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"Analysis of Stress and Displacement of Simply Supported Symmetric Cross-Ply [0/90/90/0] Square Laminated Composite Plate Under Sinusoidal Loading Using Finite Element Method"

[1]Mahaboobali Nadaf, [2]Dr. R. J. Fernandes

[1]*Assistant Professor, Department of Civil Engineering, Jain College of Engineering, Affiliated to Visvesvaraya Technological University (VTU), Belagavi, Karnataka, India.*

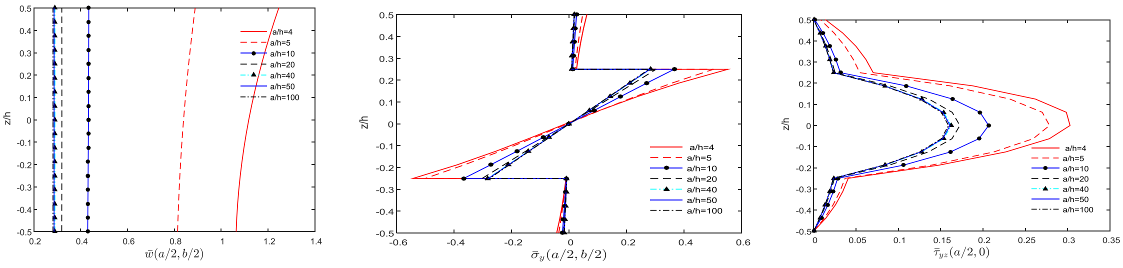
[2]*Associate Professor, Department of Civil Engineering, SDM College of Engineering and Technology, Dharwad, An Autonomous college under Visvesvaraya Technological University (VTU), Belagavi, Karnataka, India.*

ORCID ID: [1]0009-0003-4243-5782, [2]0000-0002-4253-7346

Abstract: Laminated composite plates are widely used in aerospace, military, civil infrastructure, and automotive sectors for their beneficial attributes, such as excellent fatigue resistance, strong yet lightweight construction, and customizable fiber orientation, materials, and stacking patterns. Therefore, it's crucial to analyze deflections and stress levels in these plates.In this current research, a mathematical formulation based on the higher-order deformation theory (HSDT) is employed, featuring 12 degrees of freedom per node. The focus of the analysis involves studying stress and deflection in four-layer symmetric [0/90/90/0] cross-ply laminate plates under sinusoidal loading. The obtained results are then compared with established findings in existing literature. Furthermore, a parametric investigation is conducted, exploring different side-to-thickness ratios at values of a/h equal to 4, 5, 10, 20, 40, 50, and 100. In this research, the finite element analysis was performed with HSDT using MATLAB software. The deflections were subsequently compared to analytical solutions provided by results from Turvey [11].Detailed results are presented in tables, and they exhibit a high degree of agreement with established standard results,with errors of less than 10%.

Keywords*:higher order shear deformation theory (HSDT),Displacement, Stress analysis, Finite element method, Cross-ply.*

Graphical Abstract:



1. Introduction

In classical plate theory (CLPT), the influence of transverse shear is neglected. In the first-order shear deformation theory (FSDT), transverse shear effects are considered but with a linear variation. However, in higher-order shear deformation theory, the incorporation of an appropriate displacement model adjustment leads to improved accuracy in stress resultants.An investigation into the application of the trigonometric shear deformation theory for analyzing the displacements and stress patterns in a cross-ply laminated beam under varying loads, as conducted by Tupe and Dahake [1]. The investigation of buckling analysis in a laminated composite skew plate was carried out by Mishra, Kumar, Samui and Roshni. They employed a C0 finite element (FE) model based on the higher-order shear deformation theory (HSDT) in combination with minimax probability machine regression (MPMR) and multivariate adaptive regression spline [2]. Komal Navale and Dr. Pise have introduced theories that posit a non-linear variation of transverse shear strain throughout the plate's thickness, resulting in transverse shear stress-free surfaces at both the top and bottom of the plate [3].Research conducted by Thai, Ferreira and Nguyen-Xuan, involves an investigation employing a nonlocal strain gradient meshfree plate method that integrates the nonlocal strain gradient theory (NSGT), higher-order shear deformation theory (HSDT), and meshfree technique. This study focuses on conducting bending and free vibration analyses of laminated composite and sandwich nanoplates[4].Dhuria, Grover and Goyal have undertaken a study in which they have formulated a novel higher-order hyperbolic shear deformation theory for analyzing the mechanical behavior of cross-ply and angle-ply multilayered plates. The proposed theory incorporates the secant hyperbolic function of the thickness coordinate within the displacement field, leading to non-linear displacement distributions while ensuring that both the upper and lower surfaces of the plates exhibit zero shear stresses[5]. Attia Bachiri, Ahmed Amine Daikh, and Abdelouahed Tounsi have introduced a mathematical model founded on an innovative higher shear deformation plate theory. This model serves to explore the thermo-elastic behavior of cross-ply laminated plates reinforced with carbon nanotubes (CNTRC) when subjected to thermal loading. Their study examines both functionally graded distributions (FG) and uniform distribution (UD) of carbon nanotube reinforcement material [6].Amirhadi Alesadi et al. [7] conducted free vibration and buckling analysis of cross-ply LCPs using unified formulation based on the Iso-geometric approach. J.A. Artero-Guerrero et al. [8] investigated the influence of laminate stacking sequence using the Artificial Neural Networks methodology. Durgesh Bahadur Singh et al. [9] proposed new HSDT for free vibration and buckling analysis. Y.S. Joshana et al. [10] developed an analytical model for thermo-mechanical analysis of cross-ply and angle-ply laminated composite plates.Further studies include bending analysis of laterally loaded, moderately thick, anti-symmetrically laminated rectangular plates by Turvey, G.J [11]. Pagano, N.J [12] presented exact solutions for rectangular bidirectional composites and sandwich plates. Tarun Kant and B.S. Manjunatha [13], Pandya B.N. & T. Kant [14]utilized the finite element method to analyze laminates.

