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Hybrid Nil Rapid Fuzzy Bi-Ideals of Near Rings

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Abstract: In this paper, our mains focus is on exploring the concept of hybrid nil rapid fuzzy bi-ideals of near rings, including an investigation of their properties. We also delve into the hybrid intrinsic product of these bi-ideals and establish relevant theorems. Our contribution lies in advancing the field of hybrid nil rapid fuzzy bi-ideals in near rings by deepening our understanding of these concepts.
Keywords: Hybrid nil rapid, hybrid intrinsic product, hybrid fuzzy bi-ideals, Near Ring, Hybrid Structure**.**

Introduction: 1A

In [1] M. Himaya Jaleela Begum and Jeyalakshmi, has proposed the developed the concept of Institutions Q Fuzzy bi-ideals in near-rings. [5] L.A.Zadeh developed and investigated fuzzy sets. [2] Kasi Porselvi, Ghulam Muhiuddin, Balasubramanian Elavarasan and Abdullah Assiry introduced and concept of Hybrid nil radical of a ring. [4] Young Bae Jun, Mehmet Ali Ozturk and Chul Hwan Park introduced and developed the oncept of Intuitionistic nil raricals of intuitionistic fuzzy ideals and Eulidea intuitionistic fuzzy deals in rings. [3] Elavarasn B and Jun Y.B has introduced the topic of Regularity of semigroups in terms of hybrid ideals and hybrid bi-ideals. In this paper, we introduce the concept of hybrid nil rapid fuzzy bi-ideals of near rings, including an investigation of their properties.

Preliminaries: 2B

Definion 1[2]

A set R (= φ) together with two binary operation '+' and ' .' is said to be a ring if it fulfills the following assertions.

(i) R is an abelian group under '+',
(ii) R is associative under '.',
(iii) c.(u+k)=c.u+c.k and (c+u).k=c.k+u.k for all c, u, k ∈ R.

Throughout this paper, unless stated otherwise, R denotes a ring and P(X), the power set of a set X.

Definition 2[2]

Let I be the unit interval and U be an initial universal set. Consider a mapping $\tilde{j}_\mu = (\tilde{j}, \mu): R \to P(U) \times I, x_1 \to \left(\tilde{j}(x_1), \mu(x_1)\right)$, where $\tilde{j}: R \to P(U)$ and $\mu: R \to I$. Then, $\tilde{j}_\mu$ is described as a hybrid structure in R over U.

Let all the hybrid structures collected in R over U be described by H (R). An order << in H (R) is outlines as follows : For every $\tilde{j}_\mu, \tilde{l}_\gamma \in H(R), j^{\sim} \mu l^{\sim} \gamma$ if and only if $\tilde{j}(w) \subseteq l^{\sim}(w)\, and\, \mu(w) \geq \gamma(w)$ for all w∈R. For any $x_1, x_2 \in R, \tilde{j}_\mu(x_1) = \tilde{l}_\gamma(x_2)$ if and only if $\tilde{j}_\mu(x_1) << \tilde{l}_\gamma(x_2)\, and\, \tilde{l}_\gamma(x_2) << \tilde{j}_\mu(x_1).$ Additionally, $\tilde{j}_\mu << \tilde{l}_\gamma\, and\, \tilde{l}_\gamma << \tilde{j}_\mu$ if and only if $\tilde{j}_\mu = \tilde{l}_\gamma$. It is noted that (H(R), <<) is a poset.

Definition 3[4]

Let R be a ring and S a non-empty subset of R that is closed under the operations of addition and multiplication in R. If S is itself a ring under these operations then S is called a subring of R. A subring I of a ring R is a left ideal provided.

