A Novel two-dimensional DOA Estimation Algorithm for Low Elevation Targets

[1]Sanjeev Kumar Machkuri, [2]Y Ravi Kumar

[1] Dept. of ECE, Mahatma Gandhi Institute of Technology, Hyderabad, Telangana, India
[2]Scientist-G(Retd), DLRL DRDO, Hyderabad, Telangana, India.

E-mail; [1]msanjeevkumar_ece@mgit.ac.in,[2]dr.ravikumaryeda@gmail.com

Abstract: This paper proposes a fast Two-Dimensional DOA Estimation algorithm for low-elevation targets of Very High Frequency (VHF) Array Radar using an estimation method based on the Alternating Direction Method of Multipliers (ADMM). The method firstly utilizes the uncoupling characteristics of azimuth and pitch angle under a uniform planar array, to convert the two-dimensional angle estimation problem into two one-dimensional DOA estimation problems. The target information is extracted by digital beamforming in the azimuth and elevation dimensions. Finally, the Alternating Direction Method of Multipliers is used for estimation of the azimuth and pitch angle. This method avoids the two-dimensional joint estimation, which is complex in the calculation, the complexity is greatly reduced, and the operation process does not require Eigenvalue decomposition, further improving the operation efficiency.
Keywords: DOA estimation; low elevation target; VHF array radar; ADMM algorithm.

1. Introduction

With the continuous improvement of application requirements, modern meter wave radar should have high-precision ranging and two dimensional angle measurement capability. However, when the meter wave radar detects low-altitude and ultra-low altitude targets, the radar receives the echo signal includes not only the direct wave signal scattered and by the target but also there are ground reflected multipath signals. The presence of multipath signals will lead to unreliable DOA, hence the algorithm's performance degrades or even falls.
The main reasons can be summarized as follows:

- Separating the direct and multipath waves from time and frequency domain is difficult as they are usually located in the same distance unit.
- Due to considerably wider beam width, the reflected multipath and the direct signals are a group of strongly correlated waves, and are present in the same beam width or even in the half the beam width, which seriously affects the accurate measurement of angel of arrival in meter wave radars.
- Due to narrow bandwidth of meter-wave radar, the distance unit is generally in the order of 100 meters, and the distance measurement is accurate. The degree of measurement further affects the radar measurement performance.

A lot of research has been done on the problem in recent years, many scholars at home and abroad have made a DOA Estimation on low elevation angle DOA. The methods are mainly divided into feature subspace algorithms, maximum likelihood (ML) class algorithms and compressed sensing class algorithms. The low-elevation feature subspace class algorithm is mainly based on (MUSIC) Multiple Signal Classification and the rotation-invariant subspace (Estimation of Signal Parameter via Rotational Invariance Technique (ESPRIT), the solution method for the frame. Since MUSIC algorithm compared with ESPRIT, the former algorithm has higher stability and angular resolution, hence it is favored by developers [1, 2]. Literature [3] using spatial smoothing (SS) technique restores the rank of the covariance matrix to achieve solution coherence, but the absence of effective aperture will lead to the weakening of algorithm performance during estimation, which made it difficult for this type of algorithm to measure up the standard.

The practical application requirements of meter wave radar, in literature [4] alternate projections technology and MUSIC Algorithms can be combined, to use prior information for low elevation angle estimation. But due to its cost function number is a non-convex optimization problem and one cannot guarantee that the algorithm will converge to global optimal solution. ML kind of algorithm can directly deal with coherent signals, and also the algorithm has better estimation performance subject to the condition of low signal-to-noise ratio, but the algorithm complexity increases at an exponential rate to number of targets, and the amount of calculation is huge, which cannot be fully meet real-time needs [5]. Compressed sensing algorithms use the few target characteristics in the spatial domain to directly analyze the sources with the same phase for DOA estimate. And most sparsely reconstructed classes DOA Estimation method, has reasonable estimation performance in the context of low signal-to-noise ratio and few snapshots [6], but currently the sparsely reconstructed class DOA Estimation algorithms usually have a large amount of computation.

The problem of convex optimization with separable structure can be processed in blocks to reduce the solution complexity by using Alternating Direction Method of Multipliers (ADMM). Due to its high estimation accuracy and fast convergence, it is of frequent use in signal processing, image processing, machine learning and other fields [7].

ADMM Algorithm is proposed in this article for DOA estimation of two dimensional array, a suitable technique for meter wave surface array radar. The algorithm first begins with target angle, which is roughly estimated by dimensional beamforming, and then the azimuth and elevation angles row and column beamforming is performed on the array data respectively, which are not coupled. And Fourier Extract the target data by leaf interpolation, and finally use ADMM algorithm performs azimuth and pitch angle estimation.

Compared with traditional algorithms, this proposed method uses the rough estimation data of the angle to limit range of target degrees. Reduces solution computational complexity, through the row and column beamforming processing, on the other hand improving the signal-to-noise ratio, data dimensionality reduction is achieved. Therefore avoiding the need for two dimensional combined estimation of complex computations, improves computational efficiency. Simulation result shows efficacy of proposed algorithm.

