Tuijin Jishu/Journal of Propulsion Technology
ISSN: 1001-4055
Vol. 44 No. 2 (2023)

Root Cube Mean Cordial Labeling of Some Special Graphs

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Abstract -: *Let G =(V,E)be a graph and f be a mapping from V(G) □{ 0,1,2 }. For each edge uv of G assign the label⌊√((〖f(u)〗 ^3+ 〖f(v)〗 ^3)/2)⌋, ,f is called a root cube mean cordial labeling if |v_f (i)–v_f (j)|≤ 1 and |e_f (i)–e_f (j)|≤1, where v_f (x) and e_f (x) denote the number of vertices and edges labeled with x,x□{0,1,2} respectively. A graph with a root cube mean cordial labeling is called root cube mean cordial graph. In this paper, we investigate about the existence of root cube mean cordiality of some special graph such as Coconut tree graph.*

***Keywords**: Coconut Tree graph, Root Cube Mean Cordial Labeling, Root Cube Mean Cordial Graphs.*

I. Introduction

The graphs considered here are finite, undirected and simple. The vertex set and edge set of a graph G are denoted by V(G) and E(G) respectively. Labeled graphs are useful models for a broad range of applications such as coding theory, X- ray crystallography, astronomy, circuit design etc. The concept of cordial labeling was introduced by Cahit in the year 1987. After the introduction of cordial labeling various types of cordial labeling has been studied. Motivated by the works of many researchers in the area of cordial labeling, we introduced a new type of labeling called root cube mean cordial labeling. In this paper we have discussed about the cordiality of special graphs such as Coconut tree graph.

Definition:

Coconut Tree $CT\ (m,n)$ is the graph obtained from the path P_m by adding a star graph S_n at the end vertex of a path P_m .

Main Result:

Theorem:

The Coconut Tree graph $CT(m,n)$ admits root cube mean cordial labelling.

Proof:

Let G be a Coconut Tree graph, $CT(m,n)$ with vertex set $V(T_{m,n}) = \begin{cases} u_i\,, 1 \le i \le m, \\ v_j\,, 1 \le j \le n. \end{cases}$

and edge set $E(T_{m,n}) = \begin{cases} u_i u_{i+1} : 1 \le i \le m \\ u_n v_j : 1 \le j \le n \end{cases}$

Let l be the number of vertices and number of edges.

Define $f: V(G) \to \{0,1,2\}$ as follows.

Case (i):

$m = n$

Subcase (i):

$m = 3t, \ \ n = 3t$

$m \equiv 0 (\bmod 3), \ \ n \equiv 0 (\bmod 3)$

Let $l \equiv 2 (\bmod 3)$

ie, $l = 3s + 2$

Define $f(u_i) = 2, \ \ 1 \le i \le t$

$f(u_{t+i}) = 0, \ \ 1 \le i \le t$

$f(u_{2t+i}) = 1, \ \ 1 \le i \le t$

$f(v_i) = 0, \ \ 1 \le i \le t$

$f(v_{t+i}) = 1, \ \ 1 \le i \le t$

$f(v_{2t+i}) = 2, \ \ 1 \le i \le t$

Then $v_f(0) = s + 1, \ \ v_f(1) = s + 1, \ \ v_f(2) = s + 1$

$e_f(0) = s + 1, \ e_f(2) = s, \ \ e_f(3) = s + 1$

Clearly $|v_f(i) - v_f(j)| \le 1$ and $|e_f(i) - e_f(j)| \le 1 \ \ \forall \ \ i, j \in \{0,1,2\}$

Subcase (ii):

$m = 3t + 1, \ n = 3t + 1$

$m \equiv 1 (\bmod 3), \ n \equiv 1 (\bmod 3)$

Let $l \equiv 1 (\bmod 3)$

ie, $l = 3s + 1$

Define $f(u_i) = 2, \ \ 1 \le i \le t$

$f(u_{t+i}) = 0, \ \ 1 \le i \le t$

$f(u_{2t+i}) = 1, \ \ 1 \le i \le t + 1$

$f(v_i) = 0, \ \ 1 \le i \le t + 1$

$$f(v_{t+1+i}) = 1, \quad 1 \le i \le t$$

$$f(v_{2t+1+i}) = 2, \quad 1 \le i \le t$$

Then $v_f(0) = s+1, \quad v_f(1) = s+1, \quad v_f(2) = s$

$$e_f(0) = s+1, \; e_f(1) = s, \quad e_f(2) = s$$

Clearly $|v_f(i) - v_f(j)| \le 1$ and $|e_f(i) - e_f(j)| \le 1 \quad \forall \quad i, j \in \{0,1,2\}$

