Exponential Consensus of The Stochastic Multi-Agent System by Means of Non-Fragilesampled Data Control with Markovian Leaping Parameters Under Switched Directedgraph

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Abstract:-This work addresses the issue of achieving exponential consensus in a stochastic multi-agent system with a leader, utilizing a non-fragile sample data control approach. The proposed system incorporates a Markovian jumping parameter, and a directed graph with two switches represents the communication between agents. The leader agent assumes the role of guiding the other agents in the system. The linear matrix inequality format is derived by generating an appropriate Lyapunov Krasovskii functional for the stated system, using the implementation of integral inequality. Numerical examples are provided to demonstrate the efficiency of the specified method.

Keywords*:* None-fragile; exponential; actuator faults; additive time-varying delay; sampled-data control.

1. Introduction

A specific kind of complex dynamical system called a multi-agent systems (MASs) is made up of several independent interacting agents and is designed to solve problems that isolated systems find challenging to solve. MASs can achieve all system states simultaneously by utilizing the data collected from their nearby agents [1], [2]. Numerous applications of MASs exist, including flocking, autonomous vehicles, spacecraft formation control, multi-robot, cooperative control of unmanned aircraft, and DC microgrid [3]- [6]. Developing a consensus problem-solving control protocol based on local information exchange that would enable all MAS agents to agree on particular circumstances or interests was the fundamental objective of all these experiments. In the past 10 years, consensus control for multi-agent systems (MASs) has garnered a lot of research attention due to its potential applications in several scientific and technical domains [7], [8]. Generally speaking, the main objective of consensus is to getthe agents to agree on a crucial parameter.

It is well known that physical processes always have a temporal delay in their performance and/orinstability. Consequently, the consensus problem of MAS with time delays has caused great concern in the past few years

[9], [10]. In message-passing algorithms , communication topology is essential for transmitting information from a leader agent to follower agents. A multi-agent system with a leader who guides the other agents is examined in [11] consensus. For convenience, the communication topology is represented here as an undirected graph, though it is typically regarded as a directed graph [12], [13]. [14] examines the consensus of MASs with a directed graph. Stochastic noises and disturbances are unavoidable in real-world scenarios for practical systems. Stochastic systems are constructed with the previously specified parameters to circumvent these is- sues. Stochastic dynamical systems have been the subject of several fruitful research projects recently. In [15] and [16], consensus in stochastic multi-agent systems is thoroughly examined. Unpredictable structural fluctuations, such as repairs resulting from abrupt environmental shocks, subsystem break- downs, and random component failures of systems, are commonly represented by Markov chains. Consensus of MASs with Markovian jumping parameter is investigated in [17].

Note that there are unknowns in the controller implementation that can cause instability in the final closed-loop system, so it's not always exact. To avoid this situation, the contemplated controller should be prepared to accept a certain amount of ambiguity. One subclass of robust controllers that can overcome these obstacles is called non-fragile controllers. Consensus of a non-linear multi-agent system with non-fragile control is investigated in [18]. However, in many real-world applications,,information sharing between agents can only occur under specific model conditions because of the use of digital sensors and limited bandwidth communication pathways. Instead of using continuous-time control, sampled-data control could be used to address this problem. Sampled-data systems contain continuous-time plants with discrete-time control updates. The control community has a long tradition of employing sampled-data controls in many different research areas. The consensus of a multi-agent system with sampled-data control is studied in [19] and [20]. In [21], the exponential consensus of

nonlinear multi-agent systems with Lipschitz conditions is investigated with reference to the sampled data. Explored in [22] is the phenomenon of exponential consensus in non-linear multi-agent systems,including semi-Markov switching topologies.To our best knowledge, there has not yet been a thorough examination of the exponential consensusof stochastic multi-agent systems with Markovian jumping parameters utilizing non-fragile sampled-data control. The study is motivated by this.

The main highlights of this paper are listed below:

- ❖ Considered is the issue of the leader adhering to the exponential consensus in a multi-agentsystem involving a stochastic process.
- ❖ Two switches in a directed graph facilitate agent communication. Two modes are examinedfor jumping parameters.
- ❖ To attain the acceptable condition for the multi-agent system, an appropriate variant of LKFwith double integrals is also systemized.
- ❖ Numerical examples are shown at the end to highlight the effectiveness of the derived theoretical results.

This is how the rest of the paper is organized. Section 2 contains the introduction, problem explanation, and control description. In section 3, non-fragile sampled-data control is used to construct a leader following consensus for a nonlinear multi-agent system. In section 4, a numerical result is provided to demonstrate the viability of the suggested system. The paper's conclusion is provided in section 5.