$r \in R$ and $x \in I \Rightarrow rx \in I$

I is a right ideal provided

$r \in R$ and $x \in I \Rightarrow xr \in I$

I is an ideal it is both a left and right ideal. Note that non-empty subset I of a ring R is a left (resp. right) ideal if and only if for all $a, b \in I \; and \; r \in R$:

(i) $a, b \in I) \; a - b \in I;$

(ii) $a \in I, \; r \in R \Rightarrow ra \in I \; (resp. ar \in I).$

Main Results :

Definion: 3.1

Let $\tilde{\mu}_\lambda \in H(N)$ is a hybrid fuzzy bi-ideals in N. Then, the hybrid nil rapid of $\tilde{\mu}_\lambda$ is the hybrid structure in N over U, represented by $\sqrt{\tilde{\mu}_\lambda} = \left(\sqrt{\tilde{\mu}}, \sqrt{\lambda}\right)$ where $\sqrt{\tilde{\mu}}(x) = \sup_{n \geq 1} \tilde{\mu}(xy)^n$ and $\sqrt{\lambda}(x) = \inf_{n \geq 1} \lambda(xy)^n$ for $x, y \in N$ and some $n \in N$.

Example:3.2

Let $\tilde{\mu}_\lambda \in H(N)$ over $U = [0,1]$ be given by

$\tilde{\mu}(x) = \begin{cases} [0,0.5] if \; x \in N \\ [0,0.1] if \; x \notin N \end{cases}$ and a mapping $\lambda : N \to 1$ be constant. Then $\tilde{\mu}_\lambda$ is a hybrid nil rapid fuzzy bi-ideals of near rings.

Prepositions:3.3

Let $\tilde{\mu}_\lambda, \tilde{\eta}_\gamma \in H(N)$ be hybrid fuzzy bi-ideals in N. Then, the following assertions hold:

$$(i) \; \tilde{\mu}_\lambda << \sqrt{\tilde{\mu}_\lambda} \quad (ii) \tilde{\mu}_\lambda << \tilde{\eta}_\gamma \Rightarrow \sqrt{\tilde{\mu}_\lambda} << \sqrt{\tilde{\eta}_\gamma} \quad (iii) \sqrt{\sqrt{\tilde{\mu}_\lambda}} = \sqrt{\tilde{\mu}_\lambda}$$

Proof:

Let $t, s \in N$, Then, $\sqrt{\tilde{\mu}}(s) = \sup_{k \geq 1} \tilde{\mu}(st)^k \supseteq \tilde{\mu}(st)^k \supseteq \tilde{\mu}(st)^k$ for some $k \in N$ and

$\sqrt{\lambda}(s) = \inf_{k \geq 1} \lambda(st)^k \leq \lambda(st)^k \leq \lambda(st)^k$ So, $\tilde{\mu}_\lambda << \sqrt{\tilde{\mu}_\lambda}$

(ii) Let $s \in N$, Then, $\sqrt{\tilde{\mu}}(s) = \sup_{k \geq 1} \tilde{\mu}(st)^k \subseteq \sup_{k \geq 1} \tilde{\eta}(st)^k = \sqrt{\tilde{\eta}(st)}$ and

$\sqrt{\lambda}(s) = \inf_{k \geq 1} \lambda(st)^k \geq \gamma(st)^k = \sqrt{\gamma}(s).$ So, $\sqrt{\tilde{\mu}_\lambda} << \sqrt{\tilde{\eta}_\gamma}.$

(iii) $\sqrt{\sqrt{\tilde{\mu}}}(s) = \sup_{k \geq 1} \sqrt{\tilde{\mu}}(st)^k = \sup_{k \geq 1} \sup_{r \geq 1} \left(\left((\tilde{\mu}(st))^k \right)^r = \sup_{m \geq 1} \mu(st)^m = \sqrt{\tilde{\mu}}(s)$ and

$\sqrt{\sqrt{\lambda}}(s) = \inf_{k \geq 1} \sqrt{\lambda}(st)^k = \inf_{k \geq 1} \inf_{r \geq 1} \left(\left((\lambda(st))^k \right)^r = \inf_{m \geq 1} \lambda(st)^m = \sqrt{\lambda}(s)$ so, $\sqrt{\sqrt{\tilde{\mu}_\lambda}} = \sqrt{\tilde{\mu}_\lambda}$.