Subcase (iii):

$$m = 3t+2, \quad n = 3t+2$$

$$m \equiv 2(\text{mod}\,3), \quad n \equiv 2(\text{mod}\,3)$$

Let $l \equiv 0(\text{mod}\,3)$

ie, $l = 3s$

Define $f(u_i) = 2, \quad 1 \le i \le t$

$$f(u_{t+i}) = 0, \quad 1 \le i \le t+1$$

$$f(u_{2t+1+i}) = 1, \quad 1 \le i \le t+1$$

$$f(v_i) = 0, \quad 1 \le i \le t$$

$$f(v_{t+i}) = 1, \quad 1 \le i \le t+1$$

$$f(v_{2t+1+i}) = 2, \quad 1 \le i \le t+1$$

Then $v_f(0) = s, \quad v_f(1) = s+1, \quad v_f(2) = s$

$$e_f(0) = s, \; e_f(1) = s, \quad e_f(2) = s$$

Clearly $|v_f(i) - v_f(j)| \le 1$ and $|e_f(i) - e_f(j)| \le 1 \quad \forall \quad i, j \in \{0,1,2\}$

Case (ii):

$$m < n$$

Subcase (i):

$$m = 3t, \quad n = 3t+1$$

$$m \equiv 0(\text{mod}\,3), \quad n \equiv 1(\text{mod}\,3)$$

Let $l \equiv 0(\text{mod}\,3)$

ie, $l = 3s$

Define $f(u_i) = 2, \quad 1 \le i \le t$

$f(u_{t+i})=0,\ \ 1\le i\le t$

$f(u_{2t+i})=1,\ \ 1\le i\le t$

$f(v_i)=0,\ \ 1\le i\le t$

$f(v_{t+i})=1,\ \ 1\le i\le t+1$

$f(v_{2t+1+i})=2,\ \ 1\le i\le t$

Then $v_f(0)=s,\ \ v_f(1)=s+1,\ \ v_f(2)=s$

$e_f(0)=s,\ e_f(1)=s,\ \ e_f(2)=s$

Clearly $|v_f(i)-v_f(j)|\le 1$ and $|e_f(i)-e_f(j)|\le 1\ \ \forall\ \ i,j\in\{0,1,2\}$

Subcase (ii):

$m=3t+1,\ \ n=3t+2$

$m\equiv 1(\mathrm{mod}\,3),\ \ n\equiv 2(\mathrm{mod}\,3)$

Let $l\equiv 2(\mathrm{mod}\,3)$

ie, $l=3s+2$

Define $f(u_i)=2,\ \ 1\le i\le t$

$f(u_{t+i})=0,\ \ 1\le i\le t$

$f(u_{2t+i})=1,\ \ 1\le i\le t+1$

$f(v_i)=0,\ \ 1\le i\le t+1$

$f(v_{t+1+i})=1,\ \ 1\le i\le t$

$f(v_{2t+1+i})=2,\ \ 1\le i\le t+1$

Then $v_f(0)=s+1,\ \ v_f(1)=s+1,\ \ v_f(2)=s+1$

$e_f(0)=s+1,\ e_f(1)=s+1,\ \ e_f(2)=s$

Clearly $|v_f(i)-v_f(j)|\le 1$ and $|e_f(i)-e_f(j)|\le 1\ \ \forall\ \ i,j\in\{0,1,2\}$

Subcase (iii):

$m=3t,\ \ n=3t+2$

$m\equiv 0(\mathrm{mod}\,3),\ \ n\equiv 2(\mathrm{mod}\,3)$

Let $l\equiv 1(\mathrm{mod}\,3)$

ie, $l = 3s+1$

Define $f(u_i) = 2, \ \ 1 \le i \le t$

$f(u_{t+i}) = 0, \ \ 1 \le i \le t$

$f(u_{2t+i}) = 1, \ \ 1 \le i \le t$

$f(v_i) = 0, \ \ 1 \le i \le t$

$f(v_{t+i}) = 1, \ \ 1 \le i \le t+1$

$f(v_{2t+1+i}) = 2, \ \ 1 \le i \le t+1$

Then $v_f(0) = s, \ \ v_f(1) = s+1, \ \ v_f(2) = s+1$

$e_f(0) = s, \ e_f(1) = s, \ \ e_f(2) = s+1$

Clearly $|v_f(i) - v_f(j)| \le 1$ and $|e_f(i) - e_f(j)| \le 1 \ \ \forall \ \ i, j \in \{0,1,2\}$

Case (iii):

$m > n$

Subcase (i):