Notations:Notations are utterly usual.$\mathbb{R}^n$denotes n dimensional Euclidean space and $\mathbb{R}^{n\times m}$is the set of all $n\times m$ real matrices.The symbol * denotes the symmetric term in the matrix. The transpose and inverse for the matrix $\mathcal{A}$ is denoted by $\mathcal{A}^T$ and $\mathcal{A}^{-1}$,respectively,I is the identity matrix with appropriate dimension,$\mathcal{X}>0$ is a symmetric positive definite matrix,theKronecker product of matrices $\mathcal{A}$ and $\mathcal{B}$ is $denoted$ by $\mathcal{A}\otimes\mathcal{B}$

2.Problem Description And Preliminaries

The network communication topology of the proposed stochastic multi-agent system is indicated by a switching directed graph $\mathcal{G}_\sigma = (\mathcal{V}, \in_\sigma, \mathcal{A}_\sigma)$ where $\mathcal{V} = \{v_1, v_2 \dots \dots \dots v_N\}$ denotes the set of agents,

$\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$is the set of edges and $\mathcal{A}_\sigma = [a_{ij}^\sigma]_{N\times N}$ represent the adjacency matrix. The elements in the $\mathcal{A}_\sigma$ are defined by $a_{ij}^\sigma > 0\ if\ (v_i, v_j) \in \varepsilon_\sigma\ and\ a_{ij}^\sigma > 0$ otherwise. A switching signal σ is defined by $\sigma: [0, \infty) \rightarrow \{1,2,,,,,,S\}$ where S is the number of possible topologies.The neighbour set v_i can be represented by $\mathcal{N}_i = \{j: (v_i, v_j) \in \varepsilon\}$.The degree matrix of the directed graph which can be calculated by $\mathcal{L}_\sigma = \mathcal{D} - \mathcal{A}_\sigma$ and guarantees $\sum_{j=1}^{\mathcal{N}} l_{ij}^\sigma = 0$.

Here we consider stochastic multi-agent system with $\mathcal{N}$ intellignt following agents and one leader denoted by directed graph $\overline{\mathcal{G}_\sigma}$ .The topological connection between each following agent and leader is represented through a directed graph $\mathcal{G}(\mathcal{V}, \in_\sigma, \mathcal{A}_\sigma) of\ order\ \mathcal{N}$,which is a subgraph of directed graph $\overline{\mathcal{G}_\sigma}$ .The element of the weighted adjacency matrix $\mathcal{A}_\sigma = [a_{ij}^\sigma]_{N\times N}$ satisfy that $a_{ii} = 0, a_{ij} > 0\ and\ a_{ij} = 1$ if and only if I can receive the information node j,i.e.,$(i,j) \in \varepsilon$.The leader adjacency matrix is defined as a diagonal matrix $\overline{\mathcal{B}} = diag\{b_1, b_2, \dots . b_{\mathcal{N}}\}$ where $b_i = 1$ if the followeragent i has access to the leader's state $x_0\ and\ b_i = 0$.The Leader's state information x_0 cannot be available to all the follower agents but not only a part of the agents.Let us denote$\hat{\mathcal{L}}_\sigma = \mathcal{L}_\sigma + \overline{\mathcal{B}}$

Now consider the a stochastic multi-agent system, and its dynamics among each agent are represented as

$$dx_i(t) = [\mathcal{A}(r(t)x_i(t) + \mathcal{B}(r(t)u_i(t) + f(x_i(t), t)] + \rho(t, x_i(t), r(t)dw(t), i = 1,2 \dots \dots . \mathcal{N} \quad (1)$$

Where $x_i(t) \in \mathbb{R}^n$ is a state of the ith agent;$u_i(t)$ is the control input and $f(x_i(t), t$ is a non-linear function and it assumed later

$\rho(t, x_i(t))$,$x_i(t - h(t), r(t))$ is a noise term on ith agent;$w(t)\ is\ an\ n -$dimensional Brownian motion defined on a probability space with $\mathbb{E}\{[dw(t)]^2\} = dt$

$r(t)(t > 0)$, is a continuous-time Markovian Process taking the values from the finite space S with transition probability matrix $\Pi = \Pi_{pq}\ is\ given\ by$

$$\Pr(r(t + \Delta(t)) = q \mid r(t) = p) = \begin{cases} \Pi_{pq}\Delta(t) + 0(\Delta(t)), if\ \ q \neq p \\ 1 + \Pi_{pq}\Delta(t) + 0(\Delta(t)), if\ \ q = p \end{cases}$$

where$\Delta(t) > 0, \lim_{\Delta(t)\to 0} \frac{o(\Delta(t))}{\Delta(t)} = 0\ and\ \Pi_{pq} \geq 0$ is the transition rate from mode p at time t to mode q at time $t + \Delta(t)$ if $p \neq q$ and $\Pi_{pp} = -\sum_{q=1,q\neq p}^{\mathcal{N}} \Pi_{pq}\ \forall p \in S$.

$\mathcal{A}(r(t), \mathcal{B}(r(t)$, are known constants matrices.

