Tuijin Jishu/Journal of Propulsion Technology
ISSN: 1001-4055
Vol. 44 No. 4 (2023)

ξ-Semi-Continuous Maps on ξ-topological spaces

[1]Arvind Kumar Sharma, [2]Mudassir Ahmad, [3]Mohammad Javed Alam, [4]Nazir Ahmad Ahengar, [5]Sanjay Bhajanker

[1][4]Department of Mathematics, School of Engineering, Pimpri Chinchwad University, Pune-412106, India
[2]Department of Mathematics, School of Chemical Engineering and Physical Sciences, Lovely Professional University Punjab-144411, India
[3]Department of Mathematics, School of Engineering, Presidency University Bangaluru-560064, India
[5]Department of Physics, Govt. Agrasen College Bilha, Bilaspur, C.G. – 495224 India

Email: [1]arvind02bhanu@gmail.com, [2]mdabstract85@gmail.com, [3]mohd.javedalam@presidencyuniversity.in, [4]nzrhmd97@gmail.com, [5]sanjaybhajanker@hotmail.com

Abstract. In this paper the concepts of ξ-semi-continuous maps in ξ-topological spaces are introduced and all the possible relationships of these maps have been discussed and established by making the use of some counter examples.

Keywords: ξ-continuous maps, ξ-semi-continuous maps, totally ξ-continuous maps, strongly ξ-continuous maps, strongly

1. Introduction

Continuity is most important concept in Mathematics and many different generalized forms of continuity have been studied and investigated. Levine [15] introduced weakly continuous functions and established some new results. Further, Son et al. [22] introduced weakly clopen and almost clopen functions. These authors [22] investigate that almost clopen functions are the generalized forms of perfectly continuous functions, regular set-connected functions and clopen functions. Chen et al. [6] demonstrated the dynamics on binary relations over topological spaces. The authors Arya,S. P., Gupta,R Anuradha, Baby Chacko and Singh D [2-3,21] introduced the concept of strongly continuous functions and almost perfectly continuous functions in topological spaces and established the various significant results. Benchalli S.S and Umadevi I Neeli Nour T.M [4, 20] studied the concept of totally semi-continuous functions and semi-totally continuous functions in topological spaces and verify the certain properties of the concept.

Nithyanantha and Thangavelu [19] introduced the concept of binary topology between two sets and investigate some of the basic properties, where a binary topology from X to Y is a binary structure satisfying certain axioms that are analogous to the axioms of topology. Jamal M. Mustafa [13] studied binary generalized topological spaces and investigate the various relationships of the maps so discussed with some other maps.

In this paper we study the concepts of generalized binary semi-continuous maps (ξ-semi-continuous maps), totally generalized binary continuous maps (totally ξ-continuous maps), totally generalized binary semi-continuous maps (totally ξ-semi-continuous maps), strongly generalized binary continuous maps (strongly ξ-continuous maps), strongly generalized binary semi-continuous maps (strongly ξ-semi-continuous maps) in generalized binary topological spaces (ξ-topological spaces).

The concepts of ξ-topological space ($\xi_T S$) have been discussed in section 2. In section 3, the concept of ξ-semi-continuous maps, totally ξ-continuous maps, totally ξ-semi-continuous maps, strongly ξ-continuous maps and strongly ξ-semi-continuous maps in ξ-topological spaces have been introduced and established the relationships. Throughout the paper $\wp(\Upsilon)$ denotes the power set of Υ.

2. Preliminaries

Definition 2.1: Let Υ_1 and Υ_2 be any two non-void sets. Then ξ-topology (ξ_T) from Υ_1 to Υ_2 is a binary structure $\xi \subseteq \wp(\Upsilon_1) \times \wp(\Upsilon_2)$ satisfying the conditions i.e. $(\emptyset, \emptyset)$, $(\Upsilon_1, \Upsilon_2) \in \xi$ and If $\{(L_\alpha, M_\alpha) ; \alpha \in \Gamma\}$ is a family of elements of ξ, then $(\bigcup_{\alpha\in\Gamma} L_\alpha, \bigcup_{\alpha\in\Gamma} M_\alpha) \in \xi$. If ξ is ξ_T from Υ_1 to Υ_2, then $(\Upsilon_1, \Upsilon_2, \xi)$ is called a ξ-topological space ($\xi_T S$) and the elements of ξ are called the ξ-open subsets of $(\Upsilon_1, \Upsilon_2, \xi)$. The elements of $\Upsilon_1 \times \Upsilon_2$ are called simply ξ-points.