2. Signal Model

As shown in the Figure 1, matrix of array elements are $M \times N$. The uniform area array is placed in Y-Z axis, the adjacent array elements are separated by the distance of $d = \lambda/2$, λ represents the wavelength. Under far-field conditions, the target echo arriving at each array element can be considered as the plane wave, and a target is well-defined as a projection on X-Y Plane.

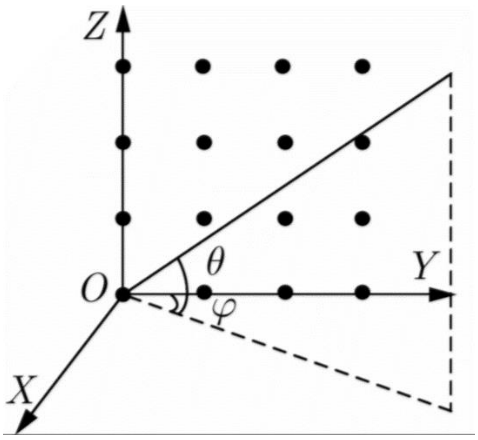

Fig 1: Geometric Model of Meter Wave Area Array Radar.

The azimuth angle φ, is a angle in positive direction of Y, the angle target making at origin with X-Y plane is the pitch angle θ. The lattice element is the reference array element, and each array element transmits narrow-band chirp signal as

$$s(t) = g(t) exp(j2\pi f_c t); \quad 0 \le t \le T ---(1)$$

Where $g(t)$ represents the complex envelope of a signal, f_c is the center frequency, T Indicates the pulse width. For the convenience of analysis, it is assumed to be only a single target in the space, and noise is additive noise, each array element is received fundamental frequency of echo at time can be represented as

$$s(m,n,t) = \sigma g\left(t - \frac{R(m,n)}{c}\right) \exp\left(-j2\pi \frac{R(m,n)}{\lambda}\right) + \sigma\rho g\left(t - \frac{R\prime(m,n)}{c}\right) \cdot \exp\left(-j2\pi \frac{R\prime(m,n)}{\lambda}\right) + w(m,n,t) - -(2)$$

Where σ is the target backscattering coefficient, ρ indicates the multipath echo specular reflection coefficient. $R(m,n)$ represents the target to the m row n array element distance, $m \in [1,M]$, $n \in [1,N]$. $R'(m,n)$ indicates multipath echo that reaches each array element distance. W (m, n, t) means the first m row n additive white Gaussian noise of array elements.

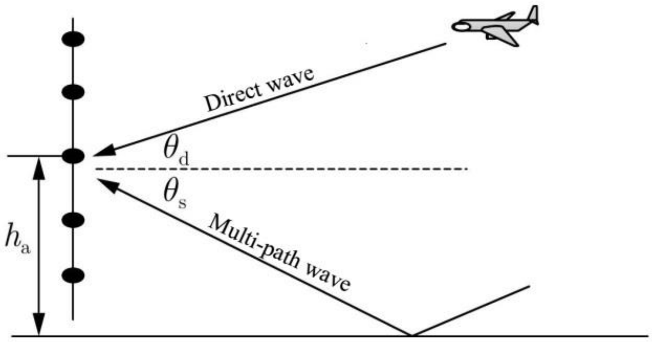

Fig 2: Ideal Multipath Propagation Model

According to the assumption of far-field conditions, let the distance between the target and the reference array element be R, $R(m,n)$ expressed as $R(m,n) = R + y_n cos\theta cos\varphi + z_m sin\theta$, in y_n means the first n array element on Y axis position, z_m is the first m row element on the Z axis position. Many studies have modeled multipath models under ideal terrain conditions, such as [1–3], the ideal plane position reflection model is as shown in figure 2. Multipath and direct wave distances there difference can be stated as $\Delta R = R'(m,n) - R(m,n) \approx 2h_a \sin\theta_d$, where h_a indicates the height of the center of array, elevation angle is the multipath direct wave θ_d with many radial elevation θ_s satisfy $\theta_s = -$ arc sin (sin(θ_d) + $2h_a$/R), when the standard distance R≫h_a hour, $\theta_d \approx -\theta_s$. Assuming the radar is elevated 10 m, as shown in the figure 3 relationship between difference of angle between multipath reflection-angle and angle of incidence of direct wave versus target distance.

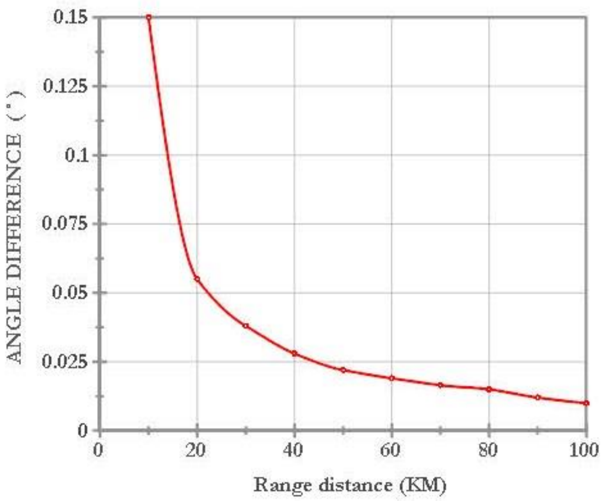

Fig 3: Relationship between angle difference and target distance

From the figure it is evident that target distance, the farther you reach, the difference between the absolute value of multipath incident angle and the direct wave incident angle will be smaller, ignoring the effect of delay on the envelope, provided that the signal is narrowband, we can write the equation as.