2. Formulationsused in analysis symmetric Cross-ply LCPs [0/90/90/0]:

The displacement model is:

$$\begin{aligned} u(x,y,z) &= u_0(x,y) + z\theta_x(x,y) + z^2 u_0^*(x,y) + z^3\theta_x^*(x,y) \\ v(x,y,z) &= v_0(x,y) + z\theta_y(x,y) + z^2 v_0^*(x,y) + z^3\theta_y^*(x,y) \\ w(x,y,z) &= w_0(x,y) + z\theta_z(x,y) + z^2 w_0^*(x,y) + z^3\theta_z^*(x,y) \end{aligned} \tag{1}$$

Where, u_0, v_0, w_0 are the in-plane and transverse displacement of a point (x,y) on the midplane. $\theta_x, \theta_y, \theta_z$ are the rotations of the normal to mid plane about x,y and z axes. $u_0^*, v_0^*, w_0^*, \theta_x^*, \theta_y^*, \theta_z^*$ are the corresponding higher order shear deformation terms and these terms are also defined at the midplane.

The strain displacement relations:

$$\varepsilon_x = \varepsilon_{x_0} + z\kappa_x + z^2\varepsilon_{x_0}^* + z^3\kappa_x^* \quad , \quad \varepsilon_y = \varepsilon_{y_0} + z\kappa_y + z^2\varepsilon_{y_0}^* + z^3\kappa_y^* \quad , \quad \varepsilon_z = \varepsilon_{z_0} + z\kappa_z^* + z^2\varepsilon_{z_0}^*$$

$\gamma_{xy} = \varepsilon_{xy_0} + z\kappa_{xy} + z^2\varepsilon^*_{xy_0} + z^3\kappa^*_{xy}$, $\gamma_{yz} = \phi_y + z\kappa_{yz} + z^2\phi^*_y + z^3\kappa^*_{yz}$, $\gamma_{xz} = \phi_x + zk_{xz} + z^2\phi^*_x + z^3\kappa^*_{xz}$
(2)

The above strain expression can be represented in matrix form as,

$$\varepsilon^L_{MB} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_{x_0} \\ \varepsilon_{y_0} \\ \varepsilon_{z_0} \\ \gamma_{xy_0} \end{Bmatrix} + z \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_z \\ \kappa_{xy} \end{Bmatrix} + z^2 \begin{Bmatrix} \varepsilon^*_{x_0} \\ \varepsilon^*_{y_0} \\ \varepsilon^*_{z_0} \\ \varepsilon^*_{xy_0} \end{Bmatrix} + z^3 \begin{Bmatrix} \kappa^*_x \\ \kappa^*_y \\ 0 \\ \kappa^*_{xy} \end{Bmatrix}$$

$$\varepsilon^L_S = \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}^L = \begin{Bmatrix} \phi_y \\ \phi_x \end{Bmatrix} + z \begin{Bmatrix} \kappa_{yz} \\ \kappa_{xz} \end{Bmatrix} + z^2 \begin{Bmatrix} \phi^*_y \\ \phi^*_x \end{Bmatrix} + z^3 \begin{Bmatrix} \kappa^*_{yz} \\ \kappa^*_{xz} \end{Bmatrix} \tag{3}$$

Constitutive relations with reference to laminate axes are obtained as follows:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix}^L = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & Q_{14} & 0 & 0 \\ Q_{21} & Q_{22} & Q_{23} & Q_{24} & 0 & 0 \\ Q_{31} & Q_{32} & Q_{33} & Q_{34} & 0 & 0 \\ Q_{41} & Q_{42} & Q_{43} & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{55} & Q_{56} \\ 0 & 0 & 0 & 0 & Q_{65} & Q_{66} \end{bmatrix}^L \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}^L \tag{4}$$

Components of stress resultants:

$$\sigma_x = Q_{11}\varepsilon_x + Q_{12}\varepsilon_y + Q_{13}\varepsilon_z + +Q_{14}\gamma_{xy}$$

$$\sigma_y = Q_{12}\varepsilon_x + Q_{22}\varepsilon_y + Q_{23}\varepsilon_z + Q_{24}\gamma_{xy}$$

$$\sigma_z = Q_{13}\varepsilon_x + Q_{32}\varepsilon_y + Q_{33}\varepsilon_z + Q_{34}\gamma_{xy}$$

$$\tau_{xy} = Q_{41}\varepsilon_x + Q_{42}\varepsilon_y + Q_{43}\varepsilon_z + Q_{44}\gamma_{xy}$$

$$\tau_{yz} = Q_{55}\gamma_{yz} + Q_{56}\gamma_{xz}$$

$$\tau_{xz} = Q_{65}\gamma_{yz} + Q_{66}\gamma_{xz}$$

3. Results and Discussions for symmetric cross-ply laminate [0/90/90/0]:

For the analysis of cross-ply LCPs under uniformly distributed transverse loading is considered. SS1 boundary conditions (BC) are considered.Material properties:

Material 1:

E_1 =40 Gpa E_2=E_3 =1 Gpa G_{12} = G_{13} = 0.6 Gpa G_{23} = 0.5 Gpa

$\vartheta_{12} = \vartheta_{13} = \vartheta_{23}$ = 0.25 Gpa Support condition = Simply supported

Loading = Sinusoidal loading.

Material 2:

E_1 =3 Gpa E_2=E_3 =1 Gpa $G_{12} = G_{13}$ = 0.5 Gpa G_{23} = 0.6 Gpa

$\vartheta_{12} = \vartheta_{13} = \vartheta_{23}$ = 0.25 Gpa Support condition = Simply supported

Loading = Sinusoidal loading.

Results reported in the table and plots are in non- dimensional form.

Example 1:

In this example, simply supported, symmetric cross ply 4 Layer [0/90/90/0] square plate under sinusoidal Load (Material 1), is analyzed and Comparison of values of (Non-Dimensional) Maximum Deflection of present HSDT model with the values of Turvey [11]are shown in Table 1. The deflection calculated from the present model continuously reduces as percentage values from 18.68% to 0.60%.