Definition 2.2: Let Υ_1 and Υ_2 be any two non-void set and (L_1, M_1), (L_2, M_2) be the elements of $\wp(\Upsilon_1) \times \wp(\Upsilon_2)$. Then $(L_1, M_1) \subseteq (L_2, M_2)$ only if $L_1 \subseteq L_2$ and $M_1 \subseteq M_2$.

Remark 2.1: Let $\{T_\alpha ; \alpha \in \Lambda\}$ be the family of ξ_T from Υ_1 to Υ_2. Then, $\bigcap_{\alpha\in\Lambda} T_\alpha$ is also ξ_T from Υ_1 to Υ_2. Further $\bigcup_{\alpha\in\Lambda} T_\alpha$ need not be ξ_T.

Definition 2.3: Let $(\Upsilon_1, \Upsilon_2, \xi)$ be a $\xi_T S$ and $L \subseteq \Upsilon_1, M \subseteq \Upsilon_2$. Then (L, M) is called ξ-closed in $(\Upsilon_1, \Upsilon_2, \xi)$ if $(\Upsilon_1 \backslash L, \Upsilon_2 \backslash M) \in \xi$.

Proposition 2.1: Let$(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$. Then (Υ_1, Υ_2) and $(\emptyset, \emptyset)$ are ξ-closed sets. Similarly if $\{(L_\alpha, M_\alpha): \alpha \in \Gamma\}$ is a family of ξ-closed sets, then $(\bigcap_{\alpha\in\Gamma} L_\alpha, \bigcap_{\alpha\in\Gamma} M_\alpha)$ is ξ-closed.

Definition 2.4: Let$(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$ and $(L, M) \subseteq (\Upsilon_1, \Upsilon_2)$. Let ${(L, M)^{1^*}}_\xi = \bigcap\{L_\alpha : (L_\alpha, M_\alpha)$ is ξ-closed set and $(L, M) \subseteq (L_\alpha, M_\alpha)\}$ and ${(L, M)^{2^*}}_\xi = \bigcap\{M_\alpha : (L_\alpha, M_\alpha)$ is ξ-closed set and $(L, M) \subseteq (L_\alpha, M_\alpha)\}$. Then $({(L, M)^{1^*}}_\xi, {(L, M)^{2^*}}_\xi)$ is ξ-closed set and $(L, M) \subseteq ({(L, M)^{1^*}}_\xi, {(L, M)^{2^*}}_\xi)$. The ordered pair $\left({(L, M)^{1^*}}_\xi, {(L, M)^{2^*}}_\xi\right)$ is called ξ-closure of (L, M) and is denoted $Cl_\xi(L, M)$ in $\xi_T S$ (X, Y, μ) where $(L, M) \subseteq (\Upsilon_1, \Upsilon_2)$.

Proposition 2.2: Let$(L, M) \subseteq (\Upsilon_1, \Upsilon_2)$. Then (L, M) is ξ-open in $(\Upsilon_1, \Upsilon_2, \xi)$ iff $(L, M) = I_\xi(L, M)$ and (L, M) is ξ-closed in $(\Upsilon_1, \Upsilon_2, \xi)$ iff $(L, M) = Cl_\xi(L, M)$.

Proposition 2.3: Let $(L, M) \subseteq (N, P) \subseteq (\Upsilon_1, \Upsilon_2)$ and $(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$. Then $Cl_\xi(\emptyset, \emptyset) = (\emptyset, \emptyset)$, $Cl_\xi(\Upsilon_1, \Upsilon_2) = (\Upsilon_1, \Upsilon_2)$, $(L, M) \subseteq Cl_\xi(L, M)$, ${(L, M)^{1^*}}_\xi \subseteq {(N, P)^{1^*}}_\xi$, ${(L, M)^{2^*}}_\xi) \subseteq {(N, P)^{2^*}}_\xi$, $Cl_\xi(L, M) \subseteq Cl_\xi(N, P)$ and $Cl_\xi(Cl_\xi(L, M)) = Cl_\xi(L, M)$.

