

On Strong Commutativity Preserving Pair of Maps

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Abstract:

The concept of strong commutativity preserving map on rings is being generalized as strong commutativity preserving pair of maps on rings and more general results are obtained there by generalizing the results of [6].

Keyword: Semi-Prime ring, 2-torsion free Semi-Prime ring, Strong commutativity preserving map.

1.Introduction

Let R be a ring. A mapping $f : R \rightarrow R$ is called commutativity preserving if $[f(x), f(y)] = 0$ whenever $[x, y] = 0$. It is said to be strong commutativity preserving map if $[f(x), f(y)] = [x, y]$ for all $x, y \in R$. A strong commutativity preserving map is commutativity preserving but the converse does not hold, In general, It is said to be a commuting map if $[f(x), x] = 0 \quad \forall x \in R$. It is said to be semi-commuting if either $f(x)x = xf(x)$ (or) $f(x).x = -xf(x)$ for every $x \in R$. It is said to be centralizing if $[f(x), x] \in Z \quad \forall x \in R$ and semi- centralizing if either $f(x)x - xf(x) \in Z$ (or) $f(x)x + xf(x) \in Z$ for every $x \in R$

M.S. Samman [6] had proved the following results.

- i. Let R be a semi-prime ring and f an epimorphism of R . Then f is centralizing if and only if it is strong commutativity preserving.
- ii. Let R be a 2-torsion free semi-prime ring and f be a centralizing anti-homomorphism of R onto itself. Then f is strong commutativity preserving.

M.Brekar [4] had proved that if R is a 2- torsion free semi-prime ring and $f: R \rightarrow R$ is a centralizing anti-homomorphism, then

(i) $S = \{ x \in R / f(x) = x \} \subseteq Z(R)$

(ii) if R is prime, and f does not map R into $Z(R)$ then $S = Z(R)$

(iii) $[x, y] = -[y, x]$

(iv) $[xy, z] = x[y, z] + [x, z]y$

(v) $[x, yz] = y[x, z] + [x, y]z$

$$(vi) [x, x] = 0$$

$$(vii) [x, y] = 0 \text{ iff } x \text{ and } y \text{ commute each other}$$

G.Gopalakrishnamoorthy and others [5] have generalized the concept of commuting maps and centralizing maps as commuting pair of automorphisms of rings and centralizing pair of automorphisms and obtained more general results.

In this paper, we generalize the concept of commutativity preserving map and strong commutativity preserving map and obtain more general results of M.S.Samman[6]

2.Basic Definition

In this section we see some basic definitions and results that we use in the equal

Definition 2.1

Let R be a ring, for $x, y \in R$, define $[x, y] = xy - yx$,

$[x, y]$ is called the commutator of x and y .

Note:

$$(i) [x, y + z] = [x, y] + [x, z] \quad \forall x, y, z \in R$$

$$(ii) [x + y, z] = [x, z] + [y, z] \quad \forall x, y, z \in R$$

$$(iii) [x, y] = -[y, x]$$

$$(iv) [xy, z] = x[y, z] + [x, z]y$$

$$(v) [x, yz] = y[x, z] + [x, y]z$$

$$(vi) [x, x] = 0$$

$$(vii) [x, y] = 0 \text{ iff } x \text{ and } y \text{ commute each other}$$

Definition 2.2

Let R be a ring and S and T be two non-trivial automorphisms of R .

They are said to be :

$$(a) \text{ Commuting pair of automorphisms of } R \text{ if } S(x)T(x) = T(x)S(x) \quad \forall x \in R$$

$$(b) \text{ Anti-Commuting pair of automorphisms if } S(x)T(x) = -T(x)S(x) \quad \forall x \in R$$

$$(c) \text{ Strong Commuting pair of automorphisms if either}$$

$$S(x)T(x) = T(x)S(x) \quad (\text{or})$$

$$S(x)T(x) = -T(x)S(x) \quad \text{for every } x \in R$$

$$(d) \text{ Centralizing pair of automorphisms if } S(x)T(x) - T(x)S(x) \in Z \quad \forall x \in R$$

$$(e) \text{ Anti - Centralizing pair of automorphisms if}$$

$$S(x)T(x) + T(x)S(x) \in Z \text{ for every } x \in R$$

$$(f) \text{ Semi-Centralizing pair of automorphisms if either}$$

$$S(x)T(x) - T(x)S(x) \in Z \quad (\text{or})$$

$$S(x)T(x) + T(x)S(x) \in Z \text{ for every } x \in R$$

We generalize the above definition as follows.