Theorem:3.4

For any hybrid fuzzy bi-ideal of N. $\sqrt{\tilde{\mu}_\lambda} = \left(\sqrt{\tilde{\mu}}, \sqrt{\lambda}\right)$ is a hybrid fuzzy bi-ideal of N.

Proof :

Let $v,u \in N$. Then, for any positive integers t, r we have

$$\min\left\{\sqrt{\tilde{\mu}}(v), \sqrt{\tilde{\mu}}(u)\right\} = \min\left\{\sup_{t\geq 1}\sqrt{\tilde{\mu}}(vs)^t, \sup_{r\geq 1}\sqrt{\tilde{\mu}}(uw)^r\right\} = \sup_{t\geq 1}\left\{\sup_{r\geq 1}\left\{\min\{\tilde{\mu}(vs)^t, \tilde{\mu}(uw)^r\}\right\}\right\}$$

and $\max\left\{\sqrt{\lambda}(v), \sqrt{\lambda}(u)\right\} = \max\left\{\inf_{t\geq 1}\sqrt{\lambda}(vs)^t, \inf_{r\geq 1}\sqrt{\lambda}(uw)^r\right\} = \inf_{t\geq 1}\left\{\inf_{r\geq 1}\left\{\max\{\lambda(vs)^t, \lambda(uw)^r\}\right\}\right\}$

Since N is commutative, all the terms in $(v+u)^{t+r}$ contain either v^t or u^r as a factor. Hence there exist

$c,d \in N\left(vs+uw\right)^{t+r} = c(vs)^t + d(uw)^t$. Thus min $\left\{\tilde{\mu}(vs)^t, \tilde{\mu}(uw)^r\right\} \subseteq$

$\min\left\{\max\{\tilde{\mu}(vs)^t, \tilde{\mu}(c)\}, \max\{\tilde{\mu}(uw)^r, \tilde{\mu}(d)\}\right\} \subseteq \min\left\{\tilde{\mu}\left(c(vs)^t, \tilde{\mu}\left(c(vs)^t\right)\right)\right\} \subseteq$

$\tilde{\mu}\left(c(vs)^t + d(uw)^r\right) = \tilde{\mu}(vs+uw)^{t+r} \subseteq \sup_{k\geq 1}\tilde{\mu}(vs+uw)^k = \sqrt{\tilde{\mu}}(vs+uw)$

max $\left\{\lambda(vs)^t, \lambda(uw)^r\right\} \geq \max\left\{\min\{\lambda(vs)^t, \lambda(c),\}\min\{\lambda(uw)^r, \lambda(d)\}\right\} \geq$

$\max\left\{\lambda\left(c(vs)^t, \lambda\left(c(vs)^t\right)\right)\right\} \geq \lambda\left(c(vs)^t + d(uw)^r\right) = \lambda(vs+uw)^{t+r} \geq \inf_{k\geq 1}\lambda(vs+uw)^k = \sqrt{\lambda}(vs+uw)$

Now, for a positive integer e,

max $\left\{\sqrt{\tilde{\mu}}(vs), \sqrt{\tilde{\mu}}(uw)\right\} = \max\left\{\sup_{e\geq 1}\tilde{\mu}(vs)^e, \sup_{e\geq 1}\tilde{\mu}(uw)^e\right\} = \sup_{e\geq 1}\left\{\max\{\tilde{\mu}(vs)^e, \tilde{\mu}(uw)^e\}\right\}$

min $\left\{\sqrt{\lambda}(vs), \sqrt{\lambda}(uw)\right\} = \min\left\{\inf_{e\geq 1}\lambda(vs)^e, \inf_{e\geq 1}\lambda(uw)^e\right\} = \inf_{e\geq 1}\left\{\min\{\lambda(vs)^e, \lambda(uw)^e\}\right\}$