Definition 2.5: Let$(\Upsilon_1, \Upsilon_2, \xi)$ be $\xi_T S$ and $(L, M) \subseteq (\Upsilon_1, \Upsilon_2)$. Let ${(L, M)^{1^0}}_\xi = \bigcup\{L_\alpha : (L_\alpha, M_\alpha)$is ξ-open set and $(L, M) \subseteq (L_\alpha, M_\alpha)\}$ and ${(L, M)^{2^0}}_\xi = \bigcup\{M_\alpha : (L_\alpha, M_\alpha)$ is ξ-open set and $(L, M) \subseteq (L_\alpha, M_\alpha)\}$. Then $({(L, M)^{1^0}}_\xi, {(L, M)^{2^0}}_\xi)$ is ξ-open set and $({(L, M)^{1^0}}_\xi, {(L, M)^{2^0}}_\xi) \subseteq (L, M)$. The ordered pair $\left({(L, M)^{1^0}}_\xi, {(L, M)^{2^0}}_\xi\right)$ is called ξ-interior of (L, M) and is denoted $I_\xi(L, M)$ in $\xi_T S$ $(\Upsilon_1, \Upsilon_2, \xi)$ where $(L, M) \subseteq (\Upsilon_1, \Upsilon_2)$.

Proposition 2.4: Let $(L, M) \subseteq (\Upsilon_1, \Upsilon_2)$. Then (L, M) is ξ-open set in $(\Upsilon_1, \Upsilon_2, \xi)$ iff $(L, M) = I_\xi(L, M)$.

Proposition 2.5: Let $(L, M) \subseteq (N, P) \subseteq (\Upsilon_1, \Upsilon_2)$ and $(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$. Then $I_\xi(\emptyset, \emptyset) = (\emptyset, \emptyset)$, $I_\xi(\Upsilon_1, \Upsilon_2) = (\Upsilon_1, \Upsilon_2)$, ${(L, M)^{1^0}}_\xi \subseteq {(N, P)^{1^0}}_\xi$, ${(L, M)^{2^0}}_\xi \subseteq {(N, P)^{2^0}}_\xi$, $I_\xi(L, M) \subseteq I_\xi(N, P)$ and $I_\xi(I_\xi(L, M)) = I_\xi(L, M)$

Definition 2.6: Let $(\Upsilon_1, \Upsilon_2, \xi)$ be ξ-topological space ($\xi_T S$) and $(Z, \mathcal{T})$ be generalized topological space ($G_T S$). Then the map $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is called ξ-continuous at $z \in Z$ if for any ξ-open set $(L, M) \in (\Upsilon_1, \Upsilon_2, \xi)$ with $\mathcal{F}(z) \in (L, M)$ then there exists $\mathcal{T}$-open G in $(Z, \mathcal{T})$ such that $z \in G$ and $\mathcal{F}(G) \subseteq (L, M)$. The mapping $\mathcal{F}$ is called ξ-continuous if it is ξ-continuous at each $z \in Z$.

Proposition 2.6: Let $(\Upsilon_1, \Upsilon_2, \xi)$ be $\xi_T S$ and $(Z, \mathcal{T})$ be $G_T S$. Then the map $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is called ξ-continuous map (ξCM) if $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$-open in $(Z, \mathcal{T})$ for every ξ-open set (L, M) in $(\Upsilon_1, \Upsilon_2, \xi)$.

3. ξ-Semi-Continuous Maps (ξSCM)

Definition 3.1: Let $(\Upsilon_1, \Upsilon_2, \xi)$ be $\xi_T S$. Then $(L, M) \subseteq (\Upsilon_1, \Upsilon_2, \xi)$ is said to ξ-semi-open set (ξSOS) if there exists ξ-open set (P, M) such that $(P, M) \subseteq (L, M) \subseteq Cl_\xi((L, M))$ or equivalently $(L, M) \subseteq Cl_\xi(I_\xi(L, M))$. The complement of ξ-semi-open set is ξ-semi-closed set denoted as (ξCOS).

Definition 3.2: Let $(\Upsilon_1, \Upsilon_2, \xi)$ be $\xi_T S$ and $(Z, \mathcal{T})$ be $G_T S$. Then the map $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is called ξ-semi-continuous map (ξSCM) if $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$-semi-open in $(Z, \mathcal{T})$ for every ξ-open set (L, M) in $(\Upsilon_1, \Upsilon_2, \xi)$.

