

Three Parameter Generalized Gaussian Type Distribution

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Abstract: In this article we introduced a three parameter generalized Gaussian type distribution. The various distributional properties like probability density function, moment generating function, the central moments etc., are derived. The estimators of the parameters are also obtained through the method of moments and maximum likelihood method of estimation. This distribution is useful in analyzing the random phenomenon arising at places like, agricultural experiments, biological studies, image processing etc.,. This distribution also includes several of the earlier distributions like unimodal, bimodal, symmetric, leptokurtic and platykurtic distributions for particular values of the parameters.

Key word: Generalized Gaussian distribution, unimodal, bimodal, symmetric, leptokurtic and platykurtic distributions, moment generating function, Estimation of the parameters.

1. Introduction:

Gaussian distribution has received considerable attention as an approximate model in reliability, life testing models, agricultural, biological and other fields of applications. The main drawback of the Gaussian distribution is its being mesokurtic. However, in many practical situations the data sets may not be mesokurtic even though they are symmetric. To overcome this drawback generalized Gaussian distribution was introduced with a shape parameter 'p' along with mean and variance parameters in order to accommodate platykurtic and leptokurtic distributions. The generalized Gaussian distribution was used by Shariff .K et. al (1995) for modeling the atmospheric noise sub band encoding of audio-video sub signals. Choi, S.Cichocki et al (2000) has used this distribution for impulsive noise direction of arrival and independent component analysis. Wu, Hep, Principi .J. (1995) has used this distribution for signal separation. Varanasi .M.K et al (1987) discussed the parameter estimation of this distribution by method of moments and maximum likelihood. Armando .J. et al (2003) developed a procedure to estimate the shape parameter in a generalized Gaussian distribution. Very little work has been reported regarding further generalization of this generalized Gaussian distribution. Hence, in this article, we introduced a three parameter generalized Gaussian type distribution which suits a suitable distribution for several unimodal and bimodal data sets. The various distributional properties of this distribution are studied. The inferential aspects of this distribution are also studied by deriving the estimators of the parameters through method of moments and maximum likelihood method of estimation. This distribution also includes several of the earlier existing distributions as particular cases for specific values of the parameters.

2. Three Parameter Generalized Gaussian Type Distribution:

A continuous random variable X is said to have a three parameter generalized Gaussian type distribution if its probability density function is of the form

$$f(x; \mu, \lambda) = \frac{c}{2\lambda \left(\Gamma\left(\frac{1}{c}\right) + \Gamma\left(\frac{3}{c}\right) \right)} \left[1 + \left(\frac{x - \mu}{\lambda} \right)^2 \right] e^{-\left| \frac{x - \mu}{\lambda} \right|^c} \quad (1)$$

$$-\infty < x < \infty; -\infty < \mu < \infty; \lambda > 0, c > 0$$

Making transformation $y = \frac{x-\mu}{\lambda}$ in (1) we get

$$f(y) = \frac{c}{2\left(\Gamma\left(\frac{1}{c}\right) + \Gamma\left(\frac{3}{c}\right)\right)} [(1+y)^2] e^{-|y|^c} \quad -\infty < x < \infty \quad (2)$$

which may be called generalized Gaussian type distribution.

3. PROPERTIES OF THREE PARAMETER GENERALIZED GAUSSIAN DISTRIBUTION:

i) It is symmetric about μ , which is the mean and median.

ii) The distribution function is

For $x \geq \mu$

$$F(x) = \left[\frac{1}{2} + \frac{1}{2\left(\Gamma\left(\frac{1}{c}\right) + \Gamma\left(\frac{3}{c}\right)\right)} \left(\gamma\left(\frac{1}{c}, \left(\frac{x-\mu}{\lambda}\right)^c\right) + \gamma\left(\frac{3}{c}, \left(\frac{x-\mu}{\lambda}\right)^c\right) \right) \right] \quad (3)$$

where $\left(\gamma\left(\frac{1}{c}, \left(\frac{x-\mu}{\lambda}\right)^c\right) \right)$ and $\left(\gamma\left(\frac{3}{c}, \left(\frac{x-\mu}{\lambda}\right)^c\right) \right)$ are incomplete gamma functions