Table 1:Non-dimensionalized transverse deflection and error (%) in a simply supported four layered symmetric cross-ply [0/90/90/0] square plate under sinusoidal load (Material 1).

a/h	Theory	$\bar{w}$	$Error$ (%)
5	HSDT12-FEM	0.8889	18.68
	Turvey [52]	0.7491	-
10	HSDT12-FEM	0.4348	6.66
	Turvey [52]	0.4077	-
20	HSDT12-FEM	0.3198	2.16
	Turvey [52]	0.3131	-
40	HSDT12-FEM	0.2902	0.60
	Turvey [52]	0.2885	-

Example 2:

In this example, simply supported, symmetric cross ply 4 Layer [0/90/90/0] square plate under sinusoidal Load (Material 2), is analyzed and Comparison of values of (Non-Dimensional) Maximum Deflection of present HSDT model with the values of Turvey [11] are shown in Table 2. The deflection calculated from the present model continuously reduces as percentage values from 2.81% to 0.06%.

Table 2:Non-dimensionalized transverse deflection and error (%) in a simply supported four layered symmetric cross-ply [0/90/90/0] square plate under sinusoidal load (Material 2).

a/h	Theory	$\bar{w}$	$Error$ (%)
5	HSDT12-FEM	2.2056	2.81
	Turvey [52]	2.1455	-
10	HSDT12-FEM	1.8707	0.72
	Turvey [52]	1.8574	-
20	HSDT12-FEM	1.7884	0.19
	Turvey [52]	1.7851	-

40	HSDT12-FEM	1.7678	0.06
	Turvey [52]	1.7669	-

It should be noted that the accuracy of the present model HSDT 12 increases when the a/h value varies from 5 to 40. It is also noted from the table that the percentage of errors decreases when the a/h value varies from 5 to 40, and these errors become negligible when the thickness ratio a/h ≥ 40.In Table 3, maximum values of displacement and stresses are given at top and bottom layer of cross-ply laminates for [0/90/90/0] having material 1 properties.

Table 3:Non-dimensionalized displacements and stresses in a simply supported four (n=4) layered symmetric cross-ply [0/90/90/0] square plate under sinusoidal load (Material 1).

a/h	Theory	z/h	$\overline{u}$	$\overline{v}$	$\overline{\sigma}_x$	$\overline{\sigma}_y$	$\overline{\tau}_{xy}$	$\overline{\tau}_{xz}$ *	$\overline{\tau}_{yz}$ *	$\overline{w}$ *
4	HSDT12-FEM	-0.5	0.1316	0.3090	-0.6378	-0.0434	0.0348	0.3223	0.3029	1.2457
		+0.5	-0.1251	-0.3028	0.6876	0.0614	-0.0334			
5	HSDT12-FEM	-0.5	0.0979	0.2024	-0.6052	-0.0362	0.0296	0.3405	0.2787	0.8889
		+0.5	-0.0946	-0.1992	0.6269	0.0471	-0.0287			
10	HSDT12-FEM	-0.5	0.0447	0.0623	-0.5624	-0.0233	0.0210	0.3862	0.2068	0.4348
		+0.5	-0.0443	-0.0619	0.5626	0.0257	-0.0209			
20	HSDT12-FEM	-0.5	0.0221	0.0245	-0.5543	-0.0188	0.0183	0.4063	0.1727	0.3198
		+0.5	-0.0220	-0.0245	0.5539	0.0194	-0.0183			
40	HSDT12-FEM	-0.5	0.0110	0.0113	-0.5527	-0.0176	0.0176	0.4123	0.1623	0.2902
		+0.5	-0.0110	-0.0113	0.5526	0.0177	-0.0175			
50	HSDT12-FEM	-0.5	0.0088	0.0090	-0.5525	-0.0174	0.0175	0.4131	0.1610	0.2866
		+0.5	-0.0088	-0.0090	0.5525	0.0175	-0.0175			
100	HSDT12-FEM	-0.5	0.0044	0.0044	-0.5523	-0.0172	0.0173	0.4141	0.1593	0.2818
		+0.5	-0.0044	-0.0044	0.5523	0.0172	-0.0173			

In the below section, from Table 4 to Table 11, through thickness variation of displacements and stresses for different a/h ratios, are presented.

Table 4: Through thickness variation of In-plane displacement ($\overline{u}$) for a simply supported symmetric cross ply 4 layered[0/90/90/0] square plate under sinusoidal load (Material 1).

z/h	a/h = 4	a/h = 5	a/h = 10	a/h = 20	a/h = 40	a/h = 50	a/h = 100
-0.500	0.1316	0.0979	0.0447	0.0221	0.0110	0.0088	0.0044
-0.438	0.0881	0.0705	0.0369	0.0190	0.0096	0.0077	0.0038
-0.375	0.0555	0.0493	0.0300	0.0161	0.0082	0.0066	0.0033
-0.313	0.0324	0.0333	0.0238	0.0132	0.0068	0.0055	0.0027
-0.250	0.0171	0.0217	0.0183	0.0105	0.0054	0.0044	0.0022
-0.250	0.0171	0.0217	0.0183	0.0105	0.0054	0.0044	0.0022

z/h	a/h = 4	a/h = 5	a/h = 10	a/h = 20	a/h = 40	a/h = 50	a/h = 100
-0.188	0.0082	0.0137	0.0134	0.0078	0.0041	0.0033	0.0016
-0.125	0.0039	0.0082	0.0088	0.0052	0.0027	0.0022	0.0011
-0.063	0.0028	0.0045	0.0044	0.0026	0.0014	0.0011	0.0005
0.000	0.0032	0.0017	0.0002	0.0000	0.0000	0.0000	0.0000
0.000	0.0032	0.0017	0.0002	0.0000	0.0000	0.0000	0.0000
0.063	0.0037	-0.0012	-0.0040	-0.0025	-0.0013	-0.0011	-0.0005
0.125	0.0026	-0.0049	-0.0083	-0.0051	-0.0027	-0.0022	-0.0011
0.188	-0.0017	-0.0103	-0.0129	-0.0078	-0.0041	-0.0033	-0.0016
0.250	-0.0106	-0.0184	-0.0179	-0.0104	-0.0054	-0.0044	-0.0022
0.250	-0.0106	-0.0184	-0.0179	-0.0104	-0.0054	-0.0044	-0.0022
0.313	-0.0259	-0.0300	-0.0234	-0.0132	-0.0068	-0.0055	-0.0027
0.375	-0.0490	-0.0460	-0.0296	-0.0160	-0.0082	-0.0066	-0.0033
0.438	-0.0816	-0.0672	-0.0365	-0.0190	-0.0096	-0.0077	-0.0038
0.500	-0.1251	-0.0946	-0.0443	-0.0220	-0.0110	-0.0088	-0.0044