$$g\left(t - \frac{R(m,n)}{c}\right) \approx g\left(t - \frac{R'(m,n)}{c}\right) = \mathrm{g(t)} ----(3)$$

Using the equation (3), (2) it can be converted and organized as

$$s\ (m,n,t) = \sigma' \left(exp\ \left(-\mathrm{j}\frac{2}{\lambda}(y_c\cos\theta_d\cos\phi + z_m\ \sin\theta_d) \right) + \ \rho'\exp\ \left(-\mathrm{j}\frac{2}{\lambda}(y_c\cos\theta_s\cos\phi + z_m\ \sin\theta_s) \right) \right).\mathrm{g(t)} + \ \mathrm{w(m,n,t)} - -(4)$$

Where $\sigma' = \sigma \exp(-\mathrm{j}2\pi fcR/c)$, $\rho' = \rho \exp(-\mathrm{j}2\pi\Delta R/\lambda)$, are the multipath coefficients.

Pulse processing is performed on the data of each array element. Metadata beamforming processing and target detection to find targets rough angle. First, beamforming is performed on the row array elements, so that the first one row array Meta data given as $S_{refN}\ (t) = [S(1,1,t), S(1,2,t),\ldots, S(1,N,t)]$, the reference row array metadata. Use the formula (5) for realizing of beam forming of reference line array element.

$$S_\varphi\ (t) = \ w_a{}^H\ (\varphi)\ S_{refN}\ (t) - - - - - - - (5)$$

$$w_a\ (\varphi) = [\exp\ (-\mathrm{j}2\pi y_1\ \cos\ \varphi), \exp\ (-\mathrm{j}2\pi y_2\ \cos\ \varphi), \ldots , \exp\ (\text{-}y_n \cos\ \varphi)\ ]^H$$

Represents the azimuth weighting vector of row array element. Secondly, the target detection is done on the azimuth beam synthesis result, and let us definite H_1 indicates the target exists hypothesis, H_0 indicates that the target does not exist hypothesis, azimuth beamforming signal $S_\varphi\ (t)$ exist H_1, H_0 Conditional Probabilistic Density.

The degree function $\boldsymbol{p}\ (S_\varphi\ (t)\ jH_1)\ \boldsymbol{p}\ (S_\varphi\ (t)\ jH_0)$ assuming noise observe zero mean, variance is σ^2 The Gaussian distribution of, the likelihood ratio function is defined as

$$\Lambda\ (S_\varphi\ (t)) = \frac{p(S_\varphi\ (t)\ |H_1)}{p(S_\varphi\ (t)\ |H_0)} = \frac{\exp\ (-(S_\varphi\ (t) - s(t))^2/(2\sigma^2))}{\exp\ (-(S_\varphi\ (t))^2/(2\sigma^2))} - - - - \ (6)$$

In above formula $s(t)$ means that the assumption H_1 is the source output value at the time. It can be seen from the above that the larger the likelihood ratio, the more likely the target exists in the signal. The literature [8] gives a binary hypothesis judgment method for radar target detection.

$$\Lambda\ (\mathrm{t})\ \underset{H_0}{\overset{H_1}{\gtrless}}\ \delta \quad - - - - - - - \ (7)$$

Where δ indicates the detection threshold, which may be adjusted according to actual false alarm probability. When the likelihood ratio is greater than the threshold value, it is determined that there is the target, then record the current azimuth beamforming angle φ, roughly estimated azimuth angle of that target, record the distance unit related to threshold value R_D. Use the azimuth to roughly estimate the weighted vector related to angle $w_a\ (\varphi_1)$ to beam form each row array element data, and extract the beam formed distance of each row array element unit R_D, the corresponding data is used as the initial data for the elevation beam synthesis, using the formula (8) beam synthesis to get pitch.

$$\mathrm{S}_\Theta\left(\mathrm{t}_{\mathrm{R_D}}\right) = \ \mathrm{w_a}{}^{\mathrm{H}}\ (\Theta)\ \mathrm{S_{DBF}}\left(\mathrm{t}_{\mathrm{R_D}}\right)\ \text{---------}\ (8)$$

Where, $w_p\ (\Theta) = [\exp\ (-\mathrm{j}2\pi z_1\ \sin\ \Theta), \exp\ (-\mathrm{j}2\pi z_2\ \sin\ \Theta), \ldots , \exp\ (\text{-j}2\pi z_m \sin\ \Theta)\ ]^T$, represents the array element pitch weighted vector, $\mathrm{S_{DBF}}\left(\mathrm{t}_{\mathrm{R_D}}\right)$ indicates the corresponding distance of each row array element after azimuth beam synthesis unit R_D data. In the same way as the detection method after azimuth beam synthesis, the target detection is performed on the elevation synthesis data, and the rough estimation of the angle of elevation is recorded angle Θ_1. Due to small bandwidth of meter wave radar, the distance unit is generally in the order of 100 meters, and direct wave multipath signal after pulse compression are located in the same location. Distance unit, to extract target information more accurately, use coarse estimated angle versus received data two dimensional beam synthesis and interpolation processing to record the distance unit corresponding to the target position R_Q.