The state feedback controller is taken as

$$u_i(t) = \sum_{j=1}^{\mathcal{N}} a_{ij}^\sigma \mathcal{K}[x_j(t_k) - x_i(t_k)], \quad i = 1,,,,,,,\mathcal{N}, \quad (2)$$

where$\boldsymbol{\mathcal{K}}$ is the feedback controller can be determined later and $a_{ij}\ is\ (i,j)th$ entry of the adjacency matrix of the communication topology $\mathcal{G}$

Here we consider a non-fragile control as

$$u_i(t) = \sum_{j=1}^{\mathcal{N}} a_{ij}^\sigma (\mathcal{K} + \Delta\mathcal{K})[x_j(t_k) - x_i(t_k)], \quad (3)$$

where$\Delta\mathcal{K}(t)$ is potential multiplicative controllers gain peturbations and its satisfies $\Delta\mathcal{K} = \mathcal{U}_1 \daleth(t) \mathcal{U}_2 \mathcal{K}$,

$\daleth^T(t)\daleth(t) \leq I$,where $\mathcal{U}_1 and\ \mathcal{U}_2$ are two given matrices and $\daleth(t)$ is unknown matrix

In addition ,it is very critic to prove that the carried states to controllers are always continuous,So the sampled-data control method is introduced in this stabilize the system.Sampling instants of the systems state variables are measured at the time instants $t_0, t_1, t_2 \dots \dots . t_k . t_{k+1} \dots \dots$ define the sampling interval

$\tau_k = t_{K+1} - t_k$between the consecutive instants t_k and t_{k+1} $such\ that$

$0 = t_0 < t_1 < \cdots < t_k < \cdots < \lim_{k\to\infty} t_k = +\infty$, Also let $\mu = max\{\tau_k\}$.Define $x_i(t_k) = x_i(t - \tau(t))$

where$\tau(t)$ is time varying delay and it satisfies $0 \le \tau(t) \le \mu, \dot{\tau}(t) = 1$

Then the non-fragile sampled data state feedback controller can be written as

$$u_i(t) = (\mathcal{K} + \Delta\mathcal{K}) \sum_{j=1}^{\mathcal{N}} a_{ij}^{\sigma}(x_j(t - \tau(t)) - x_i(t - \tau(t)) \,. \tag{4}$$

Now (1) becomes

$$dx_i(t) = [\mathcal{A}(r(t)x_i(t) + \mathcal{B}(r(t))(\mathcal{K} + \Delta\mathcal{K}) \sum_{j=1}^{\mathcal{N}} a_{ij}^{\sigma}(x_j(t - \tau(t)) - x_i(t - \tau(t)] + f(x_i(t), t)]dt + \rho(t, x_i(t), \tau(t)dw(t), i = 1,2,,,\mathcal{N} \;. \tag{5}$$

Leader of the network is labled as $i = o$.Then the dynamics of agent with leader is as follows

$$dx_0(t) = [\mathcal{A}r(t)x_0(t) + f(x_0(t), t]dt + \rho(t, x_0(t), x_0(t - h(t), r(t))dw(t)].$$

Error system is obtainedby synchronizing the (5) and (6), it can be given as $y_i(t) = x_i(t) - x_0(t)$.

Therefore from (5) and (6) we get

$$dy_i(t) = [\mathcal{A}_p(r(t)x_i(t) + \mathcal{B}_p(\mathcal{K} + \Delta\mathcal{K}) \sum_{j=1}^{\mathcal{N}} a_{ij}^{\sigma}(y_j(t - \tau(t)) - x_i(t - \tau(t)] + F(x_i(t), t)]dt + \hat{\rho}(t, y_i(t), \tau(t)dw(t), \tag{7}$$

where$F(y_i(t), t) = f(x_i(t), t) - f(x_0(t), t)$

$\hat{\rho}(t, y_i(t), \tau(t)dw(t) = \rho(t, x_i(t), p) - \rho(t, x_0(t), p)$,

For our ease ,we denote the terms $\mathcal{A}(r(t)), \mathcal{B}(r(t))$are as$\mathcal{A}_p$,$\mathcal{B}_p$ respectively

Under the properties of Kroneckerproduct,the MASs can be written as

$$dy(t) = \left[(I_N \otimes \mathcal{A}_p)y(t) + (I_N \otimes \mathcal{B}_p)(\mathcal{L}^{\sigma} \otimes \mathcal{B}_p)(1 + \alpha_1 \daleth(t)\alpha_2)\mathcal{K}y(t - \tau(t)) + F(y(t), t)\right]dt + \hat{\rho}(t, y_i(t), \tau(t)dw(t) \tag{8}$$

Where $y(t) = [y_1^T(t), y_2^T, \ldots y_n^T]^T$.

The following assumption,definition,and lemmas are very needful for deriving the results.

Assumtion 2.1.There exists a known real constant matrix $\mathcal{T}$such that the unknown nonlinear vector function $F(y_i(t), t)$ in system satisfies the following boundedness condition:

$\|F(y_i(t), t)\| \le \|\mathcal{T} y_i(t)\|$for any $y_i(t) \in \mathbb{R}^n$

A noise intensity function satisfies the following assumption

Assumption 2.2.The noise intensity function $\rho(t, x_i(t), p)$is uniformlyLipshitz continuous in terms of the following inequalityof trace inner product

$trace\{\rho^T(t, x_i(t), p)\rho(t, x_i(t), p), p)\} \le S_{ip} x_i^T(t) x_i(t)$,

where$S_{ip}(i = 1,2 \ldots\ldots N, p \in \mathbb{S})$are known constant matrices with appropriate dimension.