$m = 3t+1, \ \ n = 3t$

$m \equiv 1 (\bmod 3), \ \ n \equiv 0 (\bmod 3)$

Let $l \equiv 0 (\bmod 3)$

ie, $l = 3t$

Define $f(u_i) = 2, \ \ 1 \le i \le t$

$f(u_{t+i}) = 0, \ \ 1 \le i \le t$

$f(u_{2t+i}) = 1, \ \ 1 \le i \le t+1$

$f(v_i) = 0, \ \ 1 \le i \le t$

$f(v_{t+i}) = 1, \ \ 1 \le i \le t$

$f(v_{2t+i}) = 2, \ \ 1 \le i \le t$

Then $v_f(0) = s, \ \ v_f(1) = s+1, \ \ v_f(2) = s$

$e_f(0) = s, \ e_f(1) = s, \ \ e_f(2) = s$

Clearly $|v_f(i) - v_f(j)| \le 1$ and $|e_f(i) - e_f(j)| \le 1 \ \ \forall \ \ i, j \in \{0,1,2\}$

Subcase (ii):

$m = 3t + 2, \;\; n = 3t + 1$

$m \equiv 2(\bmod 3), \;\; n \equiv 1(\bmod 3)$

Let $l \equiv 2(\bmod 3)$

ie, $l = 3s + 2$

Define $f(u_i) = 2, \;\; 1 \le i \le t$

$f(u_{t+i}) = 0, \;\; 1 \le i \le t+1$

$f(u_{2t+1+i}) = 1, \;\; 1 \le i \le t+1$

$f(v_i) = 0, \;\; 1 \le i \le t$

$f(v_{t+i}) = 1, \;\; 1 \le i \le t$

$f(v_{2t+i}) = 2, \;\; 1 \le i \le t+1$

Then $v_f(0) = s+1, \;\; v_f(1) = s+1, \;\; v_f(2) = s+1$

$e_f(0) = s+1, \;\; e_f(1) = s, \;\; e_f(2) = s+1$

Clearly $|v_f(i) - v_f(j)| \le 1$ and $|e_f(i) - e_f(j)| \le 1 \;\; \forall \;\; i, j \in \{0,1,2\}$

Subcase (iii):

$m = 3t + 2, \;\; n = 3t$

$m \equiv 2(\bmod 3), \;\; n \equiv 0(\bmod 3)$

Let $l \equiv 1(\bmod 3)$

ie, $l = 3s + 1$

Define $f(u_i) = 2, \;\; 1 \le i \le t$

$f(u_{t+i}) = 0, \;\; 1 \le i \le t+1$

$f(u_{2t+1+i}) = 1, \;\; 1 \le i \le t+1$

$f(v_i) = 0, \;\; 1 \le i \le t$

$f(v_{t+i}) = 1, \;\; 1 \le i \le t$

$f(v_{2t+i}) = 2, \;\; 1 \le i \le t$

Then $v_f(0) = s+1, \;\; v_f(1) = s+1, \;\; v_f(2) = s$

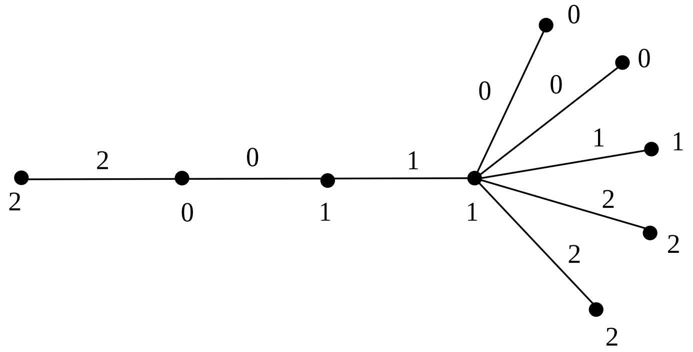
$e_f(0) = s+1, \;\; e_f(1) = s, \;\; e_f(2) = s$

Clearly $\left|v_f(i)-v_f(j)\right|\le 1$ and $\left|e_f(i)-e_f(j)\right|\le 1 \ \forall \ i,j\in\{0,1,2\}$

From all the above cases, the graph Coconut Tree CT (m, n) admit root cube mean cordial labelling.

Example:

Consider the graph $CT\ (4,5)$



Here

$v_f(0)=3$

$v_f(1)=3,\ \ v_f(2)=3$

$e_f(0)=3,\ e_f(1)=2,\ \ e_f(2)=3$

Clearly $\left|v_f(i)-v_f(j)\right|\le 1$ and $\left|e_f(i)-e_f(j)\right|\le 1 \ \forall \ i,j\in\{0,1,2\}$

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