Under the uniform area array model, the azimuth and pitch angles can be independently make an estimate. First use the azimuth angle φ_1 perform azimuth beam synthesis on each row array element, and extract the distance unit after interpolation processing R_Q data. At the time, each row array outputs data $y_r = [\mathrm{S_{1r}}\ (R_Q);\ \mathrm{S_{2r}}\ (R_Q), .., \mathrm{S_{Mr}}\ (R_Q)]^T$. . , where $\mathrm{S_{ir}}\ (R_Q)$ means the first line element azimuth wave bundled and interpolated distance units R_Q data. Use pitch angle Θ_1 and distance unit R_Q get the output data of each column array $y_c = [\mathrm{S_{1c}}\left(R_Q\right);\ \mathrm{S_{2c}}\left(R_Q\right), .., \mathrm{S_{Mc}}\left(R_Q\right)]^T$.., where $\mathrm{S_{ic}}\ (R_Q)$ express the first j array element pitch beam formed and interpolated range unit R_Q data, target data y_r, y_c written in vector form, it is represented as

$$y_r = \ (a(\Theta_d) \ + \ \rho' a(\Theta_s))\ \mathrm{S_r} \ + \ w$$
$$y_c = \ (a(\varphi_d) \ + \ \rho' a(\varphi_s))\ \mathrm{S_c} \ + \ w \qquad - - - - - - \ (9)$$

$a(\Theta_d) = \exp(-j2\pi z_M \sin\Theta_d/\boldsymbol{\lambda})$, $a(\Theta_s) = \exp(-j2\pi z_M \sin\Theta_s/\boldsymbol{\lambda})$ represent the multipath wave and direct signal pitch reception respectively, whereas steering vector,
$z_M = [0, d, 2\,d, \ldots, (M-1)d]$. $b(\varphi_d) = exp\,(-j2\pi\Upsilon_N cos\,(\varphi_d)/\lambda), b(\varphi_s) = exp\,(-j2\pi\Upsilon_N cos(\varphi_s)/\lambda)$ represents the vector direct wave and multipath signal azimuth receiving guidance, respectively, where, $\Upsilon_N = [0, d, 2d, .., (N-1)d]$. S_r represents each row array element azimuth beam synthesis target corresponds to the envelope vector, S_c represents the envelope vector corresponding to each array element pitch beamforming target, w indicates the corresponding noise vector after array synthesis.
Using $\varphi_d = \varphi_s$, in the formula (5), output signal is expanded to an over complete representation of the spatial angle

$$y_r = (a(\Theta'_d) + \rho' a(\Theta'_s))\, S'_r + w$$
$$y_c = ((1-\rho')b(\varphi'_d))\, S'_c + w \quad ------(10)$$

Where,$\Theta'_d = \Theta_{init} + [0;\ 1;\ \ldots;\ L-1]\Delta\theta$, $\varphi'_d = \varphi_{init} + [0; 1;\ \ldots;\ P-1]\Delta\varphi$, are Corresponding complete space pitch and azimuth angles, $\Theta_{init} = \Theta_1 - \Theta_{3dB}/2$, $\varphi_{init} = \varphi_1 - \varphi_{3dB}/2$, represents the initial pitch angle and azimuth set respectively , Θ_{3dB}, φ_{3dB} indicates 3 dB beam width pitch and azimuth respectively. $\Delta\theta = \Theta_{3dB}/L$, $\Delta\varphi = \varphi_{3dB}/P$, divide the interval for the pitch and azimuth angle sets, that is, the pitch and azimuth angles complete set could be estimated in accordance with rough angle and pitch angle, elevation and azimuth beam widths are determined.

Steering vector $a(\Theta'_d)$ with Θ'_d corresponds with each angle element is direct wave pitch angle steering composed of columns vector matrix, using $\Theta'_d = -\Theta'_s$ the multi-path steering vector-matrix can be obtained, $a(\Theta'_s), b(\varphi'_d)$ means with φ'_d steering vector related with each angle element is the direct wave azimuth steering vector matrix consists of columns . S'_r, S'_c represents the zero padding of incident wave in the total set of pitch and azimuth angles respectively. From the perspective of sparse recovery, the formula (10) used for solving the target azimuth and problem of pitch could be converted into an optimization problem that minimizes the target function.

$$\arg\underset{.S'_r}{min} \left\|y_r - \left(a(\Theta'_d) + \rho' a(\Theta'_s)\right)S'_r\right\|_2^2 + \eta.\, q(S'_r)$$
$$\arg\underset{.S'_c}{min} \left\|y_c - \left((1+\rho') + b(\varphi'_d)\right)S'_c\right\|_2^2 + \eta.\, q(S'_c) \quad ----(11)$$

In the above equations, $q(\cdot)$ represents the sparse constraint function, η represents the regularization parameter, and defines the sparse constraint function as $(S'_r) = \|S'_r\|_1$, $q(S'_c) = \|S'_r\|_1$, the spatial angle sparsity is limited by this function, and the objective function can be expressed as

$$\arg\underset{.S'_r}{min} \left\|y_r - \left(a(\Theta'_d) + \rho' a(\Theta'_s)\right)S'_r\right\|_2^2 + \eta.\, \|S'_r\|_1$$
$$\arg\underset{.S'_c}{min} \left\|y_c - \left((1+\rho') + b(\varphi'_d)\right)S'_c\right\|_2^2 + \eta.\, q\|S'_r\|_1 \quad ----(12)$$

