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Mean Labeling For Some Path And Cycle Related Graphs

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Abstract :
A function f is called a Mean labeling of a graph G if $f: V(G) \rightarrow \{0,1,2 \ldots, q\}$ is injection and the induced function $f^{*}: E(G) \rightarrow \{1,2,3, \ldots, q\}$ defined as

$$f^{*}(uv) = \begin{cases} \frac{f(u)+f(v)}{2} & if\ f(u)+f(v)\ \ is\ even \\ \frac{f(u)+f(v)+1}{2} & if\ f(u)+f(v)\ is\ odd \end{cases}$$

is bijective. A graph which admits mean labeling is called Mean Graph. In this paper we investigate some path and cycle based graphs are Mean Graphs.

MSC Clasiffication : 05C76, 05C78
Key Words: Laddar Graph, Planar Grid, Alternate Triangular snake, Mean Graph.

1. Introduction

Graph labeling is an Assignment of labels to edges, vertices or both subject to certain conditions. The concept of Graph labeling is introduced by Gallian. Joseph A.Gallian[3] is updating the recent topics in A Dynamic survey of Graph Labeling,2022. K.Manimekalai, K.Thirusangu[5] are proved Some Results on Pair Sum Labeling. Somasundaram.S and Ponraj.R,[8] are proved Some Results on Mean Graphs. The concept of Mean labeling is introduced by Somasundaram and Ponraj[7]. S.K. Vaidya, Lekha Bijukumar[11] are proved Some New Families of Mean Graphs. R.Ponraj, J.V.X.Parthipan and R.Kala,[6] are proved Some Results on Pair Sum Labeling. H ere we have to discuss only on connected simple graphs. In this paper we have discussed different types of graphs which satisfy the conditions of Mean labeling.

2. Definitions

2.1Definition:

A function f is called a Mean labeling of a graph G if $f: V(G) \rightarrow \{0,1,2 \ldots, q\}$ is injection and the induced function $f^{*}: E(G) \rightarrow \{1,2,3, \ldots, q\}$ defined as

$$f^{*}(uv) = \begin{cases} \frac{f(u)+f(v)}{2} & if\ f(u)+f(v)\ \ is\ even \\ \frac{f(u)+f(v)+1}{2} & if\ f(u)+f(v)\ is\ odd \end{cases}$$

is bijective.

2.2 Definition:

An Alternate Triangular snake $A(T_n)$ is obtained from a path $u_1, u_2, u_3, \ldots, u_n$ by joining u_i and u_{i+1} to new vertex V_{ie} every alternate edge if a path is replaced by C_3 .

2.3 Definition:
The ladder L_n $(n \geq 2)$ is the product graph $P_2 \times P_n$ which contains $2n$ vertices and $3n-2$ edges.

2.4 Definition:
The Planar grid P_{mn} is the product graph $P_m \times P_n$ which contains mn vertices $mn+(m-2)$ edges.

3. Results
3.1Theorem:
An Alternate Triangular Snake $A(T_n)$ is Mean Graph.
Proof:
Let $A(T_n)$ be an Alternate Triangular snake.
Let $V(G) \to \{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n, w_1, w_2, \dots w_n\}$ be a Vertex Set. Here $u_1, u_2, \dots, u_n$ be the first vertices of the Triangles and $v_1, v_2, \dots, v_n, w_1, w_2, \dots, w_n$ be the vertices of second and third vertices of Triangles respectively.
Let $E(G) \to \{e_{ii}, g_{ii}\}$ be the Edge set.
Let $e_{ii} = \{u_i v_i\}$ and $g_{ii} = \{v_i w_i\}$.
Define the function $f: V(G) \to \{0,1,2,3, \dots q\}$ as follows

$$f(u_1) = 2$$
$$f(v_1) = 0$$
$$f(w_1) = 3$$
$$f(u_i) = u_1 + 4(i-1) \qquad for\ i = 2,3 \dots, n$$
$$f(v_i) = v_1 + 4(i-1) \qquad for\ i = 2,3, \dots, n$$
$$f(w_i) = w_1 + 4(i-1) \qquad for\ i = 2,3 \dots, n$$

The Edge labels are

$$f(e_{ii}) = \frac{[f(u_i)+f(v_i)]}{2} \ or\ \frac{[f(u_i)+f(v_i)+1]}{2}$$
$$f(g_{ii}) = \frac{[f(v_i)+f(w_i)]}{2} \ or\ \frac{[f(v_i)+f(w_i)+1]}{2}$$

