Super fibonacci graceful anti – magic labeling for flower graphs and python coding


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Abstract:
A graph $G = (V, E, \phi)$ with $p$ vertices and $q$ edges. A super fibonacci graceful anti-magic labeling $\phi(G)$ of $G$ is an injective function $\phi: V \rightarrow \{F_0, F_1, F_2, \ldots, F_{q+1}\}$ such that the induced edge labeling $\phi^*(uv) = |\phi(u) - \phi(v)|$ is a bijection onto the set $\{F_2, F_3, F_4, \ldots, F_{q+1}\}$. In addition, all the vertex sums are pairwise distinct and all the edges are unique. If a graph $G$ admits a super fibonacci graceful anti magic labeling $\phi(G)$ then $G$ is called super fibonacci graceful anti-magic graph $SFGA\Gamma G$. In this article the concept of super fibonacci graceful anti-magic graph $SFGA\Gamma G$ is introduced and investigated with some flower graphs. These graphs are called super fibonacci graceful anti-magic graph $SFGA\Gamma G$.

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1. Introduction

Fibonacci graceful labeling idea was introduced by E. Barkaukas et. al. [3]. Hartsfield and Ringel introduced the concept of Anti-Magic labeling. The concept of Fibonacci Anti-magic labeling was introduced by Amenealhibi and T. Ranjani [1]. A. Rosa [12] has Published “vertex values of a graph”. It is the beginning of vertex labeling. Wang [13] shows that the Cartesian products of the cycles are Anti-Magic. In nature many plants having the number of petals in Fibonacci series like lilies and buttercups. Now this article is based on connected, undirected and Flower Graphs. Python is the most popular language across worldwide. It was introduced by GuioVan Rossum on February 20, 1991. Super Fibonacci Graceful Anti-Magic Labeling $SFGA\Gamma G$ concept is introduced here. While investigating, some Flower graphs are Super Fibonacci Graceful Anti Magic Graph $SFGA\Gamma G$. Here Python coding is generated for the Super Fibonacci Graceful Anti-Magic Labeling for some Flower graphs. Here Cherry blossom flower graph, Clematis flower graph and Rose flower graphs are introduced and we also proved the graphs are $SFGA\Gamma G$.

2. Definitions

Definition 2.1: The Fibonacci numbers $F_0, F_2, F_3, \ldots$ are defined by $F_0 = 0, F_1 = 1, F_2 = 1, \ldots$ and $F_{n+1} = F_n + F_{n-1}, n \geq 1$. The Fibonacci sequence is $1, 1, 2, 3, 5, \ldots$

Definition 2.2: A Super Fibonacci Graceful Anti-magic labeling $SFGA\Gamma G$ of $G$ is an injective function $\phi: V \rightarrow \{F_0, F_2, F_3, \ldots, F_{q+1}\}$ such that the induced edge labeling $\phi^*(uv) = |\phi(u) - \phi(v)|$ is a bijection onto the set $\{F_2, F_3, F_4, \ldots, F_{q+1}\}$. In addition, all the vertex sums are pairwise distinct and all the edges are unique. If a graph $G$ admits a Super Fibonacci Graceful Anti magic labeling $SFGA\Gamma G$ then $G$ is called Super Fibonacci Graceful Anti-Magic Graph $SFGA\Gamma G$.

Definition 2.3: A shell graph is Cycle $C_{n+1}$ with $(n - 2)$ chords sharing a common end vertex. It is denoted by $C[n + 1, n - 2]$.

Definition 2.4: A Clematis Flower graph $C_{n,m}$ is obtained by joining $n$ copies of $C_4 + e$ and $m$ copies $K_2$ with a common vertex.

Definition 2.5: A Cherry Blossom Flower graph $CB_{n,m}$ is obtained by joining $n$ copies of the cycle $C_3$ and $m$ copies $K_2$ with a common vertex.

Definition 2.6: A Rose flower graph $R_n$ is obtained by joining $n$ copies of the cycle $C_6$ with a common vertex.

Definition 2.7: The Vertex sum at one vertex is the sum of labels of edges. These edges are incident to that vertex.
3. Results

3.1. Theorem

Clematis Flower graph $C_{nm}$ is $SFGMG$ for all $n$.