After the sparse constraint function is determined, solution can be obtained by sparse recovery algorithm to get the target azimuth and pitch angle

3. Proposed Algorithm for Angle of Arrival estimation

3.1 Alternating Direction Method of Multipliers Algorithm structure

Alternating Direction Method of Multipliers Algorithm, a method suitable for solving distributed optimization problems. Computational methods, the most notable advantage of which is the ability to separate variables process, and make full use of the separable structure to solve objective function. The convergence speed is fast, and it is easy to implement in engineering (12). The objective function can be seen as LASSO class sparse reconstruction problem, the ADMM algorithm applied to LASSO in the class optimization sparse problem, its running speed and reconstruction better accuracy can be achieved than other mainstream $L1$ norm method [9-11]. According to the formula (12) one can observe that the solution of the objective function can be regarded as an problem of optimization for angle variables. ADMM is generally form of solving two-variable optimization problems with equality constraints can be represented by

$$\underset{x,z}{min} f(x) + g(z) = c$$

$$s.t\ \boldsymbol{A}x + \boldsymbol{B}z = \boldsymbol{C} \quad --------- (13)$$

Where ,$x \in R^n$, $z \in R^m$ for the variable to be optimized, $f(x) + g(z)$ The objective function to be optimized number, and $f(x)$, $g(z)$ is a convex function, linear under constraints $A \in R^{pxn}$, $B \in R^{pxm}$, $C \in R^p$. Solving optimization problem is described, Lagrangian function is constructed and the augmented, its expression is given by

$$L_\tau(x, z, \gamma) = f(x) + g(z) + (Ax + Bz - c) + \frac{\tau}{2} \| Ax + Bz - c \| \frac{2}{2} --- (14)$$

Where γ represents the Lagrange multiplier, τ represents the penalty term coefficient. According to ADMM the solution idea is to fix the other two variables when solving any variable, and make use alternate iteration method to update the parameters until convergence, the solution process is as given by (15)

$$x^k = \arg \min_x f(x) + \frac{\tau}{2} \| Ax + Bz^{k-1} - c + \zeta^{k-1} \| \frac{2}{2}$$
$$z^k = \arg \min_z g(x) + \frac{\tau}{2} \| Ax + Bz^{k-1} - c + \zeta^{k-1} \| \frac{2}{2}$$
$$\zeta^k = \zeta^{k-1} + Ax^k + Bz^k - c \quad ---------(15)$$

Where $\zeta = \frac{\gamma}{\tau}$, $k = 1,2, \ldots$

3.2 DoA Algorithm implementation

The above ADMM Algorithmic ideas are extended to one dimension DOA estimation, it is estimated with pitch angle DOA, for example, at this time $x = S'_r$ resents the angle vector to be unraveled, that is the corresponding ADMM Algorithmic solution form, introduces auxiliary vector β, then the constraints are satisfied $\beta = x$, using the corresponding formula (12), the solution form expresses objective function as

$$\min_{x, \beta} \frac{1}{2} \|y - Ax\|_2^2 + \eta. \|\beta\|_1$$
$$\text{s.t}\ \beta - x = 0 \quad ----(12)$$

where, $y = y_r$ is the array observation data, in terms of signal model, the specific form of the pitch angle estimation observation can be represented as

$$A = \exp\left(-j\frac{2\pi}{\lambda} (z_m \sin(\theta'_d)\right) + \rho' \exp\left(-j\frac{2\pi}{\lambda}(z_m \sin(\theta'_s))\right)$$

Similarly, for azimuth DOA estimated, $y = y_c, x = S'_c$, the azimuth estimation observation may be expressed by $A = \exp(-j\frac{2\pi}{\lambda} (Y_m \sin(\varphi'_d)$. Now after this point, construct the augmented Lagrangian function

$$L_\tau(x, z, \gamma) = \frac{1}{2} \| y - Ax \| \frac{2}{2} + \eta. \|\beta\|_1 + \gamma^T(\beta - x) + \frac{\tau}{2} \| \beta - x \| \frac{2}{2} --- (17)$$

According to ADMM algorithm the idea, at the beginning solve the variable x. Because L_τ is derivable, $\frac{\partial L_\tau}{\partial x} = 0$, available

$$x = (A^H A + \tau I)^{-1} (A^H y + \tau\beta - \gamma) = (A^H A + \tau I)^{-1} \left(A^H y + \tau(\beta - \zeta)\right) -- (18)$$

Where *I* represents the identity matrix, $= \frac{\gamma}{\tau}$, in this second pair beta can be solved

$$\arg \min_\beta L_\tau(x, z, \gamma) = \arg \min_\beta \left(\eta. \|\beta\|_1 + \gamma^T(\beta - x) + \frac{\tau}{2} \| \beta - x \| \frac{2}{2}\right) X \ \frac{2}{\tau}$$
$$= \arg \min_\beta \left(\frac{2\eta}{\tau} \|\beta\|_1 - \| \frac{\gamma}{\tau} \| \frac{2}{2} + \| \frac{\gamma}{\tau} + \beta - x \| \frac{2}{2}\right)$$
$$= \arg \min_\beta \left(\frac{2\eta}{\tau} \|\beta\|_1 + \| \beta - (\frac{\gamma}{\tau} + x) \| \frac{2}{2}\right)$$
$$= \Omega_{\frac{\gamma}{\tau}} \left(\frac{\gamma}{\tau} + x\right) = \Omega_{\frac{\gamma}{\tau}} (\zeta + x) ----- (19)$$

Where $\Omega_{\frac{\gamma}{\tau}}(\zeta + x)$ represents soft threshold operator, finally by using equation (15) we get ζ analytical formula.