Example 3.1: Let $Z = \{1, 2, 3\}$, $\Upsilon_1 = \{m_1, m_2\}$ and $\Upsilon_2 = \{l_1, l_2\}$. Then $\mathcal{T} = \{\emptyset, \{1\}, \{1,2\}, \{2,3\}, Z\}$ and $\xi = \{(\emptyset, \emptyset), (\{m_1\}, \{l_1\}), (\{m_1\}, \{\Upsilon_2\}), (\{m_2\}, \{\Upsilon_2\}), (\Upsilon_1, \Upsilon_2)\}$. Clearly $\mathcal{T}$ is G_T on Z and ξ is ξ_T from Υ_1 to Υ_2. Now define $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ by $\mathcal{F}(1) = (m_1, l_1)$ and $\mathcal{F}(2) = \mathcal{F}(3) = (m_2, l_2)$. Therefore $\mathcal{F}^{-1}(\emptyset, \emptyset) = \emptyset$, $\mathcal{F}^{-1}(\{m_1\}, \{l_1\}) = \{1\}$, $\mathcal{F}^{-1}(\{m_1\}, \{\Upsilon_2\}) = \{1\}$, $\mathcal{F}^{-1}(\{m_2\}, \{\Upsilon_2\}) = \{2, 3\}$ and $\mathcal{F}^{-1}(\Upsilon_1, \Upsilon_2) = Z$. This shows that the inverse image of every ξ-open set in $(\Upsilon_1, \Upsilon_2, \xi)$ is $\mathcal{T}$-semi-open in $(Z, \mathcal{T})$. Hence $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is ξSCM

Proposition 3.1: Every ξCM in ξ_TS is ξSCM

Proof: Let $(\Upsilon_1, \Upsilon_2, \xi)$ be ξ_TS and $(Z, \mathcal{T})$ be G_TS and the map $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is ξCM. Therefore $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$-open in $(Z, \mathcal{T})$ for every ξ-open set (L, M) in $(\Upsilon_1, \Upsilon_2, \xi)$. Since every $\mathcal{T}$-open set is $\mathcal{T}$-semi-open set in $(Z, \mathcal{T})$. Hence $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$-semi-open in $(Z, \mathcal{T})$ for every ξ-open set (L, M) in $(\Upsilon_1, \Upsilon_2, \xi)$. Thus $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is ξSCM.

Remark 3.1: The converse of Proposition 3.1 need not be true shown in Example 3.2.

Example 3.2: Let $Z = \{1, 2, 3, 4\}$, $\Upsilon_1 = \{m_1, m_2\}$ and $\Upsilon_2 = \{l_1, l_2\}$. Then $\mathcal{T} = \{\emptyset, \{3\}, \{3, 4\}, \{1, 2, 4\}, Z\}$ and $\xi = \{(\emptyset, \emptyset), (\{m_1\}, \{l_1\}), (\{m_1\}, \{\Upsilon_2\}), (\{m_2\}, \{\Upsilon_2\}), (\Upsilon_1, \Upsilon_2)\}$. Clearly $\mathcal{T}$ is G_T on Z and ξ is ξ_T from Υ_1 to Υ_2. Now define $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ by $\mathcal{F}(1) = (m_1, l_1)$ and $\mathcal{F}(2) = \mathcal{F}(3) = \mathcal{F}(4) = (m_2, l_2)$. Therefore $\mathcal{F}^{-1}(\emptyset, \emptyset) = \emptyset$, $\mathcal{F}^{-1}(\{m_1\}, \{l_1\}) = \{1\}$, $\mathcal{F}^{-1}(\{m_1\}, \{\Upsilon_2\}) = \{1\}$, $\mathcal{F}^{-1}(\{m_2\}, \{\Upsilon_2\}) = \{2, 3, 4\}$ and $\mathcal{F}^{-1}(\Upsilon_1, \Upsilon_2) = Z$. This shows that the inverse image of every ξ-open set in $(\Upsilon_1, \Upsilon_2, \xi)$ is $\mathcal{T}$-semi-open in $(Z, \mathcal{T})$. Hence $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is ξSCMbut not ξCM, because $\mathcal{F}^{-1}(\{m_2\}, \{l_2\}) = \{2, 3, 4\}$, where $\{2, 3, 4\}$ is $\mathcal{T}$-semi-open set but not $\mathcal{T}$-open set in $(Z, \mathcal{T})$.