Definitin 2.3 [23] The stochastic multi-agent systems (1) and (2) with the protocol (3) is said to exponentially reach mean-square leader following consensus if the error system(5) is exponentially

Mean-square stable,that is ,there exists constanntsy$\gamma > 0, \gamma_1 > 0$ such that

$\mathbb{E}\{\|\delta(t)\|^2\} \le \gamma e^{-\gamma_1 t} sup\gamma_2 \le s \le 0 \le \mathbb{E}\{\|\varphi(t\|^2\}$where$\varphi(t)$ is the initial value of (5)

Lemma 2.4(Jensen's inequality) [24] for many constant matrix $M \in \mathbb{R}^{n\times n}, M^T = M > 0$, scalars

α, β *with* $\alpha > \beta$, and vector $x: [\beta, \alpha] \to R^n$ such that the following integration are well defined then,

$$-(\alpha-\beta)\int_\beta^\alpha x^T(s)Mx(s)ds \le -\left(\int_\beta^\alpha x(s)ds\right)^T M\left(\int_\beta^\alpha x(s)ds\right).$$

Theorem 3.1. consider the stochastic non-linear multi-agent systems in (1) satisfying Assumtion 2.1 and 2.2.Suppose that there exist a symmetric matrices $\mathcal{P}_p, \mathcal{Q}, \mathcal{R}$ scalar η_p and for any known positive scalars α, μ, ϵ such that the following linear matrix inequalities

$$\mathcal{P}_p < \eta_p I, p \in \mathbb{S}_+ \quad (9)$$

$$\Theta = [\Theta]_{9\times 9} < 0, \quad (10)$$

Where

$$\Theta_{1,1} = (I_N \otimes \mathcal{P}_p)(I_N \otimes \mathcal{A}_p) + (I_N \otimes \mathcal{A}_p)^T (I_N \otimes \mathcal{P}_p)^T + \sum_{q=1}^N \Pi_{pq}(I_N \otimes \mathcal{P}_p) + (I_N \otimes \mathcal{Q}) + \eta_p (I_N \otimes S_{1p}) + (I_N \otimes \mathcal{A}_p)^T \mu^2 (I_N \otimes \mathcal{R})(I_N \otimes \mathcal{A}_p),$$

$$\Theta_{1,4} = (I_N \otimes \mathcal{P}) + (I_N \otimes \mathcal{A})^T (I_N \otimes \mathcal{R}), \Theta_{1,7} = (I_N \otimes \mathcal{T}), \Theta_{2,2} = \eta_p (I_N \otimes S_2)$$

$$\Theta_{2,8} = \mu^2 (I_N \otimes \mathcal{Y})(\mathcal{L}^\sigma \otimes \mathcal{B}_p)\mathcal{U}_1, \Theta_{3,3} = -e^{-\alpha\mu}, \Theta_{4,4} = -I + \mu^2 (I_N \otimes \mathcal{R}), \Theta_{4,9} = \mathcal{U}_2$$

$$\Theta_{5,5} = -\mu e^{-\alpha\mu}(I_N \otimes \mathcal{R}), \Theta_{6,6} = -\mu e^{-\alpha\mu}(I_N \otimes \mathcal{R})$$

$$\Theta_{7,7} = -I, \Theta_{8,8} = -\epsilon, \Theta_{9,9} = -\epsilon$$

are satisfied .Moreover the controller gain matrix is gain matrix is given by $\mathcal{R}^{-1}\mathcal{Y}$

Proof: Consider the LyapunovKarsovskii functional as

$$V_1(t, y(t), p) = y^T(t)(I_N \otimes \mathcal{P}_p)y(t), \quad (11)$$

$$V_2(t, y(t), p) = \int_{t-\mu}^t e^{\alpha(s-t)} y^T(s)(I_N \otimes \mathcal{Q})y(s)ds, \quad (12)$$

$$V_2(t, y(t), p) = \mu \int_{t-\mu}^t \int_\beta^t \dot{y}^T(s)(I_N \otimes \mathcal{R})\dot{y}(s)ds d\beta, \quad (13)$$

Based on Ito's formula

$$dV(t, y(t), p) + \alpha V(t, y(t), p)dt = [\alpha V(t, y(t), p)] + \mathfrak{L}V(t, y(t), p) + V_y(t, y(t), p)\hat{\rho}(t, y(t), dw(t)) \quad (14)$$

where

$\mathfrak{L}V(t, y(t), p = \sum_{r=1}^4 \mathfrak{L}V_r(t, y(t), p))$ and $V_y(t, y(t), p) = \frac{\partial V(t,y(t),p)}{\partial y}$.