Then the above defined function f admits Mean Labelling.
Hence An Alternate Triangular Snake is Mean Graph.
Example:

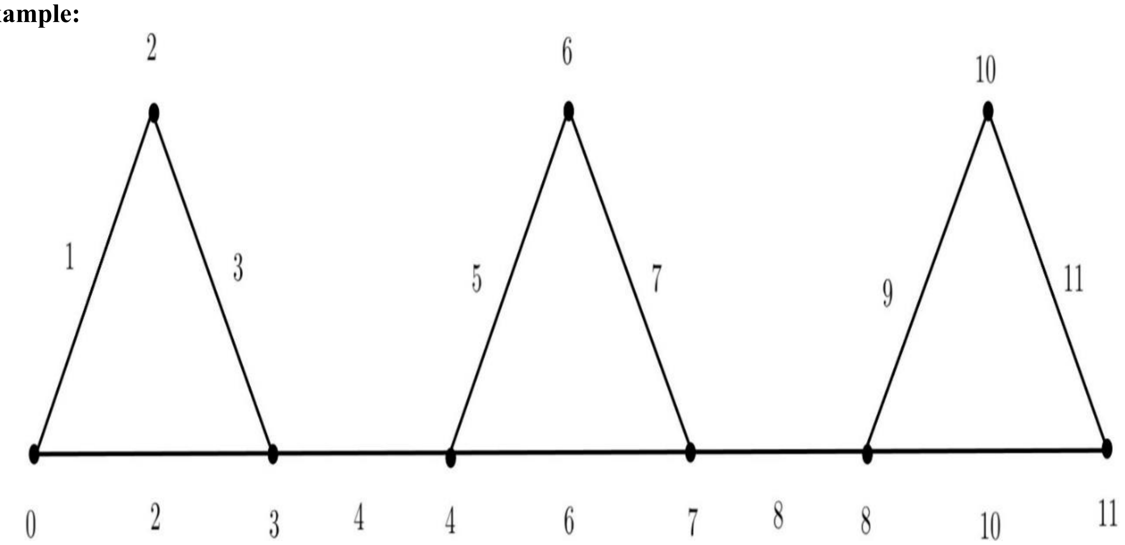

Fig 3.1: Alternate Triangular Snake $A(T_5)$

3.2 Theorem:
The Laddar L_n $(n \geq 2)$ is Mean Graph.
Proof:

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Let L_n $(n \geq 2)$ be Laddar. Let $V(G) \to \{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n.\}$ be a Vertex Set. Here $u_1, u_2, \dots, u_n$ be the vertices of the lower row and $v_1, v_2, \dots, v_n$ be the vertices of upper row . Let $E(G) \to \{e_{ij}, g_{ij}, s_{ii}\}$ be the Edge set. Let $e_{ij} = \{u_i u_j\}$, $g_{ii} = \{v_i v_j\}$ and $s_{ii} = \{u_i v_i\}$.

Define the function $f: V(G) \to \{0,1,2,3, \dots q\}$ as follows

$$f(u_i) = (i-1) \qquad for\ i = 2,3 \dots, n$$

$$f(v_i) = i + (q-n) \quad for\ i = 1,2,3, \dots, n$$

$$f(u_1) = 0.$$

The Edge labels are

$$f(e_{ij}) = \frac{[f(u_i)+f(u_j)]}{2}\ or\ \frac{[f(u_i)+f(u_j)+1]}{2}$$

$$f(g_{ij}) = \frac{[f(v_i)+f(v_j)]}{2}\ or\ \frac{[f(v_i)+f(v_j)+1]}{2}$$

$$f(s_{ii}) = \frac{[f(u_i)+f(v_i)]}{2}\ or\ \frac{[f(u_i)+f(v_i)+1]}{2}$$

Then the above defined function f admits Mean Labelling.

Hence Ladder L_n is Mean Graph.