Definition 2.3

Let R be a ring and S and T be non-trivial automorphisms of R

They are said to be

$$a) \text{ Commutativity preserving pair of maps if } [S(x), S(y)] = 0 \text{ iff } [T(x), T(y)] = 0$$

b) Strong Commutativity preserving pair of maps if either $[S(x), S(y)] \in Z$ (or) $[T(x), T(y)] \in Z$ for every $x, y \in R$

3.Main Results

Theorem 3.1

Let R be a semi-prime ring possessing two non-trivial epimorphisms S and T . They are centralizing pair of epimorphisms iff they are strong commutativity preserving pair of epimorphisms.

Proof:

Assume S and T are centralizing pair of automorphisms.

So, $[S(x), T(x)] = 0 \quad \forall x \in R$ (1)

Then $[S(x+y), T(x+y)] = 0 \quad \forall x, y \in R$

$\Rightarrow [S(x), T(x)] + [S(x), T(y)] + [S(y), T(x)] + [S(y), T(y)] = 0 \quad \forall x, y \in R$

Using (1) we get

$[S(x), T(y)] + [S(y), T(x)] = 0 \quad \forall x, y \in R$

(ie) $[S(x), T(y)] = -[S(y), T(x)] = [T(x), S(y)] \quad \forall x, y$ (2)

(ie) S and T are strong commuting pair of automorphisms

Replacing x by xy in (2) we get

$[S(xy), T(y)] = [T(xy), S(y)] \quad \forall x, y \in R$

(ie) $S(x)[S(y), T(y)] + [S(x), T(y)]S(y) = T(x)[T(y), S(y)] + [T(x), S(y)]T(y)$

Using (1) we get

$S(x)[S(y), T(x)] = T(x)[T(y), S(x)] \quad \forall x, y \in R$

Using (2) we get

$S(x)[T(y), S(x)] = T(x)[T(y), S(x)] \quad \forall x, y \in R$

(ie) $(S(x) - T(x))[T(y), S(x)] = 0 \quad \forall x, y \in R$

Since T is an epimorphism of R , we have

$(S(x) - T(x))[z, S(x)] = 0 \quad \forall x, z \in R$ (3)

Replace z by uz in (3) we get

$(S(x) - T(x))[uz, S(x)] = 0$

(ie) $(S(x) - T(x))\{u[z, S(x)] + [u, S(x)]z\} = 0$

(ie) $(S(x) - T(x))u[z, S(x)] + (S(x) - T(x))[u, S(x)]z = 0$

Using (3) we get

$(S(x) - T(x))u[z, S(x)] = 0 \quad \forall x, z, u \in R$ (4)

Replacing x by $x+u$ in (3) we get

$(S(x+u) - T(x+u))[z, S(x+u)] = 0 \quad \forall x, z, u \in R$

$(S(x) - T(x)) + (S(u) - T(u))[z, S(x+u)] = 0$

$(S(x) - T(x))[z, S(x) + S(u)] + (S(u) - T(u))[z, S(x) + S(u)] = 0$

$(S(x) - T(x))[z, S(x)] + (S(x) - T(x))[z, S(u)] + (S(u) - T(u))[z, S(x)] +$
 $(S(u) - T(u))[z, S(u)] = 0$