Then,

Min $\left\{\breve{\mu}(\upsilon s)^t, \breve{\mu}(bc)^t\right\} \subseteq \breve{\mu}\left((\upsilon s)^t(bc)^t\right) = \breve{\mu}\left((\upsilon s)(\mu w)(bc)\right)^t \subseteq$

$\sup_{k\geq 1}\left((\upsilon s)(uw)(bc)\right)^k = \sqrt{\tilde{\mu}}(\upsilon s)(uw)(bc)$

$\max\left\{\lambda(\upsilon s)^t, \lambda(bc)^t\right\} \geq \lambda\left((\upsilon s)^t(uw)^t(bc)^t\right) = \lambda\left((\upsilon s)(uw)(bc)\right)^t \geq$

$\inf_{k\geq 1}\lambda\left((\upsilon s)(uw)(bc)\right)^k = \sqrt{\lambda}(\upsilon s)(uw)(bc)$

Thus, $\sqrt{\tilde{\mu}}(\upsilon s)(uw)(bc) \supseteq \min\left\{\breve{\mu}(\upsilon s)^t, \breve{\mu}(bc)^t\right\}$

and $\sqrt{\lambda}(\upsilon s)(uw)(bc) \leq \max\left\{\lambda(\upsilon s)^t, \lambda(bc)^t\right\}$

And hence $\sqrt{\tilde{\mu}_\lambda}$ is a hybrid fuzzy bi – ideals in N.

Definition : 3.5

Let $\tilde{\mu}_\lambda, \tilde{\eta}_\gamma, \tilde{\zeta}_\delta \in H(N)$ be hybrid fuzzy bi – ideals in N. Then the hybrid intrinsic product $\tilde{\mu}_\lambda \tilde{*} \tilde{\eta}_\gamma \tilde{\zeta}_\delta$ is the hybrid structure in N stated as below: For $w \in N$, define

$$\left(\left(\tilde{\mu} \tilde{*} \tilde{\eta}\right)\tilde{*}\zeta\right)(w) = \max\left\{\inf_{1\leq i\leq s}\min\left\{\left(\tilde{\mu}(a_i), \tilde{\eta}(b_i)\right)\tilde{\zeta}(C_i)\right\}\sum_{i=1}^{S}(a_i b_i)c_i = w \text{ for some } S \in N\right\}$$

$$\left(\left(\lambda \tilde{*} \gamma\right)\tilde{*}\delta\right)(w) = \min\left\{\sup_{1\leq i\leq s}\{(\lambda(a_i), \gamma(b_i))\delta(c_i)\}\sum_{i=1}^{S}(a_i b_i)c_i = w \text{ for some } s \in N\right\}$$

If we can express $w=\sum_{i=1}^{k}(a_ib_i)c_i$ for some $a_ib_ic_i \in N$ where each $a_ib_ic_i \neq 0$ and $k \in N$ Otherwise, we define $((\tilde{\mu} \tilde{*} \tilde{\eta}) \tilde{*} \zeta)(w)=0$ and $((\lambda \tilde{*} \gamma) \tilde{*} \delta)(w)=1$ obviously the product $(\tilde{\mu}_\lambda \tilde{*} \tilde{\eta}_\gamma) \tilde{*} \tilde{\zeta}_\delta \in H(N)$ is commutative if N is commutative.

Theorem : 3.6

Let $\tilde{\mu}_\lambda, \tilde{\eta}_\gamma, \tilde{\zeta}_\delta \in H(N)$ be hybrid fuzzy bi – ideals in *N*. Then

$$\sqrt{(\tilde{\mu}_\lambda \tilde{*} \tilde{\eta}_\gamma) \tilde{*} \tilde{\zeta}_\delta} = \sqrt{(\tilde{\mu}_\lambda \cap \tilde{\eta}_\gamma) \cap \tilde{\zeta}_\delta} = \left(\sqrt{\tilde{\eta}_\lambda} \cap \sqrt{\tilde{\eta}_\gamma} \cap \sqrt{\tilde{\zeta}_\delta}\right)$$