Proof:

Let $C_{nm}$ be the Clematis Flower graph. The order of $C_{nm}$ is $p = 3n + m + 1$ and the size $q = 5n + m$. Let $V(G) = \{a_0, a_1, a_2, ..., a_m, b_1, b_2, ..., b_n, c_1, c_2, ..., c_m, d_1, d_2, ..., d_n\}$ be the vertex set. Let $a_0$ be the central vertex, $b_1, b_2, ..., b_n$ be the second vertices of $C_4 + e$, $c_1, c_2, ..., c_m$ be the third vertices of $C_4 + e$, $d_1, d_2, ..., d_n$ be the fourth vertices of $C_4 + e$ and $a_0, a_2, ..., a_m$ be the pendant vertices. The edge set $E(G) = \{e_{ij}, e_{ai}, e_{oi}, s_{ij}, p_{oi}\}$ here $e_j = (a_0, a_j)$, $e_{ii} = (b_ic_i)$, $e_{ai} = (a_0, b_i)$, $s_{ii} = (c_i, d_i)$ and $s_{oi} = (a_o, d_i)$.

Define $\phi: V \rightarrow \{0, F_1, F_2, ..., F_{n+1}\}$

$$\phi(a_0) = 0$$

$$\phi(a_j) = F_{j+1} \quad \text{for} \quad j = 1, 2, 3, ..., m$$

$$\phi(b_i) = F_{m+i-1} \quad \text{for} \quad i = 1, 2, 3, ..., n$$

$$\phi(c_i) = F_{m+i-1} \quad \text{for} \quad i = 1, 2, 3, ..., n$$

$$\phi(d_i) = F_{m+i+1} \quad \text{for} \quad i = 1, 2, 3, ..., n$$

The edge labels are

$$e_j = \phi'(a_0a_j) = |\phi(a_0) - \phi(a_j)| \quad \text{for} \quad j = 1, 2, 3, ..., m$$

$$e_{ii} = \phi'(b_ic_i) = |\phi(b_i) - \phi(c_i)| \quad \text{for} \quad i = 1, 2, 3, ..., n$$

$$e_{ai} = \phi'(a_0b_i) = |\phi(a_0) - \phi(b_i)| \quad \text{for} \quad i = 1, 2, 3, ..., n$$

$$s_{ii} = \phi'(c_id_i) = |\phi(c_i) - \phi(d_i)| \quad \text{for} \quad i = 1, 2, 3, ..., n$$

$$s_{oi} = \phi'(a_0d_i) = |\phi(a_0) - \phi(d_i)| \quad \text{for} \quad i = 1, 2, 3, ..., n$$

Here all the vertex sums are pairwise distinct and all the edges are unique.

Thus $\phi$ admits $SFGML$.

Hence Clematis Flower graph $C_{nm}$ is $SFGMG$.

Python coding and output is enclosed in the Appendix.

Example 3.1

![Clematis Flower graph $C_{35}$ is $SFGMG$](image)
3.2. Theorem
Cherry Blossom Flower graph $CB_{nm}$ is $SFGAMG$ for all $n$.

Proof:
Let $CB_{nm}$ be the Cherry Blossom Flower graph. The order of $CB_{nm}$ is $p = 2n + m + 1$ and the size $q = 3n + m$. Let $V(G) = \{a_0, a_1, a_2, \ldots, a_n, b_1, b_2, \ldots, b_m, c_1, c_2, \ldots, c_m\}$ be the vertex set. Let $a_0$ be the central vertex, $a_1, a_2, \ldots, a_n$ be the second vertices of the cycle $C_3$, $b_1, b_2, \ldots, b_m$ be the third vertices of the cycle $C_3$ and $c_1, c_2, \ldots, c_m$ be the pendant vertices. The edge set $E(G) = \{e_i, e_{i1}, e_j\}$ here $e_i = (a_0, a_i), e_{i1} = (a_i b_i)$ and $e_j = (a_0 c_j)$.

Define $\phi: V \rightarrow \{0, F_2, F_3, \ldots, F_{q+1}\}$

$\phi(a_0) = 0$
$\phi(a_i) = F_{3i}$ \hspace{3cm} for $i = 1, 2, 3, \ldots, n$
$\phi(b_i) = F_{3i+1}$ \hspace{3cm} for $i = 1, 2, 3, \ldots, n$
$\phi(c_j) = F_{3n+1+j}$ \hspace{3cm} for $j = 1, 2, 3, \ldots, m$

$n$ = No. of Copies of cycle $C_3$
$m$ = No. of Pendant Vertices

The edge labels are
$e_i = \phi^*(a_0 a_i) = |\phi(a_0) - \phi(a_i)|$ \hspace{3cm} for $i = 1, 2, 3, \ldots, n$
$e_{i1} = \phi^*(a_i b_i) = |\phi(a_i) - \phi(b_i)|$ \hspace{3cm} for $i = 1, 2, 3, \ldots, n$
$e_j = \phi^*(a_0 c_j) = |\phi(a_0) - \phi(c_j)|$ \hspace{3cm} for $j = 1, 2, 3, \ldots, m$.