One dimensional DoA estimation parameter update strategy can be stated through ADMM algorithm. This can be represented as follows,

$$X^k = (A^H A + \tau I)^{-1} \left(A^H y + \tau(\beta^{k-i} - \zeta^{k-i})\right)$$
$$\beta^k = \Omega_{\frac{\gamma}{\tau}} (\zeta^{k-i} + X^k)$$

$$\zeta^k = \zeta^{k-i} + X^k - \beta^k \ ------ (20)$$

The target angle estimated value e Θ could be get from above formula, as stated in signal model, using the formula (19). Attenuation coefficient 'ρ' can be solved, use ρ update observation list A, the target angle estimated value is given by continuously updating the solution until the algorithm converges to Θ. Azimuth angle estimation is the same as the above process, so it will not be described in detail as,

$$\rho' = (y_r - \exp(-j2\pi \ (z_m \sin(\Theta')))/(\exp(-j2\pi(z_m \sin(-\Theta'))$$

In summary, based on ADMM two dimensional fast DOA flow chart of Estimation method flow chart is represented in the figure 4.

3.3 Algorithm Complexity Analysis

Complexity of proposed algorithm is analyzed in this section. Subject to condition of homogeneous array, one can assume that azimuth and elevation beams are synthesized and search angle is the numbers are M, N, the whole set of azimuth and elevation airspace The numbers are m, n. Then based on the formula (5) find the array receive data. Complexity of the time in two dimensional beamforming is $O(MN)$, according to the formula (20) know the ADMM main calculation amount of the algorithm comes from solving the linear equation system when updating the parameters, and complexity of time is $O(m^3) + O(n^3)$. In summary, algorithm proposed here, the main time complexity is $O(MN + m^3 + n^3)$.

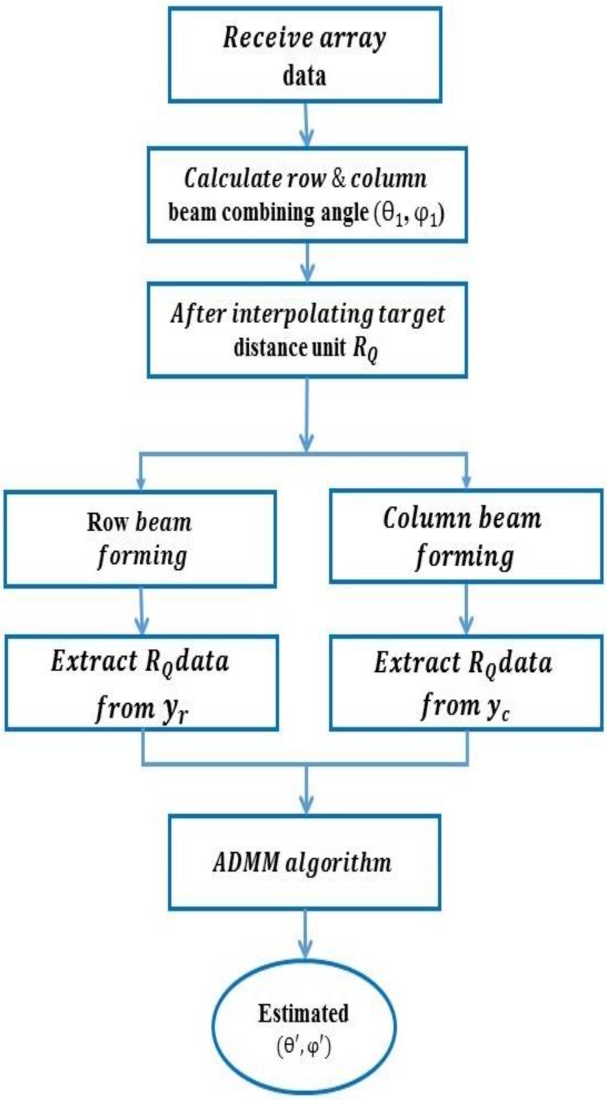

Fig 4: ADMM for two dimensional DOA estimation.

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4. Experimental simulation and analysis

This section is presented with the comparison of proposed algorithm with Spatial Smoothing-Multiple Signal Classification (SS-MUSIC)[3] and Alternating projection- Multiple Signal Classification (AP-MUSIC)[4] algorithm. The comparison of estimation accuracy and operation time verifies the proposed algorithm's effectiveness. The simulation conditions are set as follows: Number of horizontal array elements are 20, the no. of vertical array elements are 16, the array element interval 0.5 m, wavelength 1 meter, Radar Elevation 10 meters, the specular reflection coefficient is 0.95, the transmission bandwidth of the each array element 500KHZ.