Example 3.2

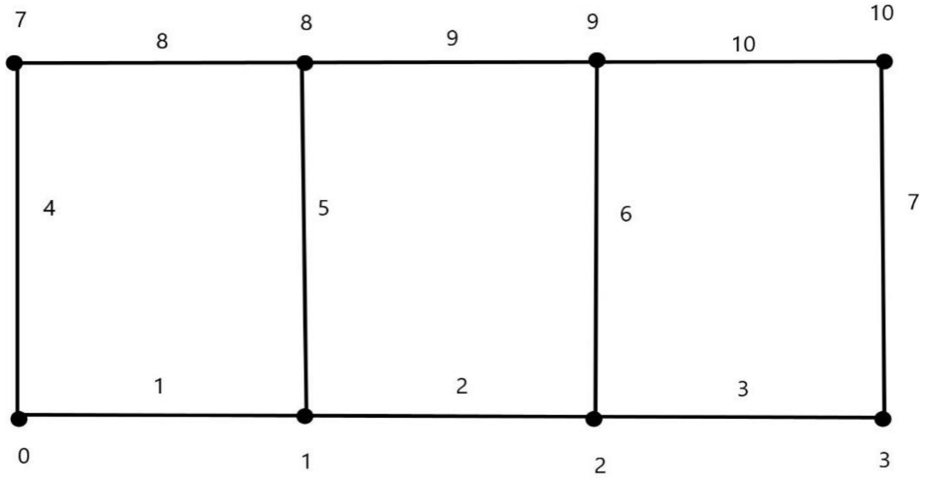

Fig 3.2 : Laddar L_4

3.3 Theorem:

The Planar grid P_{mn} is Mean Graph.

Proof:

Let P_{mn} be Planar grid. Let $V(G) \to \{u_1, u_2, \dots, u_{mn}\}$ be a Vertex Set. Here $u_1, u_2, \dots, u_{mn}$ be the vertices of Planar grid. Let $E(G) \to \{e_{ij}\}$ be the Edge set. Let $e_{ij} = \{u_{ki} u_{k(i+1)}\}$.

Define the function $f: V(G) \to \{0,1,2,3, \dots q\}$ as follows

$$f(u_{ki}) = (m-i) + (k-1)(2m-1) \qquad for\ i = 1,2, \dots m$$

$$for\ k = 1,2, \dots n$$

The Edge labels are

$$f(e_{ij}) = \frac{[f(u_{ki})+f(u_{k(i+1)})]}{2}\ or\ \frac{[f(u_k)+f(u_{k(i+1)})+1]}{2}$$

Then the above defined function f admits Mean Labelling.

Hence Planar Grid P_{mn} is Mean Graph.

Example :

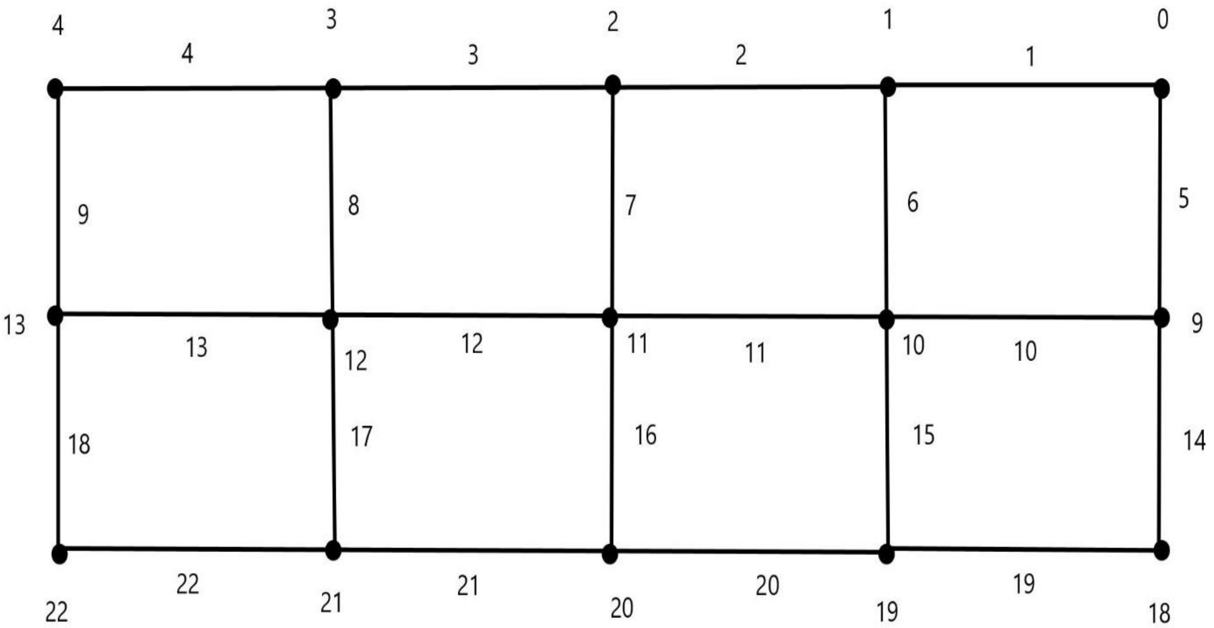

Fig 3.3: Planar grid P_{53}

3.4 Theorem

The graph $P_nAK_1 + 2e$ is mean graph.