Using (3) we get

$$(S(x) - T(x))[z, S(u)] + (S(u) - T(u))[z, S(x)] = 0 \forall x, z, u \in R$$

$$(ie) (S(x) - T(x))[z, S(u)] = -(S(u) - T(u))[z, S(x)] \dots\dots\dots(5)$$

$$\forall x, z, u \in R$$

$$(S(x) - T(x)) [z, S(u)]v(S(x) - T(x))[z, S(u)]$$

$$= -(S(u) - T(u))[z, S(x)]v(S(x) - T(x))[z, S(u)]$$

$$= -(S(u) - T(u))w [z, S(u)] \quad \text{Where } w = [z, S(x)]v(S(x) - T(x))$$

$$= 0 \quad (\text{Using (4)})$$

$$(ie) (S(x) - T(x))[z, S(u)]v(S(x) - T(x))[z, S(u)] = 0 \dots\dots\dots(6)$$

$$\forall x, z, u, v \in R$$

Since R is semi-prime we get

$$(S(x) - T(x))[z, S(u)] = 0 \quad \forall x, z, u \in R \dots\dots\dots(7)$$

Put $z = wz$ we get

$$(S(x) - T(x))[wz, S(u)] = 0$$

$$(S(x) - T(x))\{w[z, S(u)] + [w, S(u)]z\} = 0$$

$$(S(x) - T(x))w[z, S(u)] + (S(x) - T(x))[w, S(u)]z = 0$$

Using (7) we get

$$(S(x) - T(x))w[z, S(u)] = 0 \quad \forall x, z, u, w \in R \dots\dots\dots(8)$$

Pre multiplying by $[z, S(u)]$ and post multiplying by $(S(x) - T(x))$ we get

$$\text{So, } [z, S(u)] (S(x) - T(x))w[z, S(u)](S(x) - T(x)) = 0 \quad \forall x, z, u, w \in R$$

Since R is Semi-prime we get,

$$[z, S(u)](S(x) - T(x)) = 0 \quad \forall x, z, u \in R \dots\dots\dots(9)$$

From (8) and (9) we get

$$(S(x) - T(x))[z, S(u)] = [z, S(u)](S(x) - T(x)) \quad \forall x, z, u \in R$$

So by Herstein (Lemma1.1.8 [10]) we get

$$(S(x) - T(x)) \in Z, \quad \forall x \in R \dots\dots\dots(10)$$

$$\text{So, } [S(x) - T(x), y] = 0 \quad \forall x, y \in R$$

$$(ie) [S(x), y] - [T(x), y] = 0 \quad \forall x, y \in R$$

$$(ie) [S(x), y] = [T(x), y] \quad \forall x, y \in R \dots\dots\dots(11)$$

Now, $[S(x), S(y)] = [T(x), S(y)]$ (using (11))

$$= -[S(y), T(x)]$$

$$= -[T(y), T(x)] \text{ (using (11))}$$

$$= [T(x), T(y)]$$

$$(ie) [S(x), S(y)] = [T(x), T(y)] \quad \forall x, y \in R$$

This proves that S and T are strong commutativity preserving maps.

Conversely assume that S and T are strong commutativity preserving automorphism of R

$$\text{Then } [S(x), S(y)] = [T(x), T(y)] \quad \forall x, y \in R \dots\dots\dots(12)$$

Replacing y by xy we get

$$[S(x), S(xy)] = [T(x), T(xy)]$$

$$(ie) \quad S(x)[S(x), S(y)] + [S(x), S(x)]S(y) = T(x)[T(x), T(y)] + [T(x), T(x)]T(y)$$

$$(ie) \quad S(x)[S(x), S(y)] = T(x)[T(x), T(y)] \quad \forall x, y \in R \quad \dots\dots\dots(13)$$

Using (12) we get

$$S(x)[S(x), S(y)] = T(x)[S(x), S(y)]$$

$$(ie) (S(x) - T(x))[S(x), S(y)] = 0 \quad \forall x, y \in R \quad \dots\dots\dots(14)$$

Also from (12) and (13) we get

$$S(x)[T(x), T(y)] = T(x)[T(x), T(y)]$$

$$(ie) (S(x) - T(x))[T(x), T(y)] = 0 \quad \forall x, y \in R \quad \dots\dots\dots(15)$$

Replacing y by zy in (14) we get

$$(S(x) - T(x))[S(x), S(zy)] = 0 \quad \forall x, y, z \in R$$

$$(S(x) - T(x))\{S(z)[S(x), S(y)] + [S(x), S(z)]S(y)\} = 0$$

$$(S(x) - T(x))S(z)[S(x), S(y)] + (S(x) - T(x))[S(x), S(z)]S(y) = 0$$

Using (14) we get

$$(S(x) - T(x))S(z)[S(x), S(y)] = 0 \quad \forall x, y, z \in R \quad \dots\dots\dots(16)$$