For $x \leq \mu$

$$F(x) = \left[\frac{1}{2} - \frac{1}{2\left(\Gamma\left(\frac{1}{c}\right) + \Gamma\left(\frac{3}{c}\right)\right)} \left(\gamma\left(\frac{1}{c}, \left(\frac{x-\mu}{\lambda}\right)^c\right) + \gamma\left(\frac{3}{c}, \left(\frac{x-\mu}{\lambda}\right)^c\right) \right) \right] \quad (4)$$

where $\left(\gamma\left(\frac{1}{c}, \left(\frac{x-\mu}{\lambda}\right)^c\right) \right)$ and $\left(\gamma\left(\frac{3}{c}, \left(\frac{x-\mu}{\lambda}\right)^c\right) \right)$ are incomplete gamma functions

iii) The mode of the distribution is obtained by solving the equation for different values of c .

$$\left[\frac{2\left(\frac{x-\mu}{\lambda}\right)\left(\frac{1}{\lambda}\right)}{1 + \left(\frac{x-\mu}{\lambda}\right)^2} - \frac{\frac{c}{\lambda} \left|\frac{x-\mu}{\lambda}\right|^{c-1} \left|\frac{x-\mu}{\lambda}\right|}{\frac{x-\mu}{\lambda}} \right] = 0 \quad (5)$$

For $c = 1$ or $c = 2$, $x = \mu$ is the modal value.

For $c > 2$, $x = \mu$ is a saddle point having two modal values.

iv) The central moment of X are

$$\mu_{2l} = \lambda^{2l} \left(\frac{\Gamma\left(\frac{2l+1}{c}\right) + \Gamma\left(\frac{2l+3}{c}\right)}{\Gamma\left(\frac{1}{c}\right) + \Gamma\left(\frac{3}{c}\right)} \right) \quad (6)$$

v) The recurrence relation among the central moment is

$$\mu_{2l+2} = \lambda^2 \frac{\left\{ \Gamma\left(\frac{2l+3}{c}\right) + \Gamma\left(\frac{2l+5}{c}\right) \right\}}{\left\{ \Gamma\left(\frac{2l+1}{c}\right) + \Gamma\left(\frac{2l+3}{c}\right) \right\}} \mu_{2l} \quad (7)$$

vi) Mean deviation about means is

$$\text{Mean deviation about mean} = \lambda \left(\frac{\Gamma\left(\frac{2}{c}\right) + \Gamma\left(\frac{4}{c}\right)}{\Gamma\left(\frac{1}{c}\right) + \Gamma\left(\frac{3}{c}\right)} \right) \quad (8)$$

vii) The q^n absolute moment is

$$E(|x - \mu|^q) = \lambda^q \left(\frac{\Gamma\left(\frac{q+1}{c}\right) + \Gamma\left(\frac{q+3}{c}\right)}{\Gamma\left(\frac{1}{c}\right) + \Gamma\left(\frac{3}{c}\right)} \right) \quad (9)$$

viii) Kurtosis of the distribution is

$$\beta_2 = \frac{\left\{ \Gamma\left(\frac{5}{c}\right) + \Gamma\left(\frac{7}{c}\right) \right\} \left\{ \Gamma\left(\frac{1}{c}\right) + \Gamma\left(\frac{3}{c}\right) \right\}}{\left(\Gamma\left(\frac{3}{c}\right) + \Gamma\left(\frac{5}{c}\right) \right)^2} \quad (10)$$

As c increases the values of β_2 decreases

ix) The ratio of mean deviation to standard deviation is

$$\frac{M.D}{S.D} = \frac{\Gamma\left(\frac{2}{c}\right) + \Gamma\left(\frac{4}{c}\right)}{\sqrt{\left\{ \Gamma\left(\frac{1}{c}\right) + \Gamma\left(\frac{3}{c}\right) \right\} \left\{ \Gamma\left(\frac{3}{c}\right) + \Gamma\left(\frac{5}{c}\right) \right\}}} \quad (11)$$