Proof :

Let $w \in N$ and let $w=\sum_i^k = 1(a_ib_i)c_i$ where $a_ib_ic_i \neq 0$ in *N*

Then $\min\{\tilde{\mu}(a_i), \tilde{\eta}(b_i), \tilde{\zeta}(c_i)\} \subseteq \tilde{\mu}(a_i) \subseteq \tilde{\mu}((a_ib_i)c_i)$ and

$\max\{\lambda(a_i), \gamma(b_i), \delta(c_i)\} \geq \lambda(a_i) \geq \lambda((a_ib_i)c_i)\ \lambda((a_ib_i)c_i)$ for $1 \leq i \leq m$

Now,

$$\min_{1\le i\le m}\{\min\{\tilde{\mu}(a_i), \tilde{\eta}(b_i), \tilde{\zeta}(c_i)\}\} \subseteq \min_{1\le i\le m} \tilde{\mu}((a_ib_i)c_i) \subseteq \tilde{\mu}\left(\sum_{i=1}^{m}((a_ib_i)c_i)\right) = \tilde{\mu}(w)$$

$$\min_{1\le i\le m}\{\max\{\lambda(a_i), \gamma(b_i), \delta(c_i)\}\} \geq \max_{1\le i\le m} \lambda((a_ib_i)c_i) \geq \lambda\left(\sum_{i=1}^{m}((a_ib_i)c_i)\right) = \lambda(w)$$

So, $((\tilde{\mu} \tilde{*} \tilde{\eta}) \tilde{*} \tilde{\zeta})(w) \subseteq \tilde{\mu}(w)$ and $((\lambda \tilde{*} \gamma) \tilde{*} \delta(w) \geq \lambda(w))$

Hence $\tilde{\mu}_\lambda \tilde{*} \tilde{\eta}_\gamma << \tilde{\mu}_\lambda, \tilde{\eta}_\gamma \tilde{*} \tilde{\zeta}_\delta << \tilde{\eta}_\gamma$ similarly, we can show that $\tilde{\zeta}_\delta \tilde{*} \tilde{\mu}_\lambda << \tilde{\zeta}_\delta$

So, $((\tilde{\mu} \tilde{*} \tilde{\eta}) \tilde{*} \tilde{\zeta})(w) \subseteq \min\{\tilde{\mu}(w), \tilde{\eta}(w), \tilde{\zeta}(w)\} = ((\tilde{\mu} \tilde{\cap} \tilde{\eta}) \tilde{\cap} \tilde{\zeta})(w)$ and $(\lambda \tilde{*} \gamma) \tilde{*} \delta(w) \geq$ $\max\{\lambda(w), \gamma(w), \delta(w)\} = ((\lambda \vee \gamma) \vee \delta)(w)$

Therefore by proposition 3.3 $\sqrt{(\tilde{\mu}_\lambda \tilde{*} \tilde{\eta}_\gamma) \tilde{*} \tilde{\zeta}_\delta} << \sqrt{(\tilde{\mu}_\lambda \cap \tilde{\eta}_\gamma) \cap \tilde{\zeta}_\delta}$

Let $w \in N$, Then $\sqrt{(\tilde{\mu} \tilde{*} \tilde{\eta}) \tilde{*} \tilde{\zeta}}(wik) = \sup_{e\geq 1}((\tilde{\mu} \tilde{*} \tilde{\eta}) \tilde{*} \tilde{\zeta})(wik)^e \supseteq ((\tilde{\mu} \tilde{*} \tilde{\eta}) \tilde{*} \tilde{\zeta})(wik)^{3n} \supseteq$ $\min\{\tilde{\mu}(wik)^n, \tilde{\eta}(wik)^n, \tilde{\zeta}(wik)^n\} = ((\tilde{\mu} \tilde{\cap} \tilde{\eta}) \tilde{\cap} \tilde{\zeta})(wik)^n$ for all $n \geq 1$ e