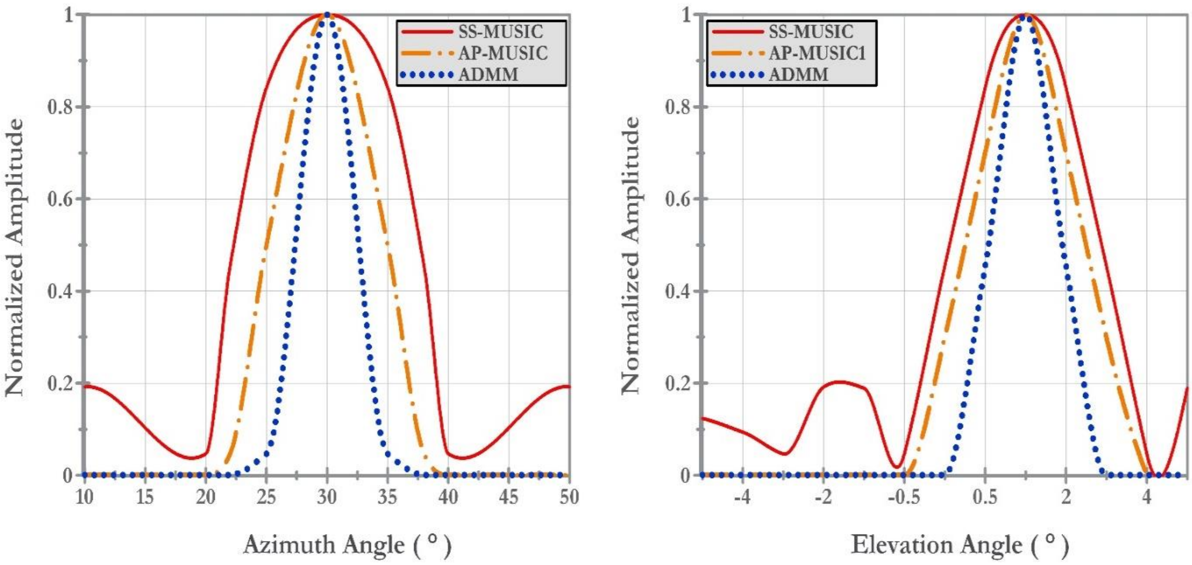

Fig 5: Target Spatial spectrum

$$\mathbf{RMSE}_{\varphi} = \sqrt{\frac{1}{D}\sum_{d=1}^{D} \left(\varphi'_d - \varphi\right)^2}$$

$$\mathbf{RMSE}_{\theta} = \sqrt{\frac{1}{D}\sum_{d=1}^{D} \left(\theta'_d - \theta\right)^2}$$

(RMSE), the root mean square error is taken as DOA Measure of estimation accuracy, azimuth and elevation angle. RMSE can be defined as in the above equations, number of Monte Carlo experiments is given by D, estimated value obtained from the second experiment are φ'_d, θ'_d respectively represent the *dth* angles. φ, θ indicates the true azimuth and pitch angle of target.

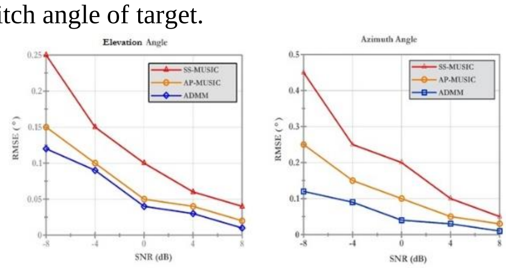

Fig 6: RMSE for different SNR measured with single snapshot

Experiment 1 is to verify effectiveness of algorithm proposed in this paper. Assuming that the target distance from radar is 50 km, the target height is 1.4 km, as given in Figure 2. Space geometric model can calculate the target pitch angle to be about 1.6°, set the target azimuth as 30° and the signal-to-noise ratio 5dB. Experimental setup for row-column beam synthesis, the angular search interval is 1°. From the beam synthesis the target azimuth angle is about 30°, so the rough azimuth angle of target is obtained by the beam synthesis. But for the pitch angle, since direct signal and multipath wave pitch angle approximately satisfy $\theta_d \approx -\theta_s$, the angle corresponds to the

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target by beamforming is in 0° nearby. The approximate position of target may be determined by beam synthesis, so the beam width can set in terms of the azimuth and elevation angle of array position, pitch angle search range. In this experiment, the azimuth search range is set as the surrounded by 28◦∼32◦, the pitch angle search range could be set as −5◦∼5◦, the interval is set for angle search is 0.1°.

Figure 5 demonstrates that results of algorithm proposed here and SS-MUSIC, AP-MUSIC. The algorithm correctly estimates the spatiotemporal spectral results. As shown in figure 5 it is evident that the target azimuth angle be estimated correctly, compared to other two algorithms. The proposed ADMM algorithm has a narrower main lobe and lower side lobes, indicating that proposed algorithm with higher DOA Estimation accuracy.