Table 5: Through thickness variation of In-plane displacement ($\overline{v}$)for a simply supported symmetric cross ply 4 layered [0/90/90/0] square plate under sinusoidal load (Material 1).

z/h	a/h = 4	a/h = 5	a/h = 10	a/h = 20	a/h = 40	a/h = 50	a/h = 100
-0.500	0.3090	0.2024	0.0623	0.0245	0.0113	0.0090	0.0044
-0.438	0.2465	0.1660	0.0535	0.0213	0.0099	0.0078	0.0039
-0.375	0.1937	0.1341	0.0451	0.0182	0.0085	0.0067	0.0033
-0.313	0.1492	0.1061	0.0371	0.0151	0.0071	0.0056	0.0028
-0.250	0.1117	0.0813	0.0294	0.0121	0.0056	0.0045	0.0022
-0.250	0.1117	0.0813	0.0294	0.0121	0.0056	0.0045	0.0022
-0.188	0.0797	0.0591	0.0219	0.0090	0.0042	0.0034	0.0017
-0.125	0.0518	0.0389	0.0145	0.0060	0.0028	0.0022	0.0011
-0.063	0.0268	0.0199	0.0073	0.0030	0.0014	0.0011	0.0006
0.000	0.0031	0.0016	0.0002	0.0000	0.0000	0.0000	0.0000
0.000	0.0031	0.0016	0.0002	0.0000	0.0000	0.0000	0.0000
0.063	-0.0206	-0.0167	-0.0069	-0.0030	-0.0014	-0.0011	-0.0006
0.125	-0.0456	-0.0356	-0.0141	-0.0060	-0.0028	-0.0022	-0.0011
0.188	-0.0735	-0.0559	-0.0215	-0.0090	-0.0042	-0.0034	-0.0017

z/h	a/h = 4	a/h = 5	a/h = 10	a/h = 20	a/h = 40	a/h = 50	a/h = 100
0.250	-0.1055	-0.0781	-0.0290	-0.0120	-0.0056	-0.0045	-0.0022
0.250	-0.1055	-0.0781	-0.0290	-0.0120	-0.0056	-0.0045	-0.0022
0.313	-0.1430	-0.1029	-0.0367	-0.0151	-0.0070	-0.0056	-0.0028
0.375	-0.1875	-0.1309	-0.0447	-0.0182	-0.0085	-0.0067	-0.0033
0.438	-0.2403	-0.1628	-0.0531	-0.0213	-0.0099	-0.0078	-0.0039
0.500	-0.3028	-0.1992	-0.0619	-0.0245	-0.0113	-0.0090	-0.0044

Table 6: Through thickness variation of transverse displacement ($\overline{w}$)for a simply supported symmetric cross ply 4 layered [0/90/90/0] square plate under sinusoidal load (Material 1).

z/h	a/h = 4	a/h = 5	a/h = 10	a/h = 20	a/h = 40	a/h = 50	a/h = 100
-0.500	1.0639	0.8138	0.4301	0.3193	0.2901	0.2865	0.2818
-0.438	1.0665	0.8157	0.4305	0.3194	0.2901	0.2866	0.2818
-0.375	1.0702	0.8179	0.4310	0.3195	0.2901	0.2866	0.2818
-0.313	1.0752	0.8205	0.4314	0.3196	0.2902	0.2866	0.2818
-0.250	1.0813	0.8235	0.4318	0.3196	0.2902	0.2866	0.2818
-0.250	1.0813	0.8235	0.4318	0.3196	0.2902	0.2866	0.2818
-0.188	1.0886	0.8269	0.4322	0.3197	0.2902	0.2866	0.2818
-0.125	1.0971	0.8306	0.4325	0.3198	0.2902	0.2866	0.2818
-0.063	1.1068	0.8348	0.4329	0.3198	0.2902	0.2866	0.2818
0.000	1.1176	0.8393	0.4332	0.3198	0.2902	0.2866	0.2818
0.000	1.1176	0.8393	0.4332	0.3198	0.2902	0.2866	0.2818
0.063	1.1296	0.8442	0.4335	0.3198	0.2902	0.2866	0.2818
0.125	1.1428	0.8495	0.4337	0.3198	0.2902	0.2866	0.2818
0.188	1.1571	0.8551	0.4340	0.3198	0.2902	0.2866	0.2818
0.250	1.1725	0.8612	0.4342	0.3198	0.2902	0.2866	0.2818
0.250	1.1725	0.8612	0.4342	0.3198	0.2902	0.2866	0.2818
0.313	1.1891	0.8675	0.4344	0.3198	0.2902	0.2866	0.2818
0.375	1.2068	0.8743	0.4345	0.3197	0.2902	0.2866	0.2818
0.438	1.2257	0.8814	0.4347	0.3196	0.2901	0.2866	0.2818
0.500	1.2457	0.8889	0.4348	0.3196	0.2901	0.2865	0.2818

Table 7: Through thickness variation of In-plane normal stress ($\overline{\sigma}_x$) for a simply supported symmetric cross ply 4 layered [0/90/90/0] square plate under sinusoidal load (Material 1).