$\sqrt{(\lambda \tilde{*} \gamma) \tilde{*} \delta}(wik) = \inf_{e\geq 1}((\lambda \tilde{*} \gamma) \tilde{*} \delta)(wik)^e \leq ((\lambda \tilde{*} \gamma) \tilde{*} \delta)(wik)^{3n} \leq$ $\max\{\lambda(wik)^n, \gamma(wik)^n, \delta(wik)^n\} = (\lambda \vee \gamma \vee \delta)(wik)^n$ for all $n \geq 1$

Therefore $\sqrt{(\tilde{\mu}_\lambda \cap \tilde{\eta}_\gamma) \cap \tilde{\zeta}_\delta} = \sqrt{(\tilde{\mu}_\lambda \tilde{*} \tilde{\eta}_\gamma) \tilde{*} \tilde{\zeta}_\delta}$

Now, by Proposition (ii) we have

$\sqrt{\tilde{\mu}_\lambda \cap \tilde{\eta}_\gamma} << \tilde{\mu}_\lambda, \sqrt{\tilde{\eta}_\gamma \cap \tilde{\zeta}_\delta} << \tilde{\eta}_\gamma$ and $\tilde{\zeta}_\delta \cap \tilde{\mu}_\lambda << \tilde{\zeta}_\delta \Rightarrow$

$\sqrt{(\tilde{\mu}_\lambda \cap \tilde{\eta}_\gamma) \cap \tilde{\zeta}_\delta} << \sqrt{\tilde{\mu}_\lambda} \cap \sqrt{\tilde{\eta}_\gamma} \cap \sqrt{\tilde{\zeta}_\delta}$

Conversely, Let $w \in N$. Then, for any two positive integers t, r we have

$$\left(\sqrt{\tilde{\mu}}\,\tilde{\cap}\sqrt{\tilde{\eta}}\right)(w)=\min\left\{\sup_{t\geq 1}\tilde{\mu}(wi)^{t},\sup_{r\geq 1}\tilde{\mu}(wi)^{r}\right\}=\sup_{t\geq 1}\left\{\sup_{t\geq 1}\left\{\min\left\{\tilde{\mu}(wi)^{t},\tilde{\eta}(wi)^{r}\right\}\right\}\right\}$$

Similarly

$$\left(\sqrt{\tilde{\eta}}\,\tilde{\cap}\sqrt{\tilde{\zeta}}\right)(w)=\sup_{t\geq 1}\left\{\sup_{t\geq 1}\left\{\tilde{\eta}(wi)^{t},\tilde{\zeta}(wi)^{r}\right\}\right\},\left(\sqrt{\tilde{\zeta}}\,\tilde{\cap}\sqrt{\tilde{\mu}}\right)(w)=$$

$$\sup_{t\geq 1}\left\{\sup_{t\geq 1}\left\{\min\left\{\tilde{\zeta}(wi)^{t},\tilde{\mu}(wi)^{r}\right\}\right\}\right\}$$

$$\left(\sqrt{\lambda}\vee\sqrt{\gamma}\right)=\max\left\{\inf_{t\geq 1}\lambda(wi)^{t},\inf_{r\geq 1}\gamma(wi)^{r}\right\}=\inf_{t\geq 1}\left\{\inf_{t\geq 1}\left\{\max\left\{\lambda(wi)^{t},\gamma(wi)^{r}\right\}\right\}\right\}$$

similarly

$$\left(\sqrt{\gamma}\vee\sqrt{\delta}\right)(w)=\inf_{t\geq 1}\left\{\inf_{t\geq 1}\left\{\max\left\{\gamma(wi)^{r}\right\}\right\}\right\},\ \left(\sqrt{\delta}\vee\sqrt{\lambda}(w)\right)=\inf_{t\geq 1}\left\{\inf_{t\geq 1}\left\{\max\left\{\delta(wi)^{r}\right\}\right\}\right\}$$