Definition 3.3: Let $(\Upsilon_1, \Upsilon_2, \xi)$ be ξ_TS and $(Z, \mathcal{T})$ be G_TS. Then the map $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is called totally ξ-continuous map (TξCM) if $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$-clopen in $(Z, \mathcal{T})$ for every ξ-open set (L, M) in $(\Upsilon_1, \Upsilon_2, \xi)$.

Example 3.3: In Example 3.1, the $\mathcal{T}$-clopen sets in (Z, τ) are $\emptyset, \{1\}, \{2,3\}, Z$. Therefore $\mathcal{F}^{-1}(\emptyset, \emptyset) = \emptyset$, $\mathcal{F}^{-1}(\{m_1\}, \{l_1\}) = \{1\}$, $\mathcal{F}^{-1}(\{m_1\}, \{\Upsilon_2\}) = \{1\}$, $\mathcal{F}^{-1}(\{m_2\}, \{\Upsilon_2\}) = \{2, 3\}$ and $\mathcal{F}^{-1}(\Upsilon_1, \Upsilon_2) = Z$. This shows that the inverse image of every ξ-open set in $(\Upsilon_1, \Upsilon_2, \xi)$ is $\mathcal{T}$-clopen in $(Z, \mathcal{T})$. Hence $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is TξCM

Definition 3.4: Let $(\Upsilon_1, \Upsilon_2, \xi)$ be ξ_TS and $(Z, \mathcal{T})$ be G_TS. Then the map $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is called totally ξ-semi-continuous map (TξCM) if $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$-semi-clopen in $(Z, \mathcal{T})$ for every ξ-open set (L, M) in $(\Upsilon_1, \Upsilon_2, \xi)$.

Example 3.4: In Example 3.1, the $\mathcal{T}$-semi-clopen sets in (Z, τ) are $\emptyset, \{1\}, \{2,3\}, Z$. This shows that the inverse image of every ξ-open set in $(\Upsilon_1, \Upsilon_2, \xi)$ is $\mathcal{T}$-semi-clopen in $(Z, \mathcal{T})$. Hence $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is TξSCM

Proposition 3.2: Every TξCM in ξ_TS is TξSCM

Proof: Let $(\Upsilon_1, \Upsilon_2, \xi)$ be ξ_TS and $(Z, \mathcal{T})$ be G_TS and the map $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is TξCM. Therefore $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$-clopen in $(Z, \mathcal{T})$ for every ξ-open set (L, M) in $(\Upsilon_1, \Upsilon_2, \xi)$. Since every $\mathcal{T}$-clopen set is $\mathcal{T}$-semi-clopen set in $(Z, \mathcal{T})$. Hence $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$-semi-clopen in $(Z, \mathcal{T})$ for every ξ-open set (L, M) in $(\Upsilon_1, \Upsilon_2, \xi)$.. Thus $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is TξSCM.

Remark 3.2: The converse of Proposition 3.2 need not be true shown in Example 3.5.

Example 3.5: Let $Z = \{1, 2, 3, 4\}$, $\Upsilon_1 = \{m_1, m_2\}$ and $\Upsilon_2 = \{l_1, l_2\}$. Then $\mathcal{T} = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}, \{1,3,4\}, \{1,2,4\}\ Z\}$ and $\xi = \{(\emptyset, \emptyset), (\{m_1\}, \{l_1\}), (\{m_1\}, \{\Upsilon_2\}), (\{m_2\}, \{\Upsilon_2\}), (\Upsilon_1, \Upsilon_2)\}$. Clearly $\mathcal{T}$ is G_T on Z and ξ is ξ_T from Υ_1 to Υ_2. Now define $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ by $\mathcal{F}(1) = (m_1, l_1)$ and $\mathcal{F}(2) = \mathcal{F}(3) = \mathcal{F}(4) = (m_2, l_2)$. Therefore $\mathcal{F}^{-1}(\emptyset, \emptyset) = \emptyset$, $\mathcal{F}^{-1}(\{m_1\}, \{l_1\}) = \{1\}$, $\mathcal{F}^{-1}(\{m_1\}, \{\Upsilon_2\}) = \{1\}$, $\mathcal{F}^{-1}(\{m_2\}, \{\Upsilon_2\}) = \{2, 3, 4\}$ and $\mathcal{F}^{-1}(\Upsilon_1, \Upsilon_2) = Z$. This shows that the inverse image of every ξ-open set in $(\Upsilon_1, \Upsilon_2, \xi)$ is $\mathcal{T}$-semi-clopen in $(Z, \mathcal{T})$. Hence $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is TξSCMbut not TξCM, because $\mathcal{F}^{-1}(\{m_2\}, \{l_2\}) = \{2, 3, 4\}$, where $\{2, 3, 4\}$ is $\mathcal{T}$-semi-clopen set but not $\mathcal{T}$-clopen set in $(Z, \mathcal{T})$.