z/h	a/h = 4	a/h = 5	a/h = 10	a/h = 20	a/h = 40	a/h = 50	a/h = 100
-0.500	-0.6378	-0.6052	-0.5624	-0.5543	-0.5527	-0.5525	-0.5523
-0.438	-0.4335	-0.4399	-0.4643	-0.4775	-0.4817	-0.4823	-0.4830
-0.375	-0.2823	-0.3121	-0.3774	-0.4038	-0.4115	-0.4124	-0.4137
-0.313	-0.1763	-0.2163	-0.3002	-0.3326	-0.3419	-0.3431	-0.3446
-0.250	-0.1077	-0.1470	-0.2311	-0.2636	-0.2729	-0.2740	-0.2756
-0.250	-0.0029	-0.0047	-0.0075	-0.0083	-0.0085	-0.0085	-0.0086
-0.188	0.0002	-0.0019	-0.0052	-0.0061	-0.0064	-0.0064	-0.0064
-0.125	0.0025	0.0003	-0.0030	-0.0040	-0.0042	-0.0042	-0.0043
-0.063	0.0044	0.0023	-0.0009	-0.0018	-0.0021	-0.0021	-0.0021
0.000	0.0061	0.0041	0.0011	0.0003	0.0001	0.0000	0.0000
0.000	0.0061	0.0041	0.0011	0.0003	0.0001	0.0000	0.0000
0.063	0.0078	0.0060	0.0032	0.0024	0.0022	0.0022	0.0022
0.125	0.0099	0.0080	0.0053	0.0045	0.0043	0.0043	0.0043
0.188	0.0126	0.0105	0.0075	0.0067	0.0065	0.0065	0.0064
0.250	0.0162	0.0135	0.0098	0.0089	0.0087	0.0086	0.0086
0.250	0.0482	0.1164	0.2274	0.2629	0.2728	0.2740	0.2756
0.313	0.1373	0.1956	0.2973	0.3320	0.3418	0.3430	0.3446
0.375	0.2683	0.3033	0.3753	0.4032	0.4114	0.4124	0.4137
0.438	0.4491	0.4453	0.4632	0.4771	0.4816	0.4822	0.4829
0.500	0.6876	0.6269	0.5626	0.5539	0.5526	0.5525	0.5523

Table 8: Through thickness variation of In-plane normal stress ($\bar{\sigma}_y$) for a simply supported symmetric cross ply 4 layered [0/90/90/0] square plate under sinusoidal load (Material 1).

z/h	a/h = 4	a/h = 5	a/h = 10	a/h = 20	a/h = 40	a/h = 50	a/h = 100
-0.500	-0.0434	-0.0362	-0.0233	-0.0188	-0.0176	-0.0174	-0.0172
-0.438	-0.0330	-0.0286	-0.0196	-0.0163	-0.0153	-0.0152	-0.0151
-0.375	-0.0243	-0.0220	-0.0162	-0.0138	-0.0131	-0.0130	-0.0129
-0.313	-0.0170	-0.0162	-0.0130	-0.0114	-0.0109	-0.0108	-0.0107
-0.250	-0.0109	-0.0112	-0.0100	-0.0090	-0.0087	-0.0087	-0.0086
-0.250	-0.5431	-0.5006	-0.3667	-0.3026	-0.2830	-0.2805	-0.2772
-0.188	-0.3893	-0.3642	-0.2728	-0.2264	-0.2121	-0.2103	-0.2079
-0.125	-0.2542	-0.2392	-0.1810	-0.1508	-0.1414	-0.1402	-0.1386
-0.063	-0.1311	-0.1214	-0.0908	-0.0755	-0.0707	-0.0701	-0.0693

z/h	a/h = 4	a/h = 5	a/h = 10	a/h = 20	a/h = 40	a/h = 50	a/h = 100
0.000	-0.0128	-0.0068	-0.0012	-0.0003	-0.0001	0.0000	0.0000
0.000	-0.0128	-0.0068	-0.0012	-0.0003	-0.0001	0.0000	0.0000
0.063	0.1077	0.1087	0.0884	0.0749	0.0706	0.0700	0.0693
0.125	0.2373	0.2292	0.1789	0.1503	0.1413	0.1401	0.1386
0.188	0.3830	0.3588	0.2709	0.2259	0.2120	0.2102	0.2079
0.250	0.5519	0.5016	0.3652	0.3021	0.2829	0.2805	0.2772
0.250	0.0255	0.0206	0.0123	0.0096	0.0088	0.0087	0.0086
0.313	0.0322	0.0259	0.0154	0.0120	0.0110	0.0109	0.0108
0.375	0.0403	0.0319	0.0186	0.0144	0.0133	0.0131	0.0129
0.438	0.0500	0.0389	0.0220	0.0169	0.0155	0.0153	0.0151
0.500	0.0614	0.0471	0.0257	0.0194	0.0177	0.0175	0.0172

Table 9: Through thickness variation of In-plane shear stress ($\bar{\tau}_{xy}$) for a simply supported symmetric cross ply 4 layered [0/90/90/0] square plate under sinusoidal load (Material 1).

z/h	a/h = 4	a/h = 5	a/h = 10	a/h = 20	a/h = 40	a/h = 50	a/h = 100
-0.500	0.0348	0.0296	0.0210	0.0183	0.0176	0.0175	0.0173
-0.438	0.0265	0.0233	0.0178	0.0159	0.0153	0.0153	0.0152
-0.375	0.0197	0.0181	0.0148	0.0135	0.0131	0.0131	0.0130
-0.313	0.0144	0.0138	0.0120	0.0112	0.0109	0.0109	0.0108
-0.250	0.0102	0.0102	0.0094	0.0089	0.0087	0.0087	0.0087
-0.250	0.0102	0.0102	0.0094	0.0089	0.0087	0.0087	0.0087
-0.188	0.0070	0.0072	0.0069	0.0066	0.0065	0.0065	0.0065
-0.125	0.0045	0.0047	0.0046	0.0044	0.0043	0.0043	0.0043
-0.063	0.0024	0.0025	0.0023	0.0022	0.0022	0.0022	0.0022
0.000	0.0006	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000
0.000	0.0006	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000
0.063	-0.0012	-0.0017	-0.0021	-0.0022	-0.0022	-0.0022	-0.0022
0.125	-0.0032	-0.0039	-0.0044	-0.0044	-0.0043	-0.0043	-0.0043
0.188	-0.0058	-0.0064	-0.0067	-0.0066	-0.0065	-0.0065	-0.0065
0.250	-0.0090	-0.0094	-0.0092	-0.0088	-0.0087	-0.0087	-0.0087
0.250	-0.0090	-0.0094	-0.0092	-0.0088	-0.0087	-0.0087	-0.0087
0.313	-0.0131	-0.0129	-0.0118	-0.0111	-0.0109	-0.0109	-0.0108
0.375	-0.0184	-0.0173	-0.0146	-0.0134	-0.0131	-0.0130	-0.0130

z/h	a/h = 4	a/h = 5	a/h = 10	a/h = 20	a/h = 40	a/h = 50	a/h = 100
0.438	-0.0251	-0.0225	-0.0176	-0.0158	-0.0153	-0.0152	-0.0152
0.500	-0.0334	-0.0287	-0.0209	-0.0183	-0.0175	-0.0175	-0.0173