By manipulating the derivative of $V(t, y(t), p)\alpha$ through the trajectories of the system, we get

$$\mathfrak{L}V_1(t, y(t), p) = 2y^T(t)(I_N \otimes \mathcal{P}_p)[(I_N \otimes \mathcal{A}_p)y(t) + (I_N \otimes \mathcal{B}_p)(\mathcal{L}^\sigma \otimes \mathcal{B}_p)(1 + \alpha_1 \daleth(t)\alpha_2)\mathcal{K}y(t - \tau(t)) + F(y(t), t)] + \sum_{q=1}^N \Pi_{pq} y^T(t)(I_N \otimes \mathcal{P}_p)y(t) + trace\{\hat{y}^T(t, y(t), p)(I_N \otimes \mathcal{P}_p)\hat{\rho}(t, y(t), p)\} - \alpha V\hat{\rho}(t, y(t), p) + \alpha V_1(t, y(t), p), p) \quad . \quad (15)$$

$$\mathfrak{L}V_2(t, y(t), p) = y^T(t)(I_N \otimes \mathcal{Q})y(t) - e^{-\alpha\mu} y^T(t-\mu) - \alpha V_2(t, y(t), p), \quad (16)$$

$$\mathfrak{L}V_3(t, y(t), p) = \mu^2 \dot{y}^T(t)(I_N \otimes \mathcal{R})\dot{y}(t) - \mu e^{-\alpha\mu}\int_{t-\mu}^t \dot{y}^T(s)(I_N \otimes \mathcal{R})\dot{y}(s)ds - \alpha V_2(t, y(t), p), \quad (17)$$

The integral terms in (15) can be written as

$-\mu e^{-\alpha\mu}\int_{t-\mu}^{t}\dot{y}^{T}(s)\,(I_N\otimes\mathcal{R})\dot{y}(s)ds\leq$

$-\mu e^{-\alpha\mu}\int_{t-\mu}^{t-\tau(t)}\int_{t-\mu}^{t}\dot{y}^{T}(s)\,(I_N\otimes\mathcal{R})\dot{y}(s)ds-\mu\int_{t-\tau(t)}^{t}\int_{t-\mu}^{t}\dot{y}^{T}(s)\,(I_N\otimes\mathcal{R})\dot{y}(s)ds$ (18)

By using Jensen's inequality we get

$-\mu e^{-\alpha\mu}\int_{t-\mu}^{t-\tau(t)}\dot{y}^{T}(s)\,(I_N\otimes\mathcal{R})\dot{y}(s)ds\leq-\mu e^{-\alpha\mu}\left(\int_{t-\mu}^{t-\tau(t)}\dot{y}^{T}(s)\,\dot{y}(s)ds\right)^{T}(I_N\otimes\mathcal{R})\int_{t-\mu}^{t-\tau(t)}\dot{y}^{T}(s)\,\dot{y}(s)ds,$ (19)

$,-\mu e^{-\alpha\mu}\int_{t-\mu}^{t-}\dot{y}^{T}(s)\,(I_N\otimes\mathcal{R})\dot{y}(s)ds\leq-\mu e^{-\alpha\mu}\left(\int_{t-\mu}^{t}\dot{y}^{T}(s)\,\dot{y}(s)ds\right)^{T}(I_N\otimes\mathcal{R})\int_{t-\mu}^{t}\dot{y}^{T}(s)\,\dot{y}(s)ds$ (20)

From the assumption 2.1 we get

$y^{T}(t)(I_N\otimes\mathcal{T})(I_N\otimes\mathcal{T})^{T}y(t)-F^{T}(y(t),t)F(y(t),t)>0,$ (21)

From the assumption 2.2 we get

$trace\{\bar{\rho}^{T}\}(I_N\otimes\mathcal{P}_p)\bar{\rho}\leq\eta_p trace\{\bar{\rho}^{T}\bar{\rho}),$

$\leq\eta_p y^{T}(t)(I_N\otimes\mathcal{S}_{ip})y(t)+\eta_p y^{T}(t-\tau(t))(I_N\otimes\mathcal{S}_{2p})y(t-\tau(t))$ (22)

where$\bar{\rho}=\hat{\rho}(t,y_i(t),p),\eta_p$is unknown positive scalar $\mathcal{S}_{1p},\mathcal{S}_{2p}$ is a known constant

By incorporating (13)-(20) and taking the mathematical expectation,we get

$\frac{\mathbb{E}\{dV(t,y(t),p)+\alpha V(t,y(t),p)\}}{dt}\leq\mathbb{E}[\alpha V(t,y(t),p)]+\mathfrak{L}V(t,y(t),p)\},$

$\leq\{\zeta_1^{t}(t)\Theta\zeta_1(t)\},$ (23)

where$\zeta_1^{t}(t)=[y^{T}(t-\tau(t))y^{T}(t-\mu)F(y(t),t)]\int_{t-\mu}^{t-\tau(t)}\dot{y}(s)ds\int_{t-\tau(t)}^{t}\dot{y}(s)ds$ obviously from the

$\mathcal{P}_p<\eta_p I,p\in\mathbb{S}_+,\Theta=[\Theta]_{9\times 9}<0$ is hold.From(21) we have,

$\mathbb{E}\{\mathfrak{L}V(t,y(t),p)\}\leq-\alpha\mathbb{E}\{V(t,y(t),p)\},$ (24)

Combaining (12) with (22) and integrating we have

$\mathbb{E}\{V(t,y(t),p)\}\leq e^{-\alpha(t-s)}\mathbb{E}\{V(s,y(s),p)\},$ (25)

