 138701-1 – 138701-4.
[16] H. Gao, J. Lam, G. Chen, New criteria for synchronization stability of general complex dynamical networks with coupling delays, Physics Letters A, 360 (2006) 263 – 273.
[17] Z. Fen, H. Gao,W. Zhen, New synchronization stability of complex networks with an interval time-varying coupling delay, IEEE Transactions on Circuits and Systems II, 56 (2009) 499 – 503.
[18] Z. X. Li, J. H. Park, Sampling-interval-dependent synchronization of complex dynamical networks with distributed coupling delay, Nonlinear Dynamics, 78 (2014) 341 – 348.
[19] H. Yan, H. Zhang, M. Q. Meng, Delay-range-dependent robust H¥ control for uncertain systems with interval timevarying delays, Neurocomputing, 73 (2010) 1235 – 1243.
[20] H. Q. Zheng, Y. W. Jing, Robust H¥ observer-based control for synchronization of a class of complex dynamical networks, Chinese Physics B, 20 (2011) 060504..
[21] L. Su, H. Shen, Mixed H¥/passive synchronization for complex dynamical networks with sampled-data control, Applied Mathematics and Computation, 259 (2015) 931 – 942.
[22] W. H. Chen, Z. Jiang, X. Lu, S. Luo, H¥ synchronization for complex dynamical networks with coupling delays using distributed impulsive control, Nonlinear Analysis: Hybrid Systems, 17 (2015) 111 – 127.
[23] J. C. Willems, Dissipative dynamical systems part I: General theory, Archive for Rational Mechanics and Analysis, 45 (1972) 321 – 351.
[24] C. K. Ahn, P. Shi, Dissipativity analysis for fixed-point interfered digital filters, Signal Processing, 109 (2015) 148– 153.
[25] Y. Sun, B. T. Cui, Dissipativity analysis of neural networks with time-varying delays, International Journal of Automation and computing, 5 (2008) 290 – 295.
[26] Y. Sun, B. T. Cui, Dissipativity analysis of neural networks with time-varying delays, International Journal of Automation and computing, 5 (2008) 290 – 295.
[27] S. Xu, W. X. Zheng, and Y. Zou, Passivity analysis of neural networks with time-varying delays, IEEE Transactions on Circuits Systems II, 56 (2009) 325 – 329.
[28] A. N. Langville, W. J. Stewart, The Kronecker product and stochastic autonomous networks, Journal of Computational and Applied Mathematics, 167 (2004) 429 – 447.
[29] W. Qian, S. Cong, T. Li, S. Fei, Improved stability conditions for systems with interval time-varying delay, International Journal of Control, Automation and Systems, 10 (2012) 1146 – 1152.

[30] W. I. Lee, S. Y. Lee, P. G. Park, Improved criteria on robust stability and H¥ performance for linear systems with interval time-varying delays via new triple integral functional, Applied Mathematics and Computation, 243 (2014) 570 – 577.
[31] P. Y. Park, W. I. Lee, S. Y. Lee, Auxiliary function-based integral inequalities for quadratic functions and their applications to time-delay systems, Journal of the Franklin Institute, 35 (2015) 1378 – 1396.
[32] C. Ma, Q. Zeng, L. Zhang, Y. Zhu, Passivity and passification for Markov jump genetic regulatory networks with time-varying delays, Neurocomputing, 136 (2014) 321 – 326.
[33] L. Sheng, M. Gao, W. Zhang, Dissipative control for Markov jump non-linear stochastic systems based on TS fuzzy model, International Journal of Systems Science, 45 (2014) 1213 – 1224..
[34] R. Li, J. Cao, Z. Tu, Passivity analysis of memristive neural networks with probabilistic time-varying delays, Neurocomputing,191 (2016) 249 – 262.
[35] X. Cai, K. Shi, S. Zhong, J. Wang, Y. Tang, Dissipative analysis for high speed train systems via looped-functional and relaxed condition methods, Applied Mathematical Modelling, 96 (2021) 570 – 583.
[36] S. Dong, H. Zhu, S. Zhong, K. Shi, Y. Liu, New study on fixed-time synchronization control of delayed inertial memristive neural networks, Applied Mathematics and Computation, 339 (2021) 126035.
[37] Q. Cheng, J. Cao, Global synchronization of complex networks with discrete-time delays and stochastic disturbances, Neural Computing and Applications, 20 (2011) 1167 – 79.
[38] Z. G. Wu, P. Shi, H. Su, J. Chu, Sampled-data exponential synchronization of complex dynamical networks with time-varying coupling delay, IEEE Transactions on Neural Networks and Learning Systems, 24 (2013) 1177 – 87.
[39] K. Shi, J. Wang, Y. Tang, S. Zhong, Reliable asynchronous sampled-data filtering of T-S fuzzy uncertain delayed neural networks with stochastic switched topologies, Fuzzy Sets and Systems, 381 (2020) 1 – 25.
[40] X. Cai, J. Wang, S. Zhong, K. Shi, Y. Tang, Fuzzy quantized sampled-data control for extended dissipative analysis of TS fuzzy system and its application to WPGSs, Journal of the Franklin Institute, 358 (2021) 1350 – 1375.
[41] X. Cai, K. Shi, S. Zhong, X. Pang, Dissipative Sampled-Data Control for High-Speed Train SystemsWith Quantized Measurements, IEEE Transactions on Intelligent Transportation Systems, DOI: 10.1109/TITS.2021.3052940