Table 10: Through thickness variation of Transverse shear stress ($\bar{\tau}_{xz}$) for a simply supported symmetric cross ply 4 layered [0/90/90/0] square plate under sinusoidal load (Material 1).

z/h	a/h = 4	a/h = 5	a/h = 10	a/h = 20	a/h = 40	a/h = 50	a/h = 100
-0.500	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.438	0.1201	0.1215	0.1254	0.1267	0.1270	0.1270	0.1271
-0.375	0.2056	0.2126	0.2288	0.2350	0.2367	0.2369	0.2372
-0.313	0.2673	0.2810	0.3127	0.3255	0.3292	0.3297	0.3303
-0.250	0.3140	0.3331	0.3791	0.3988	0.4047	0.4054	0.4065
-0.250	0.3140	0.3331	0.3791	0.3988	0.4047	0.4054	0.4065
-0.188	0.3173	0.3363	0.3823	0.4021	0.4080	0.4088	0.4098
-0.125	0.3196	0.3386	0.3846	0.4045	0.4104	0.4112	0.4122
-0.063	0.3212	0.3399	0.3859	0.4059	0.4119	0.4126	0.4136
0.000	0.3221	0.3405	0.3862	0.4063	0.4123	0.4131	0.4141
0.000	0.3221	0.3405	0.3862	0.4063	0.4123	0.4131	0.4141
0.063	0.3223	0.3402	0.3856	0.4058	0.4118	0.4126	0.4136
0.125	0.3217	0.3391	0.3841	0.4043	0.4104	0.4111	0.4122
0.188	0.3199	0.3369	0.3816	0.4018	0.4080	0.4087	0.4098
0.250	0.3168	0.3335	0.3781	0.3984	0.4046	0.4054	0.4064
0.250	0.3168	0.3335	0.3781	0.3984	0.4046	0.4054	0.4064
0.313	0.3048	0.3017	0.3144	0.3254	0.3291	0.3296	0.3303
0.375	0.2592	0.2428	0.2319	0.2351	0.2367	0.2369	0.2372
0.438	0.1674	0.1485	0.1285	0.1269	0.1270	0.1271	0.1271
0.500	0.0151	0.0088	0.0014	0.0002	0.0001	0.0000	0.0000

Table 11: Through thickness variation of Transverse shear stress ($\bar{\tau}_{yz}$) for a simply supported symmetric cross ply 4 layered [0/90/90/0] square plate under sinusoidal load (Material 1).

z/h	a/h = 4	a/h = 5	a/h = 10	a/h = 20	a/h = 40	a/h = 50	a/h = 100
-0.500	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.438	0.0145	0.0128	0.0092	0.0078	0.0074	0.0073	0.0072
-0.375	0.0258	0.0230	0.0169	0.0144	0.0137	0.0136	0.0135

z/h	a/h = 4	a/h = 5	a/h = 10	a/h = 20	a/h = 40	a/h = 50	a/h = 100
-0.313	0.0344	0.0310	0.0232	0.0200	0.0191	0.0190	0.0188
-0.250	0.0408	0.0370	0.0283	0.0246	0.0235	0.0233	0.0231
-0.250	0.0408	0.0370	0.0283	0.0246	0.0235	0.0233	0.0231
-0.188	0.1484	0.1393	0.1062	0.0894	0.0842	0.0836	0.0827
-0.125	0.2270	0.2130	0.1617	0.1357	0.1276	0.1266	0.1252
-0.063	0.2785	0.2594	0.1952	0.1634	0.1537	0.1524	0.1508
0.000	0.3029	0.2787	0.2068	0.1727	0.1623	0.1610	0.1593
0.000	0.3029	0.2787	0.2068	0.1727	0.1623	0.1610	0.1593
0.063	0.2985	0.2701	0.1965	0.1636	0.1537	0.1525	0.1508
0.125	0.2620	0.2319	0.1641	0.1360	0.1277	0.1267	0.1253
0.188	0.1882	0.1613	0.1093	0.0900	0.0844	0.0837	0.0827
0.250	0.0703	0.0544	0.0315	0.0252	0.0236	0.0234	0.0232
0.250	0.0703	0.0544	0.0315	0.0252	0.0236	0.0234	0.0232
0.313	0.0617	0.0468	0.0260	0.0206	0.0192	0.0190	0.0188
0.375	0.0499	0.0369	0.0193	0.0149	0.0138	0.0137	0.0135
0.438	0.0343	0.0241	0.0111	0.0081	0.0074	0.0074	0.0072
0.500	0.0141	0.0081	0.0014	0.0002	0.0001	0.0000	0.0000

In the below section, from Figure 1 to Figure 8, through thickness variation of displacements and stresses for different a/h ratios,for a simply supported symmetric cross ply 4 layered **[0/90/90/0]** square plate under sinusoidal load **(Material 1).** are presented.