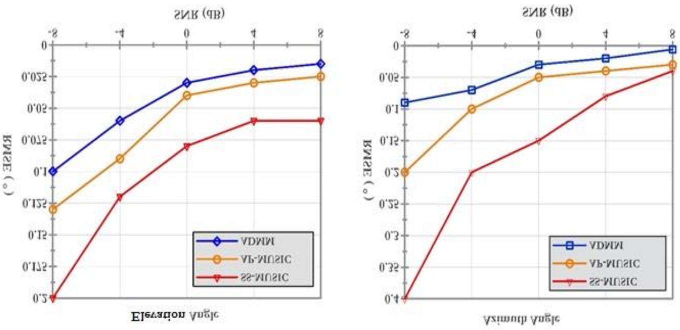

Fig 7: RMSE for different SNR measured with 20 snapshots

For the purpose of verifying performance of angle measurement of proposed algorithm, Experiment 2 has been conducted. Figure 6 shows the performance of DOA estimation, each algorithm with a single snapshot, where Figures 6(a),(b) can be noticed intuitively that the algorithm of this work is closer to actual angle and has better two-dimensional DOA estimated performance compared to other two.

Figure 7, the algorithm estimation performance compared under condition of the 20 snapshots, the RSME variation of algorithm with different signal-to-noise ratios (SNR). On an average, the proposed algorithm's performance is improved by about 53% compared to the SS-MUSIC, and improved by about 100% compared to the AP-MUSUC algorithm.

In Figure 8, we study RMSE azimuth and elevation estimation curve to different snapshot numbers, at the time of SNR = 0dB. It is obvious that RMSE performance is improved considering increasing snapshot numbers. And it is shown that the RMSEs of the methods with ADMM take an advantage over the SS-MUSIC and AP-MUSIC.

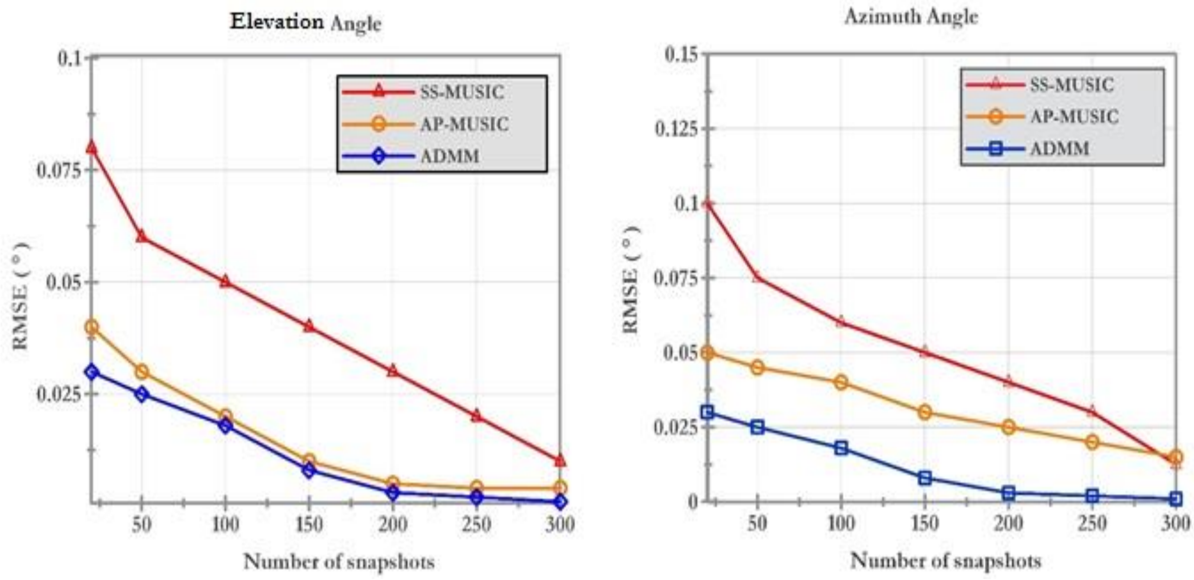

Fig 8: RMSE with different numbers of snapshots

In order to compare the computational efficiency between the algorithms, table 1 gives the time required for a single run of each algorithm. One can notice from the Table 1, the computational efficiency of algorithm proposed in the paper is obviously better than AP-MUSIC, the reason is method proposed in the study does not require to perform Eigen decomposition, and the convergence speed is fast. Because in SS-MUSIC, the procedure does not involve an iterative process, the operation speed is fast, and the process needs to perform Eigen decomposition. When array is large, the Eigen decomposition operation time will be greatly increased, so under the idea of ensuring accuracy requirements, proposed algorithm has more advantage.

Table 1: Algorithm running time

Sl#	Algorithm	Running time
1	ADMM	0.0189
2	AP-MUSIC	0.0598
3	SS-MUSIC	0.0040

5. Conclusion

This study proposes a fast two-dimensional angle of arrival estimation technique for meter wave array radar. The algorithm transforms the two-dimensional DOA estimation into two one-dimensional DOA estimations through the row and column beamforming technology, which avoids the complex calculation of two-dimensional joint estimation and greatly lessens the complexity. Using the ADMM algorithm for angle estimation does not need any Eigen decomposition and it does not lose the effective aperture of array under condition of ensuring the angle measurement accuracy. Convergence speed of algorithm is better than existing, therefore proposed algorithm has more efficient and has wider scope of application.

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