Definition 3.5: Let $(\Upsilon_1, \Upsilon_2, \xi)$ be ξ_TS and $(Z, \mathcal{T})$ be G_TS. Then the map $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is called strongly ξ-continuous map (SξCM) if $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$-clopen in $(Z, \mathcal{T})$ for every ξ-set (L, M) in $(\Upsilon_1, \Upsilon_2, \xi)$.

Example 3.6: In Example 3.1, the $\mathcal{T}$-clopen sets in $(Z, \mathcal{T})$ are $\emptyset, \{1\}, \{2,3\}, Z$. Therefore $\mathcal{F}^{-1}(\emptyset, \emptyset) = \emptyset$, $\mathcal{F}^{-1}(\{m_1\}, \{l_1\}) = \{1\}$, $\mathcal{F}^{-1}(\{m_1\}, \{\Upsilon_2\}) = \{1\}$, $\mathcal{F}^{-1}(\{m_2\}, \{\Upsilon_2\}) = \{2,3\}$, $\mathcal{F}^{-1}(\{\emptyset\}, \{l_1\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{\emptyset\}, \{l_2\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{\emptyset\}, \{\Upsilon\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{m_1\}, \{\emptyset\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{m_1\}, \{l_2\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{m_1\}, \{\Upsilon_2\}) = \{1\}$, $\mathcal{F}^{-1}(\{m_2\}, \emptyset) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{m_2\}, \{l_1\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{m_2\}, \{l_2\}) = \{2,3\}$,

$\mathcal{F}^{-1}(\{\Upsilon_1\},\{\emptyset\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{\Upsilon_1\},\{l_1\}) = \{1\}$, $\mathcal{F}^{-1}(\{\Upsilon_1\},\{l_2\}) = \{2,3\}$ and $\mathcal{F}^{-1}(\Upsilon_1,\Upsilon_2) = Z$. This shows that the inverse image of every ξ-set (L, M) in $(\Upsilon_1,\Upsilon_2,\xi)$ is $\mathcal{T}$-clopen in $(Z,\mathcal{T})$. Hence f is SξCM.

Definition 3.6: Let $(\Upsilon_1,\Upsilon_2,\xi)$ be ξ_TS and $(Z,\mathcal{T})$ be G_TS . Then the map $\mathcal{F}:(Z,\mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is called strongly ξ-semi-continuous map (SξSCM) if $\mathcal{F}^{-1}(L,M)$ is $\mathcal{T}$-semi-clopen in $(Z,\mathcal{T})$ for every ξ-set (L, M) in $(\Upsilon_1,\Upsilon_2,\xi)$.

Example 3.7: In Example 3.6 $\emptyset$, $\{1\}$, $\{2,3\}$, Z are $\mathcal{T}$-clopen sets in $(Z,\mathcal{T})$ and the inverse image of every ξ-set (L, M) in $(\Upsilon_1,\Upsilon_2,\xi)$ is $\mathcal{T}$-semi-clopen in $(Z,\mathcal{T})$. Hence f is SξSCM