Proof:

Let $P_nAK_1 + 2e$ be a graph. Let $V(G) \to \{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n, w_{1,} w_2\}$ be a Vertex Set. Here $u_1, u_2, \dots, u_n$ be the first vertices of the path and $v_1, v_2, \dots, v_n$ be the pendant vertices attaching by the path vertices and $w_{1,} w_2$ be the vertices attached by first and last vertex of path. Let $E(G) \to \{e_{ii}, g_{ii},, s_{ij}\}$ be the Edge set. Let $e_{ii} = \{u_i v_i\}, s_{ij} = \{u_i u_j\}$, and $g_{ij} = \{u_i w_i\}$.

Define the function $f: V(G) \to \{0,1,2,3, \dots q\}$ as follows

$$f(w_1) = 0$$
$$f(w_2) = q$$
$$f(u_i) = (q-1) - (2i-2) \quad for\ i = 1,2,3, \dots, n.$$
$$f(v_i) = (q-1) - (2i-1) \quad for\ i = 1,2,3, \dots, n.$$

The Edge labels are

$$f(s_{ij}) = \frac{[f(u_i)+f(u_j)]}{2} \ or\ \frac{[f(u_i)+f(u_j)+1]}{2}$$
$$f(g_{ij}) = \frac{[f(v_i)+f(w_j)]}{2} \ or\ \frac{[f(v_i)+f(v_j)+1]}{2}$$
$$f(e_{ii}) = \frac{[f(u_i)+f(v_i)]}{2} \ or\ \frac{[f(u_i)+f(v_i)+1]}{2}$$

Then the above defined function f admits Mean Labelling.

Hence $P_nAK_1 + 2e$ is Mean Graph.

Example :

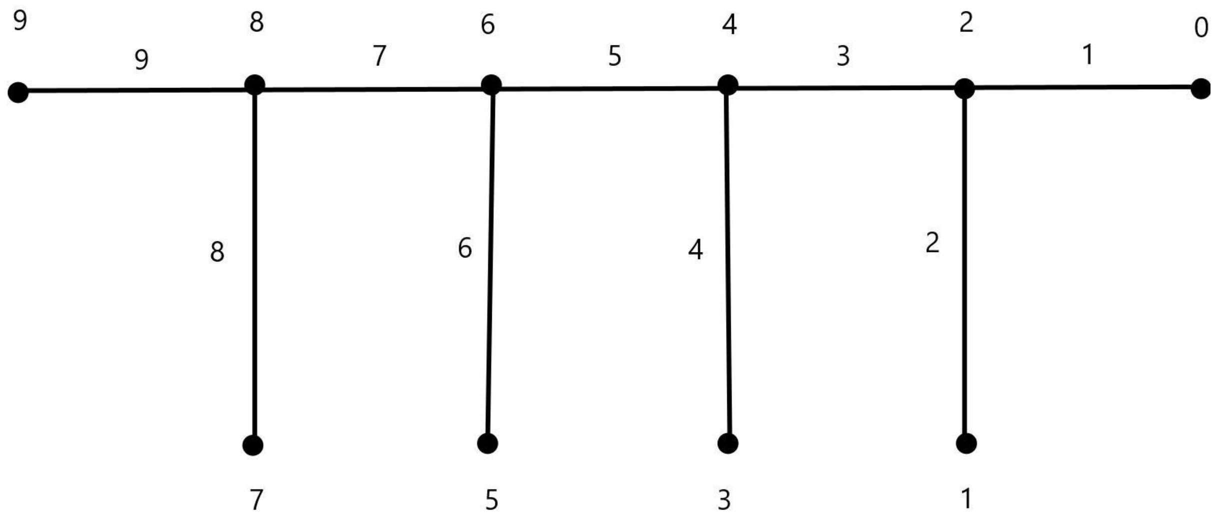

Fig 3.4 $P_4AK_1 + 2e$

4. Conclusion

We proved that cycle related graphs and path related graphs are Mean graphs. We extend the study to other families of Graph.

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