Since S is an automorphism of R, we have

$$(S(x) - T(x))w[S(x), S(y)] = 0 \quad \forall x, y, w \in R \quad \dots\dots\dots(17)$$

Put $y = s^{-1}(T(x))$ in (17) we get

$$(S(x) - T(x))w[S(x), T(x)] = 0 \quad \forall x, w \in R \quad \dots\dots\dots(18)$$

Put $w = T(x)$ in (18) we get

$$(S(x) - T(x))T(x)w[S(x), T(x)] = 0$$

$$(ie) (S(x)T(x) - T(x)^2)w[S(x), T(x)] = 0 \quad \forall x, w \in R \quad \dots\dots\dots(19)$$

Pre multiplying (18) by T(x) we get

$$T(x)(S(x) - T(x))w[S(x), T(x)] = 0$$

$$(ie) (T(x)S(x) - T(x)^2)w[S(x), T(x)] = 0 \quad \forall x, w \in R \quad \dots\dots\dots(20)$$

(19) - (20) gives

$$(S(x)T(x) - T(x)S(x))w[S(x), T(x)] = 0 \quad \forall x, w \in R$$

$$(ie) [S(x), T(x)]w[S(x), T(x)] = 0 \quad \forall x, w \in R$$

Since R is semi-prime we have

$$[S(x), T(x)] = 0 \quad \forall x \in R$$

So S and T are commuting pair of automorphisms of R and so S and T are centralizing pair of automorphisms of R

Remark 3.2

Let R be a ring and $f: R \rightarrow R$ be an anti-homomorphisms. Then f is commutativity preserving.

Proof

Let $f: R \rightarrow R$ be an anti-homomorphism

Let $x, y \in R$

Suppose $[x, y] = 0$

$$\Rightarrow xy - yx = 0$$

$$\Rightarrow xy = yx$$

$$\Rightarrow f(xy) = f(yx)$$

$$\Rightarrow f(y)f(x) = f(x)f(y)$$

$$\Rightarrow [f(x); f(y)] = 0$$

(ie) f is commutativity preserving.

Theorem 3.3

Let R be a 2-torsion free semi-prime ring and S and T be a centralizing pair of anti homomorphisms of R onto itself. Then they are strong commutativity preserving pair.

Proof:

Let S and T be a centralizing pair of anti homomorphisms of R onto itself

We shall prove that S and T are commuting pair

Since S and T are centralizing pair

$$[S(x), T(x)] \in Z \quad \forall x \in R$$

Suppose $\exists x_0 \in R$ such that $[S(x_0), T(x_0)] = 0$ (1)

From (1) we have

$$[S(x_0 + y), T(x_0 + y)] \in Z, y \in R$$

$$(ie) [S(x_0), T(x_0)] + [S(x_0), T(y)] + [S(y), T(x_0)] + [S(y), T(y)] \in Z, \forall y \in R$$

Using (1) we get

$$[S(x_0), T(y)] + [S(y), T(x_0)] \in Z, \forall y \in R$$

$$So [S(x_0), [S(x_0), T(y)] + [S(y), T(x_0)]] = 0, \forall y \in R$$

Putting $y = x_0^2$, we get

$$[S(x_0), [S(x_0), T(x_0^2)] + [S(x_0^2), T(x_0)]] = 0$$

$$(ie) [S(x_0), T(x_0)] [S(x_0), T(x_0)] + [S(x_0), T(x_0)] T(x_0) +$$

$$S(x_0) [S(x_0), T(x_0)] + [S(x_0), T(x_0)] T(x_0) = 0$$

Using (1) we get

$$[S(x_0), 2T(x_0) [S(x_0), T(x_0)] + 2S(x_0) [S(x_0), T(x_0)]] = 0$$

$$2[S(x_0), T(x_0) [S(x_0), T(x_0)] + 2[S(x_0), S(x_0) [S(x_0), T(x_0)]] = 0$$

$$2\{T(x_0) [S(x_0), [S(x_0), T(x_0)]] + [S(x_0), T(x_0)] [S(x_0), T(x_0)]\}$$

$$+ 2\{S(x_0) [S(x_0), [S(x_0), T(x_0)]] + [S(x_0), S(x_0)] [S(x_0), T(x_0)]\} = 0$$