x) The hazard rate function of distribution is

$$h(x) = \frac{\frac{c}{\lambda} \left[1 + \left(\frac{x-\mu}{\lambda} \right)^2 \right] e^{-\left| \frac{x-\mu}{\lambda} \right|^c}}{\Gamma\left(\frac{1}{c}\right) + \Gamma\left(\frac{3}{c}\right) - \gamma\left(\frac{1}{c}, \left(\frac{x-\mu}{\lambda} \right)^c\right)} \quad x > 0 \quad (12)$$

xi) Survival rate function $S(x)$ is

$$\begin{aligned} S(x) &= 1 - F(x) \\ &= \frac{1}{2} - \frac{1}{2 \left(\Gamma\left(\frac{1}{c}\right) + \Gamma\left(\frac{3}{c}\right) \right)} \left(\gamma\left(\frac{1}{c}, \left| \frac{x-\mu}{\lambda} \right|^c\right) \right) \\ &\quad + \gamma\left(\frac{3}{c}, \left| \frac{x-\mu}{\lambda} \right|^c\right) \end{aligned} \quad (13)$$

xii) The moment generating function of a random variable X is defined as

$$M_x(t) = \frac{(2\pi)^{\frac{1-c}{2}} c^{\frac{1}{2}} k}{\pi t} G_{1,c}^{c,1} \left\{ \left(\frac{t}{c} \right)^c \middle| \frac{1}{c}, \dots, 1 \right\} + \frac{(2\pi)^{\frac{1-c}{2}} c^{\frac{5}{2}}}{\pi t^3} G_{1,c}^{c,1} \left\{ \left(\frac{t}{c} \right)^c \middle| \frac{3}{c}, \dots, \frac{c+2}{c} \right\} \quad (14)$$

Where G is a Meijers function given in the Julian Cheng et. al (2003).

xiii) The probability density function of the variate $Y=X^2$ is

$$f(x) = \frac{c}{2\lambda \left(\Gamma\left(\frac{1}{c}\right) + \Gamma\left(\frac{3}{c}\right) \right) \sqrt{y}} \left[1 + \left(\frac{\sqrt{y}}{\lambda} \right)^2 \right] e^{-\left(\frac{\sqrt{y}}{\lambda} \right)^c} \quad 0 < y < \infty \quad (15)$$

xiv) The mean value of the variable Y is

$$E(Y) = \lambda^2 \frac{\Gamma\left(\frac{5}{c}\right) + \Gamma\left(\frac{3}{c}\right)}{\Gamma\left(\frac{1}{c}\right) + \Gamma\left(\frac{3}{c}\right)} \quad (16)$$

xv) The l^{th} raw moment of Y is

$$\mu'_t = \lambda^{2t} \left[\frac{\Gamma\left(\frac{2l+1}{c}\right) + \Gamma\left(\frac{2l+3}{c}\right)}{\Gamma\left(\frac{1}{c}\right) + \Gamma\left(\frac{3}{c}\right)} \right] \quad (17)$$

xvi) The Kurtosis of the variate Y is

$$\beta_2 = \left[\frac{\left(\Gamma\left(\frac{9}{c}\right) + \Gamma\left(\frac{11}{c}\right) \right) \left(\Gamma\left(\frac{1}{c}\right) + \Gamma\left(\frac{3}{c}\right) \right)^3}{\left(\left(\Gamma\left(\frac{5}{c}\right) + \Gamma\left(\frac{7}{c}\right) \left(\Gamma\left(\frac{1}{c}\right) + \Gamma\left(\frac{3}{c}\right) - \left(\Gamma\left(\frac{5}{c}\right) + \Gamma\left(\frac{3}{c}\right) \right) \right) \right)^2} \right)^2} \right. \\ - 4 \frac{\left(\Gamma\left(\frac{7}{c}\right) + \Gamma\left(\frac{9}{c}\right) \right) \left(\Gamma\left(\frac{5}{c}\right) + \Gamma\left(\frac{3}{c}\right) \right) \left(\Gamma\left(\frac{1}{c}\right) + \Gamma\left(\frac{3}{c}\right) \right)^2}{\left(\Gamma\left(\frac{5}{c}\right) + \Gamma\left(\frac{7}{c}\right) \left(\Gamma\left(\frac{1}{c}\right) + \Gamma\left(\frac{3}{c}\right) - \left(\Gamma\left(\frac{5}{c}\right) + \Gamma\left(\frac{3}{c}\right) \right) \right) \right)^2} \\ + 6 \frac{\left(\Gamma\left(\frac{7}{c}\right) + \Gamma\left(\frac{9}{c}\right) \right) \left(\Gamma\left(\frac{7}{c}\right) + \Gamma\left(\frac{9}{c}\right) \right) \left(\Gamma\left(\frac{5}{c}\right) + \Gamma\left(\frac{3}{c}\right) \right)^2}{\left(\Gamma\left(\frac{5}{c}\right) + \Gamma\left(\frac{7}{c}\right) \left(\Gamma\left(\frac{1}{c}\right) + \Gamma\left(\frac{3}{c}\right) - \left(\Gamma\left(\frac{5}{c}\right) + \Gamma\left(\frac{3}{c}\right) \right) \right) \right)^2} \\ \left. - 3 \frac{\left(\Gamma\left(\frac{5}{c}\right) + \Gamma\left(\frac{3}{c}\right) \right)^4}{\left(\Gamma\left(\frac{5}{c}\right) + \Gamma\left(\frac{7}{c}\right) \left(\Gamma\left(\frac{1}{c}\right) + \Gamma\left(\frac{3}{c}\right) - \left(\Gamma\left(\frac{5}{c}\right) + \Gamma\left(\frac{3}{c}\right) \right) \right)^2} \right] \quad (18)$$