$V(t,y(t),p)=\int_{i=1}^{3}V(t,y(t),p)$, that there exist a positive scalars β_1,β_2such that

$\beta_1\|y(t)\|^2\leq V(t,y(t),p),V(s,y(s),p)\geq\beta_1\|y(s)\|^2,$ (26)

Where $\beta_1=\min(\lambda_{min}(\mathcal{P}_p),p\in\mathbb{S})$ and$\beta_2=\max(\lambda_{max}(\mathcal{P}_p),p\in\mathbb{S})+\mu\lambda_{max}(\mathcal{Q})+\mu\lambda_{max}(\mathcal{R})$

Then from (23) and (24) we get

$\mathbb{E}\{\|y(t)\|^2\}\leq\gamma e^{-\alpha(t-s)}sup\mathbb{E}\{\|\varphi(s)\|^2\},$ (27)

where ,$\gamma=\frac{\beta_2}{\beta_1}$.Therefore,from Definition 2.1,the closed-loop error system (7) is exponentially stable in mean-square,which shows that the leader-following exponential consensus problem of the stochastic multi-agent system is derived.This completes the proof.

Remark 3.2.consider the stochastic multi-agent system (8) without Markovian jumping parameters,then

$dy(t)=[(I_N\otimes\mathcal{A}_p)y(t)+(I_N\otimes\mathcal{B}_p)(\mathcal{L}^{\sigma}\otimes\mathcal{B}_p)(1+\alpha_1\daleth(t)\alpha_2)\mathcal{K}y(t-\tau(t))+F(y(t),t)]dt+$
$\hat{\rho}(t,y_i(t),\tau(t)dw(t)$ (28)

The exponential consensus of the stochastic multi-agent system without Markovian jumping parameters via Non-fragile sampled-data control is derived in the following theorem

Theorem 3.3:Consider the stochastic non-linear multi-agent systems in (28) satisfying Assumtion 2.1 and 2.2.Suppose that there exist a symmetric matrices $\mathcal{P}_p, \mathcal{Q}, \mathcal{R}$ scalar η_p and for any known positive scalars α, μ, ϵ such that the following linear matrix inequalities,

$$\Theta = [\Theta]_{9\times 9} < 0$$

where,

$$\Theta_{1,1} = (I_N \otimes \mathcal{P})(I_N \otimes \mathcal{A}) + (I_N \otimes \mathcal{P})^T(I_N \otimes \mathcal{A})^T + (I_N \otimes \mathcal{Q}) + \eta_p(I_N \otimes S_1)$$

$+(I_N \otimes \mathcal{A})^T \mu^2 (I_N \otimes \mathcal{R})(I_N \otimes \mathcal{A})$,

$\Theta_{1,4} = (I_N \otimes \mathcal{P}) + (I_N \otimes \mathcal{A})^T(I_N \otimes \mathcal{R}), \Theta_{1,7} = (I_N \otimes \mathcal{T}), \Theta_{2,2} = \eta_p (I_N \otimes S_2)$

$\Theta_{2,8} = \mu^2(I_N \otimes \mathcal{Y})(\mathcal{L}^\sigma \otimes \mathcal{B}_p)\mathcal{U}_1, \Theta_{3,3} = -e^{-\alpha\mu}, \Theta_{4,4} = -I + \mu^2(I_N \otimes \mathcal{R}), \Theta_{4,9} = \mathcal{U}_2$

$\Theta_{5,5} = -\mu e^{-\alpha\mu}(I_N \otimes \mathcal{R}), \Theta_{6,6} = -\mu e^{-\alpha\mu}(I_N \otimes \mathcal{R})$

$\Theta_{7,7} = -I, \Theta_{8,8} = -\in, \Theta_{9,9} = -\epsilon$

are satisfied .Moreover the controller gain matrix is gain matrix is given by $\mathcal{K} = \mathcal{R}^{-1}\mathcal{Y}$

Proof: Consider same LKF and same procedure in Theorem 3.1

Remark:3.4 consider a linear Stochastic multi-agent system,

$$dx_i(t) = [\mathcal{A}(r(t)x_i(t) + \mathcal{B}(r(t)u_i(t) +)] + \rho(t, x_i(t), r(t)dw(t), i = 1,2 \dots \dots \mathcal{N}, \quad (30)$$

Leader of the network is labelled as $i = 0$.Then the dynamics of agents with leader is as follows

$$dx_0(t) = [\mathcal{A}(r(t)x_0(t)] + \rho(t, x_0(t), x_0(t - h(t))r(t)dw(t), \quad (31)$$

Error system can be given as $y_i(t) = x_i(t) - x_0(t)$

$$dy_i(t) = [\mathcal{A}_p y_i(t) + \mathcal{B}_p(\mathcal{K} + \Delta\mathcal{K}) \textstyle\sum_{j=1}^{\mathcal{N}} a_{ij}^\sigma (y_j(t - \tau(t)) - y_i(t - \tau(t)]dt + \hat{\rho}(t, y_i(t), p)dw(t), \quad (32)$$