Using (1) we get

$$2[S(x_0), T(x_0)]^2 = 0$$

Since R is 2-torsion free, we get

$$[S(x_0), T(x_0)]^2 = 0$$

Since the center of a semi-prime ring contains no non-zero nilpotent elements, we have

$$[S(x_0), T(x_0)] = 0, \text{ a contradiction}$$

So $[S(x), T(x)] = 0 \quad \forall x \in R$ (2) I.e, S and T are commuting pairs.

$$Now [S(x + y), T(x + y)] = 0, \forall x, y \in R$$

$$(ie) \quad [S(x), T(x)] + [S(x), T(y)] + [S(y), T(x)] + [S(y), T(y)] = 0$$

Using (2) we get

$$[S(x), T(y)] + [S(y), T(x)] = 0, \forall x, y \in R$$

$$(ie) \quad [S(x), T(y)] = -[S(y), T(x)] = [T(x), S(y)] \dots \dots \dots (3)$$

$$\forall x, y \in R$$

$$\text{Then, } [T(xy), S(x)] = [S(xy), T(x)]$$

$$\text{Now } [S(xy), T(x)] = S(x)[S(y), T(x)] + [S(x), T(x)]S(y)$$

$$[S(xy), T(x)] = S(x)[S(y), T(x)] \dots \dots \dots (4)$$

$$\text{Also } [T(xy), S(x)] = [T(y)T(x), S(x)] \quad (\because T \text{ is anti-homomorphism})$$

$$= T(y)[T(x), S(x)] + [T(y), S(x)]T(x)$$

Using (2) we get

$$[T(xy), S(x)] = [T(y), S(x)]T(x) \quad \forall x, y \in R \dots \dots \dots (5)$$

From (3), (4) and (5) we get

$$[T(y), S(x)]T(x) = S(x)[S(y), T(x)] \quad \forall x, y \in R$$

Using (3) we get

$$[T(y), S(x)]T(x) = S(x)[T(y), S(x)] \quad \forall x, y \in R$$

Since T is onto

$$[z, S(x)]T(x) = S(x)[z, S(x)] \quad \forall x, z \in R \dots \dots \dots (6)$$

Replacing z by u S(z) in (6) we get

$$[uS(z), S(x)]T(x) = S(x)[uS(z), S(x)]$$

$$u[S(z), S(x)]T(x) + [u, S(x)]S(z)T(x) = S(x)u[S(z), S(x)] + S(x)[u, S(x)]S(z)$$

Using (6) we get

$$uS(x)[S(z), S(x)] + [u, S(x)]S(z)T(x)$$

$$= S(x)u[S(z), S(x)] + [u, S(x)]T(x)S(z)$$

$$(uS(x) - S(x)u)[S(z), S(x)] + [u, S(x)](S(z)T(x) - T(x)S(z))$$

$$[u, S(x)][S(z), S(x)] + [u, S(x)][S(z), T(x)] = 0 \quad \forall x, z, u \in R \dots \dots \dots (7)$$

Using (3) we get

$$[u, S(x)][S(z), S(x)] + [u, S(x)][T(z), S(x)] = 0$$

$$[u, S(x)]\{[S(z), S(x)] + [T(z), S(x)]\} = 0$$

$$[u, S(x)][S(z) + T(z), S(x)] = 0 \quad \forall x, z, u \in R \dots \dots \dots (8)$$

Replacing u by uw in (8) we get

$$[uw, S(u)][S(z) + T(z), S(x)] = 0$$

$$u[w, S(x)][S(z) + T(z), S(x)] + [u, S(x)]w[S(z) + T(z), S(x)] = 0$$

Using (8) we get

$$[u, S(x)]w[S(z) + T(z), S(x)] = 0 \quad \forall x, z, u, w \in R \dots \dots \dots (9)$$

Put $u = S(z) + T(z)$

$$[S(z) + T(z), S(x)]w[S(z) + T(z), S(x)] = 0$$

Since R is semi-prime ring

$$[S(z) + T(z), S(x)] = 0 \quad \forall x, z \in R \dots \dots \dots (10)$$

$$(ie) \quad [S(z), S(x)] + [T(z), S(x)] = 0 \quad \forall x, z \in R$$

$$(ie) \quad [S(x), S(z)] + [S(x), T(z)] = 0 \quad \forall x, z \in R$$

$$(ie) [S(x), S(z)] = -[S(x), T(z)] \quad \forall x, z \in R \dots\dots\dots(11)$$

Interchanging S and T we get

$$[T(x), T(z)] = -[T(x), S(z)] \quad \forall x, z \in R \dots\dots\dots(12)$$

Using (8) we get

$$[S(x), S(z)] = [T(x), T(z)] \quad \forall x, z \in R$$

(ie) S and T are strong Commutativity Preserving Pair of maps of R.