4. Estimation Of The Parameters

In this section we consider the estimation of the parameters of the three parameter generalized Gaussian type distribution.

Method of Moments:

For obtaining the moment estimators we equate the sample moments with the population moments. The parameter c can be estimated by equating the sample kurtosis to the theoretical kurtosis.

$$\frac{\left\{ \Gamma\left(\frac{5}{c}\right) + \Gamma\left(\frac{7}{c}\right) \right\} \left\{ \Gamma\left(\frac{1}{c}\right) + \Gamma\left(\frac{3}{c}\right) \right\}}{\left(\Gamma\left(\frac{3}{c}\right) + \Gamma\left(\frac{5}{c}\right) \right)^2} = \frac{n \sum_{i=0}^n (x_i - \bar{x})^4}{\left(\sum_{i=0}^n (x_i - \bar{x})^2 \right)^2} \quad (19)$$

Solving the equation (19) iteratively using numerical methods like, Newton Raphsons, we obtain the moment estimator for the parameter c . The moment estimators of μ and λ are given by,

$$\hat{\mu} = \bar{x} \text{ and } \hat{\lambda} = s \sqrt{\frac{\Gamma(\frac{1}{c}) + \Gamma(\frac{3}{c})}{\Gamma(\frac{3}{c}) + \Gamma(\frac{5}{c})}} \text{ Where } s \text{ is the sample standard deviation.}$$

The sample mean $\bar{X}$ is an unbiased estimator for the parameter μ .

$$\text{The var}(\bar{X}) = \text{var}\left(\frac{1}{n} \sum_{i=1}^n x_i\right) = \frac{1}{n} \lambda^2 \left(\frac{\Gamma(\frac{3}{c}) + \Gamma(\frac{5}{c})}{\Gamma(\frac{1}{c}) + \Gamma(\frac{3}{c})} \right) \quad (20)$$

The moment estimator for the parameter λ^2 is

$$\hat{\lambda}^2 = \left(\frac{\Gamma(\frac{1}{c}) + \Gamma(\frac{3}{c})}{\Gamma(\frac{3}{c}) + \Gamma(\frac{5}{c})} \right) s^2$$

$\hat{\lambda}^2$ is not unbiased estimator of λ^2 . An unbiased estimator of λ^2 is

$$\frac{n}{n-1} \left(\frac{\Gamma(\frac{1}{c}) + \Gamma(\frac{3}{c})}{\Gamma(\frac{3}{c}) + \Gamma(\frac{5}{c})} \right) s^2$$

the variance of $\hat{\lambda}^2$ is

$$\text{var}(\hat{\lambda}^2) = \text{var}\left[\left(\frac{\Gamma(\frac{1}{c}) + \Gamma(\frac{3}{c})}{\Gamma(\frac{3}{c}) + \Gamma(\frac{5}{c})}\right) s^2\right] = \left[\left(\frac{\Gamma(\frac{1}{c}) + \Gamma(\frac{3}{c})}{\Gamma(\frac{3}{c}) + \Gamma(\frac{5}{c})}\right)\right]^2 \text{var}(s^2) \quad (21)$$