Then,by the properties of Kronecker product

$$dy(t) = [(I_N \otimes \mathcal{A}_p)y(t) + (I_N \otimes \mathcal{B}_p)(\mathcal{L}^\sigma \otimes \mathcal{B}_p)(1 + \alpha_1 \daleth(t)\alpha_2)\mathcal{K}y(t - \tau(t)) + F(y(t), t)]dt + \hat{\rho}(t, y_i(t), \tau(t)dw(t). \quad (33)$$

The exponential consensus of linear stochastic MASs can be derived from the following system

Theorem 3.5 Consider the stochastic non-linear multi-agent systems in (33) satisfying Assumtion 2.2.Suppose that there exist a symmetric matrices $\mathcal{P}_p, \mathcal{Q}, \mathcal{R}$ scalar η_p and for any known positive scalars α, μ, ϵ such that the following linear matrix inequalities,

$$\mathcal{P}_p < \eta_p I, p \in \mathbb{S}_+, \quad (34)$$

$$\Theta = [\Theta]_{8\times 8} < 0, \quad (35)$$

where

$\Theta_{1,1} = (I_N \otimes \mathcal{P}_p)(I_N \otimes \mathcal{A}_p) + (I_N \otimes \mathcal{A}_p)^T(I_N \otimes \mathcal{P}_p)^T + \sum_{q=1}^N \Pi_{pq}(I_N \otimes \mathcal{P}_p) + (I_N \otimes \mathcal{Q}) + \eta_p (I_N \otimes S_{1p}) + (I_N \otimes \mathcal{A}_p)^T \mu^2 (I_N \otimes \mathcal{R})(I_N \otimes \mathcal{A}_p)$,

$\Theta_{1,7} = (I_N \otimes \mathcal{T}), \Theta_{2,2} = \eta_{p(I_N \otimes S_{2p})}, \Theta_{2,8} = \mu^2(I_N \otimes \mathcal{Y})(\mathcal{L}^\sigma \otimes \mathcal{B}_p)\mathcal{U}_1, \Theta_{3,3} = -e^{-\alpha\mu}$,

$\Theta_{4,4} = -\mu e^{-\alpha\mu}(I_N \otimes \mathcal{R}), \Theta_{5,5} = -\mu e^{-\alpha\mu}(I_N \otimes \mathcal{R}), \Theta_{6,6} = -I, \Theta_{7,7} = -\in\ \Theta_{8,8} = -\epsilon$

Are satisfied .Moreover the controller gain matrix is gain matrix is given by$\mathcal{R}^{-1}\mathcal{Y}$

Proof:Consider the LyapunovKarasii functional as,

$V_1(t, y(t), p) = y^T(t)\big(I_N \otimes \mathcal{P}_p\big)y(t),$

$V_2(t, y(t), p) = \int_{t-\mu}^{t} e^{\alpha(s-t)} y^T(s)\,(I_N \otimes \mathcal{Q})y(s)ds,$

$V_2(t, y(t), p) = \mu \int_{t-\mu}^{t} \int_{\beta}^{t} \dot{y}^T(s)\,(I_N \otimes \mathcal{R})\dot{y}(s)dsd\beta,$

The same procedure follows from theorem 3.1

4. Numerical examples

In this section, we provide a numerical example to show the validity and feasibility of the proposed control scheme. We consider a leader-follower stochastic input delayed multi-agent system with one leader, four follower agents, and two jump nodes.

Example 4.1 The coefficient matrices of stochastic input delayed multi-agent sysyem (5) are given as follows:**Mode 1:**

$A_1 = \begin{bmatrix} -1.8 & 0.4 \\ 0.1 & -1 \end{bmatrix}, B_1 = \begin{bmatrix} 0.4 & 0.3 \\ 0 & -1 \end{bmatrix}, S_{11} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, S_{21} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$

Mode 2:

$A_2 = \begin{bmatrix} -0.9 & 1 \\ 1 & -1.4 \end{bmatrix}, B_2 = \begin{bmatrix} 2.3 & 0.4 \\ 0.1 & -1 \end{bmatrix}, S_{12} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$

And the transition probability matrix is $\Pi = \begin{bmatrix} -0.5 & 0.5 \\ 0.2 & -0.2 \end{bmatrix}$

Our main task is to validate the suggested exponential consensus protocol for stochastic multi-agent system (1) with the precedent parameter values. For this reason, the constraints denoted in Theorem 3.1 are solved by using known numerical programming, then the feedback gain matrix is obtained as

$\mathcal{K} = \begin{bmatrix} -0.0127 & -0.0270 \\ -0.0047 & 0.0128 \end{bmatrix}.$

5. Conclusion

This study discusses Markovian jumping parameters and the leader following the exponential consensus of a stochastic multi-agent system with non-fragile sampled-data control. The Laplacian matrix can be used to determine the relationship between the leader and other followers, represented by the directed graph in a communication graph. Sufficient requirements have been deduced for stochastic multi-agent systems to achieve an exponential decay rate in consensus based on the algebraic graph theory, or Lyapunov theory. The consensus criterion and controller gain matrix for a non-linear multi-agent system are derived using well-known numerical programming. Terminally, a numerical example has been provided to demonstrate the validity and correctness of the inferred conditions.