Proposition 3.3: Every SξCM in ξ_TS is SξSCM

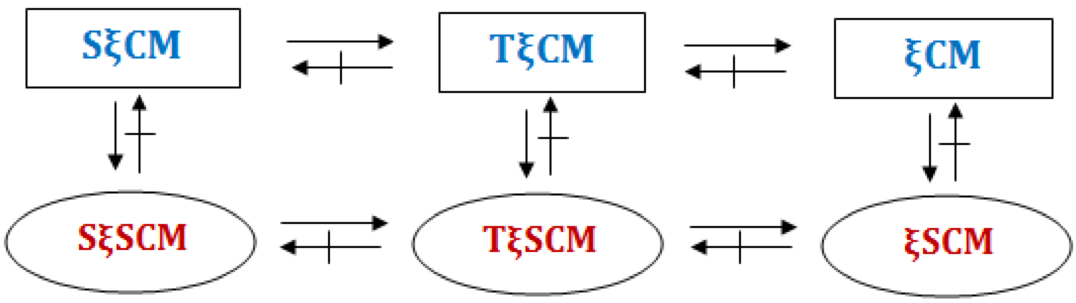
Proof: Let $(\Upsilon_1,\Upsilon_2,\xi)$ is ξ_TS and $(Z,\mathcal{T})$ be G_TS and the map $\mathcal{F}:(Z,\mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is SξCM. Therefore $\mathcal{F}^{-1}(L,M)$ is $\mathcal{T}$-clopen in $(Z,\mathcal{T})$ for every ξ-set (L, M) in $(\Upsilon_1,\Upsilon_2,\xi)$. Since every $\mathcal{T}$-clopen set is $\mathcal{T}$-semi-clopen set in $(Z,\mathcal{T})$. Hence $\mathcal{F}^{-1}(L,M)$ is $\mathcal{T}$-semi-clopen in $(Z,\mathcal{T})$ for every ξ-set (L, M) in $(\Upsilon_1,\Upsilon_2,\xi)$.. Thus $\mathcal{F}:(Z,\mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is SξSCM.

Remark 3.3: The converse of Proposition 3.3 need not be true shown in Example 3.8

Example 3.8: Let $Z = \{1,2,3,4\}$, $\Upsilon_1 = \{m_1, m_2\}$ and $\Upsilon_2 = \{l_1, l_2\}$. Then $\mathcal{T} = \{\emptyset,\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\},\{1,3,4\},\{1,2,4\},Z\}$ and $\xi = \{(\emptyset,\emptyset),$ $(\{m_1\},\{l_1\}),(\{m_1\},\{\Upsilon_2\}),(\{m_2\},\{\Upsilon_2\}),$ $(\Upsilon_1,\Upsilon_2)\}$. Clearly $\mathcal{T}$ is G_T on Z and ξ is ξ_T from Υ_1 to Υ_2. Now define $\mathcal{F}:(Z,\mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ by $\mathcal{F}(1) = (m_1, l_1)$and $\mathcal{F}(2) = \mathcal{F}(3) = \mathcal{F}(4) = (m_2, l_2)$. Therefore $\mathcal{T}$-semi clopen sets in $(Z,\mathcal{T})$ are $\emptyset,\{1\},\{2\},\{3\},$ $\{1,2,4\},\{1,3,4\},\{2,3,4\},Z$. Now $\mathcal{F}^{-1}(\emptyset,\emptyset) = \emptyset$, $\mathcal{F}^{-1}(\{m_1\},\{l_1\}) = \{1\}$, $\mathcal{F}^{-1}(\{m_1\},\{\Upsilon_2\}) = \{1\}$, $\mathcal{F}^{-1}(\{m_2\},\{\Upsilon_2\}) = \{2,3,4\},$ $\mathcal{F}^{-1}(\{\emptyset\},\{l_1\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{\emptyset\},\{l_2\}) = \{\emptyset\}, \mathcal{F}^{-1}(\{\emptyset\},\{\Upsilon_2\}) = \{\emptyset\},$ $\mathcal{F}^{-1}(\{m_1\},\{\emptyset\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{m_1\},\{l_2\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{m_1\},\{\Upsilon_2\}) = \{1\}$, $\mathcal{F}^{-1}(\{m_2\},\emptyset) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{m_2\},\{l_1\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{m_2\},\{l_2\}) = \{2,3,4\}$, $\mathcal{F}^{-1}(\{\Upsilon_1\},\{\emptyset\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{\Upsilon_1\},\{l_1\}) = \{1\}, \mathcal{F}^{-1}(\{\Upsilon_1\},\{l_2\}) = \{2,3,4\}$ and $\mathcal{F}^{-1}(\Upsilon_1,\Upsilon_2) = Z$. This shows that the inverse image of every ξ-set in $(\Upsilon_1,\Upsilon_2,\xi)$ is $\mathcal{T}$-semi clopen sets in $(Z,\mathcal{T})$. Hence $\mathcal{F}:(Z,\mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is SξSCM but not SξCM because $\{2,3,4\}$ is $\mathcal{T}$-semi-clopen sets in $(Z,\mathcal{T})$ but not $\mathcal{T}$-clopen sets in $(Z,\mathcal{T})$.