$$\text{We have } \text{var}(s^2) = \frac{\mu_4 - \mu_2^2}{n} + \frac{2(\mu_4 - \mu_2^2)}{n^2} + \frac{\mu_4 - 3\mu_2^2}{n^3}$$

where μ_i is the i^{th} central momentsubstituting

$$\mu_4 = \lambda^4 \left[\frac{\Gamma(\frac{5}{c}) + \Gamma(\frac{7}{c})}{\Gamma(\frac{1}{c}) + \Gamma(\frac{3}{c})} \right] \text{ and}$$

$$\mu_2 = \lambda^2 \lambda^4 \left[\frac{\Gamma(\frac{3}{c}) + \Gamma(\frac{5}{c})}{\Gamma(\frac{1}{c}) + \Gamma(\frac{3}{c})} \right] \text{ in equation (21) we get var}(\hat{\lambda}^2)$$

MAXIMUM LIKELIHOOD METHOD OF ESTIMATION

For obtaining the maximum likelihood estimators of the parameters consider the random sample $x_1, x_2, \dots, x_n$ drawn from the population whose p.d.f. is as given in equation (1). Then the likelihood function of the sample is

$$L = \left(\frac{c}{2\lambda} \frac{1}{\Gamma(\frac{1}{c}) + \Gamma(\frac{3}{c})} \right)^n \prod_{i=1}^n \left[\left[1 + \left(\frac{x - \mu}{\lambda} \right)^2 \right] e^{-\sum_{i=1}^n \left| \frac{x - \mu}{\lambda} \right|^c} \right] \quad (22)$$

The log likelihood function of the sample is

$$\begin{aligned} \log L &= n \log c - n \log 2 - n \log \lambda - n \log \left(\Gamma\left(\frac{1}{c}\right) + \Gamma\left(\frac{3}{c}\right) \right) + \sum_{i=1}^n \log \left[1 + \left(\frac{x - \mu}{\lambda} \right)^2 \right] \\ &\quad - \sum_{i=1}^n \left| \frac{x - \mu}{\lambda} \right|^c \end{aligned} \quad (23)$$

For obtaining the Maximum likelihood estimate of μ , we differentiate $\log L$ with respect to μ and equate it to zero. But in equation (23) the function $\log L$ is differentiable with respect to only when c is even. But in the case when c is odd we obtain the Maximum likelihood estimator as in case of Laplace distribution (Keynes (1911)) i.e. when c is odd, we find μ which maximizes $\log L$. From equation (23) $\log L$ is maximum if $\sum_{i=1}^n \left| \frac{x - \mu}{\lambda} \right|^c$ is minimum when c is odd. The function $\sum_{i=1}^n \left| \frac{x - \mu}{\lambda} \right|^c$ is minimum only when μ is the median. Therefore the Maximum likelihood estimator of μ is the median of the distribution when c is odd. In case of c being even, we differentiate $\log L$ w. r. t μ and equate it to zero.

This implies

$$\frac{d \log L}{d \mu} = \frac{2}{\lambda} \sum_{i=1}^n \frac{\frac{x - \mu}{\lambda}}{\left(\frac{x - \mu}{\lambda} \right)^2} + \frac{c}{\lambda} \sum_{i=1}^n \frac{\left| \frac{x - \mu}{\lambda} \right|^{c-1} \left| \frac{x - \mu}{\lambda} \right|}{\left(\frac{x - \mu}{\lambda} \right)} = 0 \quad (24)$$

To derive maximum likelihood estimator of λ , consider the derivative of L with respect λ and equate to zero

$$\frac{\log L}{\partial \lambda} = -\frac{n}{\lambda} - \frac{2}{\lambda} \sum_{i=1}^n \frac{\left(\frac{x - \mu}{\lambda} \right)^2}{1 + \left(\frac{x - \mu}{\lambda} \right)^2} + \frac{c}{\lambda^2} \sum_{i=1}^n \frac{\left| \frac{x - \mu}{\lambda} \right|^{c-1} \left| \frac{x - \mu}{\lambda} \right|}{\left(\frac{x - \mu}{\lambda} \right)} x_i - \mu$$