Here all the vertex sums are pairwise distinct and all the edges are unique.

Thus $\phi$ admits $SFGAML$.

Hence the Cherry Blossom Flower graph $CB_{nm}$ is $SFGAMG$.

Python coding and output is enclosed in the Appendix.

Example 3.2

3.3. Theorem
Rose flower graph $R_n$ is $SFGAMG$ for all $n$.

Proof:
Let $R_n$ be Rose flower graph. The order of the Rose flower graph $R_n$ is $p = 6n$ and size is $q = 6n$. The vertex set $V(G) = \{a_0, a_1, a_2, \ldots, a_n, b_1, b_2, \ldots, b_n, c_1, c_2, \ldots, c_n, d_1, d_2, \ldots, d_n, e_1, e_2, \ldots, e_n\}$. Let $a_0$ be the central vertex, $a_1, a_2, \ldots, a_n$ be the second vertices of the cycle $C_6$, $b_1, b_2, \ldots, b_n$ be the third vertices of the cycle $C_6$, $c_1, c_2, \ldots, c_n$ be the fourth vertices of the cycle $C_6$, $d_1, d_2, \ldots, d_n$ be the fifth vertices of the cycle $C_6$ and $e_1, e_2, \ldots, e_n$ be the sixth vertices of the cycle $C_6$.
\( E(G) = \{u_i, v_i, w_i, x_i, y_i, z_i\} \) here \( u_i = \{a_0a_i\}, v_i = \{a_ib_i\}, w_i = \{b_ic_i\}, x_i = \{c_id_i\}, y_i = \{d_ie_i\}, z_i = \{a_0e_i\} \).

Define \( \phi: V \rightarrow \{0, F_1, F_2, ..., F_{q+1}\} \)

\( \phi(a_0) = 0 \)

\( \phi(a_i) = F_{6i+1} \) for \( i = 1, 2, ..., n \)

\( \phi(b_i) = F_{6i-1} \) for \( i = 1, 2, ..., n \)

\( \phi(c_i) = F_{6i} \) for \( i = 1, 2, ..., n \)

\( \phi(d_i) = F_{6i-2} \) for \( i = 1, 2, ..., n \)

\( \phi(e_i) = F_{6i-4} \) for \( i = 1, 2, ..., n \)

The edge labels are

\( u_i = \phi^*(a_0a_i) = |\phi(a_0) - \phi(a_i)| \) for \( i = 1, 2, 3, ..., n \)

\( v_i = \phi^*(a_ib_i) = |\phi(a_i) - \phi(b_i)| \) for \( i = 1, 2, 3, ..., n \)

\( w_i = \phi^*(b_ic_i) = |\phi(b_i) - \phi(c_i)| \) for \( i = 1, 2, 3, ..., n \)

\( x_i = \phi^*(c_id_i) = |\phi(c_i) - \phi(d_i)| \) for \( i = 1, 2, 3, ..., n \)

\( y_i = \phi^*(d_ie_i) = |\phi(d_i) - \phi(e_i)| \) for \( i = 1, 2, 3, ..., n \)

\( z_i = \phi^*(a_0e_i) = |\phi(a_0) - \phi(e_i)| \) for \( i = 1, 2, 3, ..., n \)

Here all the vertex sums are pairwise distinct and all the edges are unique.

Thus \( \phi \) admits SFGAML. Hence the graph \( R_n \) is SFGAMG.

Python coding and output is enclosed in the Appendix.

**Example 3.3**

3. **Conclusion:**

In this article the concept of Super Fibonacci Graceful Anti-Magic Labeling is introduced and explained. The Flower graphs demonstrated and proved that they are Super Fibonacci Graceful Anti - Magic Graphs. Python coding is generated for all functions in Flower graphs and also for Super Fibonacci Graceful Anti-Magic Labeling. Thus the real life Flowers are represented in Super Fibonacci Graceful Anti-Magic Graphs. In future different concept of labeling can also be developed.
APPENDIX:

1. Python Coding and output for Clematis Flower graph $C_{nm}$.

2. Python Coding and output for Cherry Blossom Flower graph $CB_{nm}$.
3. Python Coding and output for Rose Flower graph $R_n$.

```python
m=0
input("Enter the n Value: ")

for i in range (0):
    print ("{0[1]}".translate(subscript))
    print ("{0[2]}".translate(subscript))
    print ("{0[3]}".translate(subscript))
    print ("{0[4]}".translate(subscript))
    print ("{0[5]}".translate(subscript))
    print ("{0[6]}".translate(subscript))
    print ("{0[7]}".translate(subscript))

```

References