Now,

$$\cap\left\{\tilde{\mu}(wi)^{t},\tilde{\eta}(wi)^{r}\right\}\subseteq\cap\left\{\tilde{\mu}(wi)^{tr},\tilde{\eta}(wi)^{tr}\right\}=\left(\tilde{\mu}\,\tilde{\cap}\,\tilde{\eta}\right)(wi)^{tr}\subseteq\sup_{k\geq 1}\left(\tilde{\mu}\,\tilde{\cap}\,\tilde{\eta}\right)(wi)^{k}=$$

$$\sqrt{\left(\tilde{\mu}\,\tilde{\cap}\,\tilde{\eta}\right)}(w)\Rightarrow\left(\sqrt{\tilde{\mu}}\cap\sqrt{\tilde{\eta}}\right)(w)\subseteq\sqrt{\left(\tilde{\mu}\,\tilde{\cap}\,\tilde{\eta}\right)}(w)$$

similarly $\left(\sqrt{\tilde{\mu}}\cap\sqrt{\tilde{\zeta}}\right)(w)\subseteq\sqrt{\tilde{\mu}\,\tilde{\cap}\,\tilde{\zeta}}(w)\,and\left(\sqrt{\tilde{\zeta}}\cap\sqrt{\tilde{\mu}}\right)(w)\subseteq\sqrt{\left(\tilde{\zeta}\,\tilde{\cap}\,\tilde{\mu}\right)}(w)$

Additionally, $\vee\left\{\lambda(wi)^{t},\gamma(wi)^{r}\right\}\geq\vee\left\{\lambda(wi)^{tr}\right\}=(\lambda\vee\gamma)(wi)^{tr}\geq$

$$\inf_{k\geq 1}\left(\tilde{\mu}\,\tilde{\cap}\,\tilde{\eta}\right)(wi)^{k}=\sqrt{\lambda\vee\gamma}(w)\Rightarrow\left(\sqrt{\lambda}\vee\sqrt{\gamma}\right)(w)\geq\sqrt{(\lambda\vee\gamma)}(w)$$

Similarly $\left(\sqrt{\gamma}\vee\sqrt{\delta}\right)(w)\geq\sqrt{(\gamma\vee\delta)}(w)$ and $\left(\sqrt{\delta}\vee\sqrt{\mu}\right)(w)\subseteq\sqrt{(\delta\vee\mu)}(w)$

So $\sqrt{\tilde{\mu}_{\lambda}}\cap\sqrt{\tilde{\eta}_{\gamma}}\cap\sqrt{\tilde{\zeta}_{\delta}}=\sqrt{(\tilde{\mu}_{\lambda}\cap\tilde{\eta}_{\gamma})\cap\tilde{\zeta}_{\delta}}$

Corollary : 3.7

Let $\tilde{\mu}_{\gamma},\tilde{\eta}_{\gamma},\tilde{\zeta}_{\delta}\in H(N)$ be a hybrid fuzzy bi – ideals in N. Then $\sqrt{\tilde{\mu}_{\lambda}{}^{n}}=\sqrt{\tilde{\mu}_{\lambda}},\sqrt{\tilde{\eta}_{\gamma}{}^{l}}$ $\sqrt{\tilde{\eta}_{\gamma}},\sqrt{\tilde{\zeta}_{\delta}{}^{m}}=\sqrt{\tilde{\zeta}_{\delta}}$ for all $n,m,l\geq 1$ where $\tilde{\mu}_{\lambda}{}^{n}=\tilde{\mu}_{\lambda}*...\tilde{\mu}_{\lambda}$ (n times) $\tilde{\mu}_{\gamma}{}^{m}=\tilde{\eta}_{\gamma}*\tilde{\eta}_{\gamma}$ (m times) $\tilde{\zeta}_{\delta}{}^{l}=\tilde{\zeta}_{\delta}*\tilde{\zeta}_{\delta}$ (1 times)