Remark 3.4

Brekar has Proved the following result.

Let R be a 2-torsion free semi-prime ring and let $f: R \rightarrow R$ be a centralizing anti-homomorphism.

Then a) $S = \{x \in R / f(x) = x\} \subseteq Z(R)$

b) If R is prime and f does not map R into $Z(R)$ then $S = Z(R)$

We generalize this result as

Theorem 3.5

Let R be a 2-torsion free semi-prime ring and let S and T are two non-trivial Centralizing anti-homomorphisms of R onto itself.

Then a) $S = \{x \in R / S(x) = T(x)\} \subseteq Z(R)$

b) If R is prime and S and T does not map R into $Z(R)$ then $S = Z(R)$

From (10) of the Previous theorem

$$[S(z) + T(z), S(x)] = 0 \quad \forall x, z \in R$$

Since S is onto

$$[S(z) + T(z), y] = 0 \quad \forall y, z \in R$$

(ie) $S(z) + T(z) \in Z(R) \quad \forall z \in R$

So for all $z \in S, S(z) + T(z) = 2S(z) \in Z(R)$

Since R is 2-torsion free semi-prime ring

$$z \in Z(R) \quad \forall z \in R$$

Hence $S \subseteq Z(R)$

Thus (a) is proved

(b) Assume that R is prime and let $z \in Z(R)$

If $T(z) = 0$, Then $z = 0$ (T is an automorphism) and so $S(z) = 0 = T(z)$

$$so z \in S \text{ if } z = 0 \in Z(R)$$

So assume $z \neq 0$ Since $z \in Z(R)$ and S and T are automorphisms of R,

$S(z) \in Z(R)$ and $T(z) \in Z(R)$

From (6) of the previous theorem, we have

$$[u, S(x)]T(x) = S(x)[u, S(x)] \quad \forall x, y \in R \dots\dots\dots(1)$$

Replace x by zx in (1) we get

$$[u, S(zx)]T(zx) = S(zx)[u, S(zx)]$$

$$S(z)[u, S(x)]T(zx) + [u, S(z)]S(x)T(zx) = S(zx)S(z)[u, S(x)] + S(zx)[u, S(z)]S(x)$$

Since $S(z) \in Z(R)$ we have

$$S(z)[u, S(x)]T(z)T(x) = S(z)S(x)S(z)[u, S(x)] \quad \forall x, u \in R$$

Since $T(z), S(z) \in Z(R)$ we get

$$[u, S(x)]T(x)T(z)S(z) = S(x)[u, S(x)]S(z^2) \quad \forall x, u \in R, z \in Z(R)$$

Again using (1) we get

$$[u, S(x)]T(x)T(z)S(z) = [u, S(x)]T(x)S(z^2)$$

$$(ie) [u, S(x)]T(x)(T(z)S(z) - S(z^2)) = 0 \quad \forall x, u \in R, z \in Z(R)$$

Since S is an automorphism of R

$$[u, w]T(x)(T(z)S(z) - S(z^2)) = 0 \quad \forall x, u, w \in R, z \in Z(R)$$

Since R is prime, any non-zero central element is not a zerodivisor

$$either T(z)S(z) - S(z^2) = 0 \text{ (or) } [u, w]T(x) = 0$$

Since $T \neq 0$ $T(x) \neq 0 \quad \forall x$ and also $[u, v] \neq 0 \quad \forall u, v$ ($\because R$ is not commutative)

$$Hence T(z)S(z) - S(z) = 0$$

$$(ie) (T(z) - S(z))S(z) = 0 \text{ since } z \neq 0 \text{ } f(z) \neq 0$$

$$Hence T(z) - S(z) = 0$$

$$(ie) T(z) = S(z) \text{ So } z \in S, S = Z(R)$$

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