Equating $\frac{\log L}{\partial \lambda}$ to zero, we get

$$\frac{n}{\lambda} + \frac{2}{\lambda} \sum_{i=1}^n \frac{\left(\frac{x - \mu}{\lambda} \right)^2}{1 + \left(\frac{x - \mu}{\lambda} \right)^2} - \frac{c}{\lambda^2} \sum_{i=1}^n \frac{\left| \frac{x - \mu}{\lambda} \right|^{c-1} \left| \frac{x - \mu}{\lambda} \right|}{\left(\frac{x - \mu}{\lambda} \right)} = 0 \quad (25)$$

To derive maximum likelihood estimator of c , consider the derivative of L w.r.t c and equate it to zero

$$\begin{aligned} \frac{\log L}{\partial c} &= \frac{n}{c} - n \frac{\frac{\Gamma\left(\frac{1}{c}\right)\varphi(0)\left(\frac{1}{c}\right) - 3\Gamma\left(\frac{3}{c}\right)\varphi(0)\left(\frac{3}{c}\right)}{c^2}}{\Gamma\left(\frac{1}{c}\right) + \Gamma\left(\frac{3}{c}\right)} - \sum_{i=1}^n \left| \frac{x - \mu}{\lambda} \right|^c \left| \frac{x_i - \mu}{\lambda} \right| \\ \frac{n}{c} - n \frac{\frac{\Gamma\left(\frac{1}{c}\right)\varphi(0)\left(\frac{1}{c}\right) - 3\Gamma\left(\frac{3}{c}\right)\varphi(0)\left(\frac{3}{c}\right)}{c^2}}{\Gamma\left(\frac{1}{c}\right) + \Gamma\left(\frac{3}{c}\right)} - \sum_{i=1}^n \left| \frac{x - \mu}{\lambda} \right|^c \left| \frac{x_i - \mu}{\lambda} \right| &= 0 \end{aligned} \quad (26)$$

where $\varphi(0)\left(\frac{1}{c}\right) = \frac{\Gamma'\left(\frac{1}{c}\right)}{\Gamma\left(\frac{1}{c}\right)}$ and $\varphi(0)\left(\frac{3}{c}\right) = \frac{\Gamma'\left(\frac{3}{c}\right)}{\Gamma\left(\frac{3}{c}\right)}$ are the digamma functions.

Solving the equations (24), (25) and (26) simultaneously for μ , λ and c using numerical methods like Newton Raphson's method, we can obtain the maximum likelihood estimators of the parameters μ , λ and c .

5. General Application

Some of the applications of these distributions are generally considered for the variables, (which depict two modes and nearly symmetric about their central through), like division of attitude in a two party election system (Rangekar, 1966), the duration of the post Partum Amenorrhea (Singh and Bhaduri, 1977), circular data having more than one preferred direction (Rao, 1965), and per acre yield of the operational holding of small, medium and larger farmers etc.,. In addition to these there are many general varieties in agricultural, Biological,

chemical and Space experiments, which gives a good, fit to the distributions that arising from this family. We have considered the fitting of a three parameter generalized Gaussian type distribution to the data on length of the fish in a catchment of the Bay of Bengal (Rao, 2004). The data on the length of the fish in a catch is given in table -1. From this data, we observe that the data is having bimodality and closer to symmetry. By assuming that the variate under consideration follows a three parameter type distribution of the form given in equation (1), we have obtained the maximum likelihood estimators of the parameters are

$$\hat{\mathcal{C}}=2, \hat{\lambda} = 1.02 \text{ and } \hat{\mu} = 9.99$$

Table 1 observed and expected Frequencies

Length of the fish (c.m)	Frequencies	
	Observed	Expected
6	1	-
7	7	9
8	22	22
9	16	15
10	7	7
11	16	16
12	23	21
13	7	10
14	1	-

The estimated frequencies are also given in Table-1. By conducting a chi-square test for goodness of fit of the distribution, (the calculated χ^2 value is 0.7683, at 5% level of significance), this data gives a good fit to the three parameter generalized Gaussian type distribution.

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