4. Conclusion

In this paper, a very useful concept of the concept of ξ-semi-continuous maps, totally ξ-continuous maps, totally ξ-semi-continuous maps, strongly ξ-continuous maps, strongly ξ-semi-continuous maps, semi-totally ξ-open maps and pre-totally ξ-open maps in ξ_TS have been introduced and established the relationships between these maps and some other maps by making the use of some counter examples. Conclusion is illustrated in the following figure



References

[1] Ahengar N.A. and J.K. Maitra, On g-binary continuity, Journal of Emerging Technologies and Inovative Research, 7, 240-244, (2018).

[2] Arya,S. P. and Gupta,R. On strongly continuous functions, Kyungpook Math. J., 14, 131-143, (1974).

[3] Anuradha N. and Baby Chacko, Some Properties of Almost Perfectly Continuous Functions in Topological Spaces, International Mathematical Forum 10(3), 143-156 (2015).

[4] Benchalli **S.S.** and Umadevi I Neeli "Semi-Totally Continuous Functions in Topological Spaces" International Mathematical Forum 6(10), 479-492, (2011).

[5] Bhattacharya,S, On Generalized Regular Closed Sets ,Int . J. Contemp. Math. Sciences, 6 (3) 145-152 (2011).
[6] Chen, C.C., Conejero, J.A., Kostic, M., Murillo-Arcila., M., Dynamics on Binary Relations over Topological Spaces. Symmetry 2018, 10: 211. https://doi.org/10.3390/sym10060211
[7] Csaszar, A. Generalized topology, generalized continuity, Acta Math. Hungar, 96, 351-357 (2002).
[8] Devi, R., Balachandran K., Maki, H. Semi-generalized closed maps and generalized semi-closed maps, Mem. Fac. Sci. Kochi Univ. Ser. A. Math, 14, 41-54 (1993).
[9] Egenhofer, MJ. Reasoning about binary topological relations. Symposium on Spatial Databases SSD 1991: Advances in Spatial Databases, 141-160, (1991).
[10] Engelking R. Generel Topology, Polish Scientific Publishers, Warszawa (1977).
[11] Gevorgyan, PS. Groups of binary operations and binary G-spaces. Topology and its Applications, 201, 18–28, (2016).
[12] Hatir E, Noiri T. Decompositions of continuity and complete continuity. Acta Math Hungary , 4, 281–287, (2006).
[13] Jamal M. Mustafa, On Binary Generalized Topological Spaces, Refaad General Letters in Mathematics, 2(3), 111-116 (2017).
[14] Kuratowski, K., Topologie I, Warszawa, (1930).
[15] Levine N. A decomposition of continuity in topological spaces. Am Math Mon, 68, 44–6, (1961).
[16] Levine, N. Semi open sets and semi continuity in topological spaces, Amer. Math. Monthly, 70, 36-41, (1963).
[17] Levine, N. Generalized closed sets in Topology, Rend. Cir. Mat. Palermo, 2, 89-96, (1970).
[18] Njastad, O, On some classes of nearly open sets, Pacific J. Math, 15, 961–970, (1965).
[19] NithyananthaJothi S., and P. Thangavelu, Topology between two sets, Journal of Mathematical Sciences & Computer Applications, 1(3), 95-107 (2011)
[20] Nour T.M, Totally semi-continuous functions, Indian J. Pure Appl.Math, 26(7), 675 – 678 (1995).
[21] Singh D., Almost Perfectly continuous functions, Quaest Math , 33, 1-11 (2010).
[22] Son MJ, Park JH, Lim KM. Weakly clopen functions. Chaos, Solitons& Fractals, 33, 1746–55, (2007).