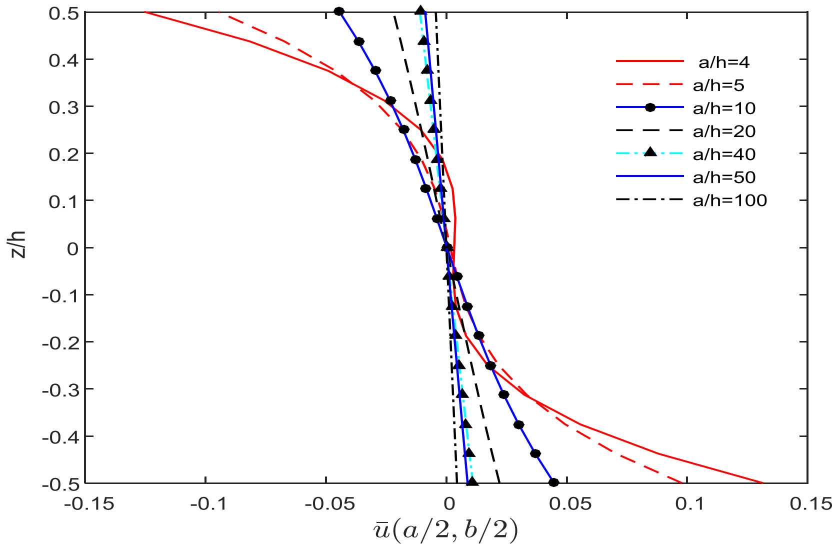


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Figure 1: Through thickness variation of In-plane displacement ($\overline{u}$) for a simply supported symmetric cross ply 4 layered [0/90/90/0] square plate subjected to sinusoidal load (Material 1)

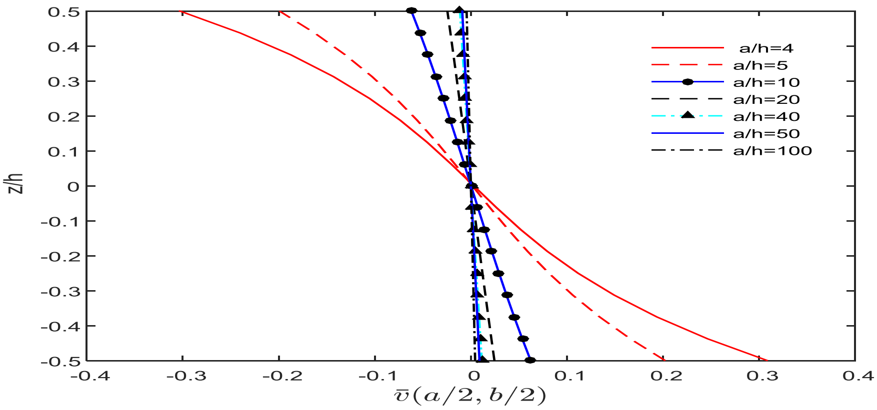

Figure 2: Through thickness variation of In-plane displacement ($\overline{v}$) for a simply supported symmetric cross ply 4 layered [0/90/90/0] square plate subjected to sinusoidal load (Material 1)

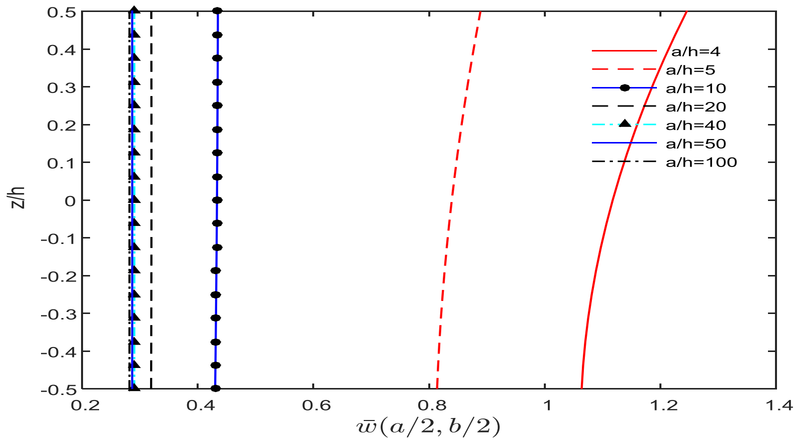

Figure 3: Through thickness variation of Transverse displacement ($\overline{w}$) for a simply supported symmetric cross ply 4 layered [0/90/90/0] square plate subjected to sinusoidal load (Material 1)

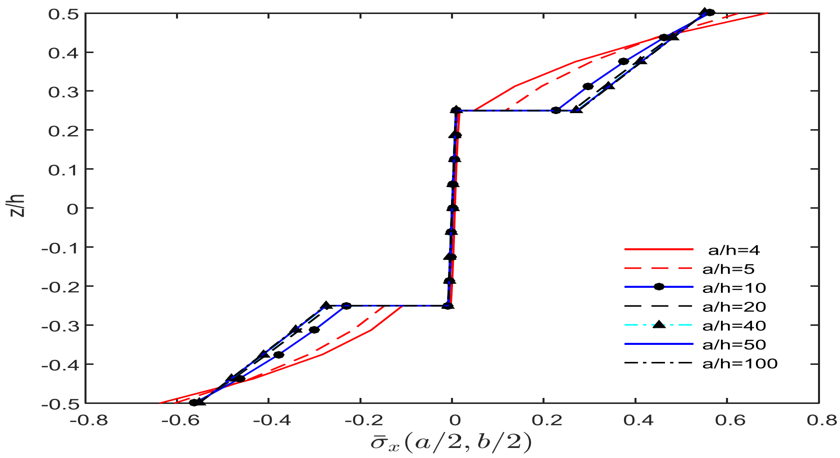

Figure 4: Through thickness variation of In-plane normal stress ($\bar{\sigma}_x$) for a simply supported symmetric cross ply 4 layered [0/90/90/0] square plate subjected to sinusoidal load (Material 1)