Proof :

Taking $\tilde{\mu}_{\lambda}=\tilde{\eta}_{\delta}=\tilde{\mu}_{\delta}$ in above theorem we have $\sqrt{\tilde{\mu}_{\lambda}\,\tilde{*}\,\tilde{\mu}_{\lambda}}=\sqrt{\tilde{\mu}_{\lambda}}\sqrt{\tilde{\eta}_{\gamma}\,\tilde{*}\,\tilde{\eta}_{\gamma}}=\sqrt{\tilde{\eta}_{\gamma}}$ and $\sqrt{\tilde{\zeta}_{\delta}\,\tilde{*}\,\tilde{\zeta}_{\delta}}=\sqrt{\tilde{\zeta}_{\delta}}$ by mathematical induction principle. Put n =1 $\sqrt{\tilde{\mu}_{\lambda}{}^{n}}=\sqrt{\tilde{\mu}_{\lambda}{}^{1}}=\sqrt{\tilde{\mu}_{\lambda}}$ by mathematical induction principle. Put n = 1 $\sqrt{\tilde{\mu}_{\lambda}{}^{n}}=\sqrt{\tilde{\mu}_{\lambda}{}^{1}}=\sqrt{\tilde{\mu}_{\lambda}}$ Therefore it is result is true for n = 1. Assume if it is true for n = k, so $\sqrt{\tilde{\mu}_{\lambda}{}^{k}}=\sqrt{\tilde{\mu}_{\lambda}}$.

To prove the result if true for n = k + 1 $\sqrt{\tilde{\mu}_{y}{}^{k+1}}=\sqrt{\tilde{\mu}_{y}{}^{k}\cdot\tilde{\mu}_{\lambda}}=\sqrt{\sqrt{\tilde{\mu}_{\lambda}}\cdot\sqrt{\tilde{\mu}_{\lambda}}}=\sqrt{\tilde{\mu}_{\lambda}}$

Hence it is true for k+1. Similarly it is true for $\sqrt{\tilde{\eta}_{\gamma}{}^{l}}=\sqrt{\tilde{\eta}_{\gamma}}$ and $\sqrt{\tilde{\zeta}_{\delta}{}^{m}}=\sqrt{\tilde{\zeta}_{\delta}}$

Corollary: 3.8

Let $\tilde{\mu}_\lambda, \tilde{\eta}_\gamma, \tilde{\zeta}_\delta \in H(N)$ be a hybrid fuzzy bi – ideals in *N* if $\tilde{\mu}_\gamma^{\,n} << \tilde{\eta}_\gamma^{\,l} << \tilde{\zeta}_\delta$ and $\tilde{\zeta}_\delta^{\,m} << \tilde{\mu}_\lambda$ for some k, l, m then $\sqrt{\tilde{\mu}_\lambda} << \sqrt{\tilde{\eta}_\gamma}, \sqrt{\tilde{\eta}_\gamma} << \sqrt{\tilde{\zeta}_\delta}, \sqrt{\tilde{\zeta}_\delta} << \sqrt{\tilde{\mu}_\lambda}.$

Proof :

By applying corollary $\sqrt{\tilde{\mu}_\lambda^{\,k}} = \sqrt{\tilde{\mu}_\lambda}, \sqrt{\tilde{\eta}_\gamma}, \sqrt{\tilde{\zeta}_\delta^{\,m}} = \sqrt{\tilde{\zeta}_\delta}$ for all k,l, $m \geq 1$

Then by Proposition (ii) we get $\sqrt{\tilde{\mu}_\lambda^{\,k}} = \sqrt{\tilde{\mu}_\lambda}, \sqrt{\tilde{\eta}_\gamma^{\,l}} = \sqrt{\tilde{\eta}_\gamma}, \sqrt{\tilde{\zeta}_\delta^{\,m}} = \sqrt{\tilde{\zeta}_\delta}$ for all k, l, $m \geq 1$

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