Refrences

[1] R. Olfati-Saber, J. A. Fax, R. M. Murray, Consensus and cooperation in networked multi-agent systems, Proc.IEEE, 95, (2007) 215-233.

[2] W. Ren, R. W. Beard, Consensus seeking in multiagent systems under dynamically changing interaction topologies,IEEE Trans. Autom. Control, 50 (2005) 655-661.
[3] A. Okubo, Dynamical aspects of animal grouping: swarms, schools, flocks, and herds, Adv. Biophys., 22, (1986)1-94.
[4] W. Ren, R. Beard, E. Atkins, Information consensus in multivehicle cooperative control, IEEE Control Syst. Mag.,27, (2007) 71-82.
[5] A. Jadbabaie, J. Lin, A. Morse, Coordination of groups of mobile autonomous agents using nearest neighborrules,IEEE Trans. Autom. Control, 48, (2003) 988-1001.
[6] L. Zheng, Y. Yao, M. Deng, T. Yau, Decentralized detection in ad hoc sensor networks with low data rate intersensor communication, IEEE Trans. Inf. Theory, 58, (2012) 3215-3224.
[7] W. Ren, R. Beard, Consensus seeking in multi-agent systems under dynamically changing interaction topologies,IEEE Trans. Automat. Control, 50, (2005) 655-661.
[8] P. Ming, J. Liu, S. Tan, G. Wang, L. Shang, C. Jia, Consensus stabilization of stochastic multi-agent system withMarkovian switching topologies and stochastic communication noise, Journal of the Franklin Institute, 352, (2015)3684-3700.
[9] H. Ren, F. Deng, Mean square consensus of leader-following multi-agent systems with measurement noises andtime delays, ISA Transactions, 71 (2017) 76-83.
[10] Tian Yu-Ping, Liu Cheng-Lin, Consensus of multi-agent systems with diverse input and communication delays,IEEE Trans Automat Control, 53, (2008) 2122-8.
[11] M. Ziyang, W. Ren, Y. Cao, Z. You, Leaderless and leader-following consensus with communication and inputdelays under a directed network topology, IEEE Trans Syst Man Cybern B, 41 (2010) 75-88.
[12] W. Yu, G. Chen, M. Cao J. Kurths, Second-order consensus for multiagent systems with directed topologies and nonlinear dynamics, IEEE Trans. Syst. Man Cybern B Cybern, 40, (2010) 881-891.
[13] Z. K. Li, G. H. Wen, Z. S. Duan, W. Ren, Designing fully distributed consensus protocols for linear multi-agentsystems with directed graphs, IEEE Trans. Autom. Control, 60 (2015) 1152-1157.
[14] B. Cui, C. Zhao, T. Ma, C. Feng, Leader-following consensus of nonlinear multi-agent systems with switchingtopologies and unreliable communications, Neural Computing Appli., 27 (2016) 909-915.
[15] L. Ma, Z. Wang,H. K. Lam, Event-triggered mean-square consensus control for time-varying stochastic multi-agent system with sensor saturations, IEEE Trans. Aut. Control, 62 (2017) 3524-3531.
[16] D. Ding, Z. Wang, B. Shen, G. Wei, Event-triggered consensus control for discrete-time stochastic multi-agentsystems: The input-to-state stability in probability, Automatica, 62 (2015) 284-291.
[17] A. Hu, J. Cao, M. Hu, L. Guo, Event-triggered consensus of Markovian jumping multi-agent systems via stochastic sampling, IET Control theory Appli., 9 (2015) 1964-1972.
[18] T. Wu, J. Hu, D. Chen, Non-fragile consensus control for nonlinear multi-agent systems with uniform quantizationsand deception attacks via output feedback approach, Nonlinear Dyn (2018) doi.org/10.1007/s11071-019-04787-z.
[19] G. Guo, L. Ding, Q. L. Han, A distributed event-triggered transmission strategy for sampled-data consensus ofmulti-agent systems, Automatica, 50 (2014) 1489-1496.
[20] W. Yu, L. Zhou, X. Yu, J. Lu, R. Lu, Consensus in multi-agent systems with second-order dynamics and sampled data, IEEE Trans. Ind. Informat., 9 (2013) 2137-2146.
[21] J. Fu, G. Wen , W. Yu, T. Huang, J. Cao, Exponential consensus of multiagent systems WwthLipschitz nonlinearities sing sampled-data information, IEE trans, circuits, syst., (2018).
[22] J. Dai, G. Guo, Exponential consensus of non-linear multiagent systems with semi-Markov switching topologies,IET control theory appl. (2017) doi: 10.1049/iet-cta.2017.0562 www.ietdl.org.
[23] R. Sakthivel, S. Kanakalakshmi, B. Kaviarasan, Yong-Ki Ma, A. Leelamani, Finite-time consensus of input delayed multi-agent systems via non-fragile controller subject to switching topology, Neurocomputing (2018).
[24] K. Gu, J. Chen, V. L. Kharitonov, Stability of time-delay systems, New York: Springer Science and BusinessMedia (2003).