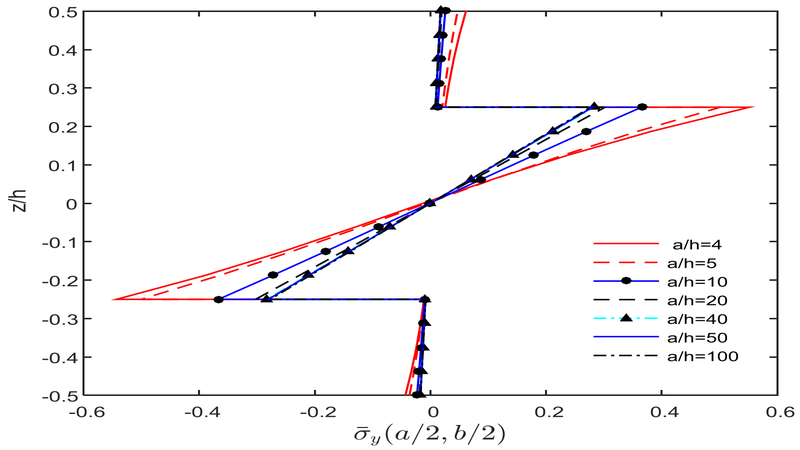

Figure 5: Through thickness variation of In-plane normal stress ($\bar{\sigma}_y$) for a simply supported symmetric cross ply 4 layered [0/90/90/0] square plate subjected to sinusoidal load (Material 1)

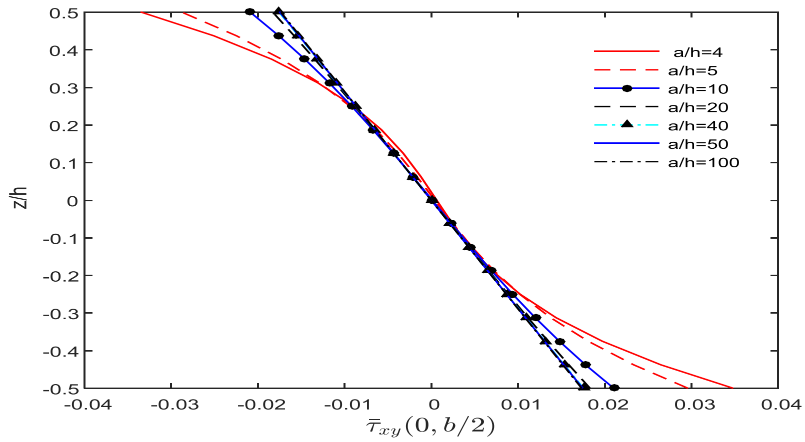

Figure 6: Through thickness variation of In-plane shear stress ($\bar{\tau}_{xy}$) for a simply supported symmetric cross ply 4 layered [0/90/90/0] square plate subjected to sinusoidal load (Material 1)

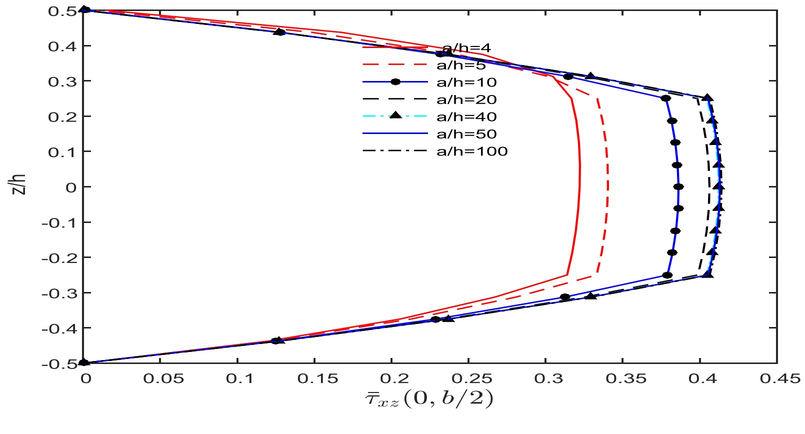

Figure 7: Through thickness variation of Transverse shear stress ($\bar{\tau}_{xz}$) for a simply supported symmetric cross ply 4 layered [0/90/90/0] square plate subjected to sinusoidal load (Material 1)

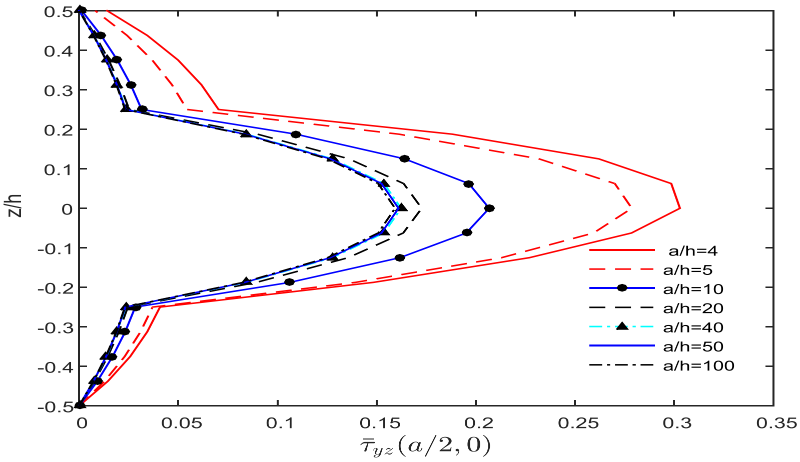

Figure 8: Through thickness variation of Transverse shear stress ($\bar{\tau}_{yz}$) for a simply supported symmetric cross ply 4 layered (0/90/90/0) square plate subjected to sinusoidal load (Material 1)

4. Conclusions

In the analysis of a simply supported symmetric cross ply 4 Layer [0/90/90/0] square plate subjected to a sinusoidal load and SS1 boundary conditions, the model relies on the Higher Order Shear Deformation Theory (HSDT) 12. The level of accuracy is noted to enhance as the ratio of the plate's length to its thickness (a/h) varies within the range of 5 to 40.These errors become negligible when the thickness ratio reaches or exceeds a/h $\geq$ 20.The deflection calculated from the present model continuously reduces as percentage values from 18.68% to 0.60% for material 1 and from 2.81% to 0.06% for material 2.This suggests that as the a/h ratio increases, the HSDT 12 model's accuracy improves, leading to diminished discrepancies and more exact deflection predictions. Precise deflection estimation is crucial for guaranteeing the structural stability and operational efficiency of the plates.

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