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Rotating Convection of a Chemical Reacting Fluid with the Effect of Helical Force: Linear and Weakly Non-Linear Analysis

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Abstract

Employing both linear and weakly non-linear analyses, the inquiry focuses on the oscillatory rotating convection with chemical reaction and helical force. The governing non-dimensionless equations for linear stability analysis have been solved using the normal mode approach, which results in an eigenvalue problem. While the weakly non-linear theory is investigated by multiple scale analysis. For linear stability analysis, the eigenvalue problem is solved using the One-term Galerkin approach, which yields the analytic formula for Rayleigh number. The problem of the commencement of convection has been addressed and resolved using the bvp4c procedure in MATLAB R2023b. For all physical parameters, neutral curves are constructed for both oscillatory and stable instability. They show no observable e ect on stationary convec tion in this Prandtl value. However, Taylor number, helical force parameter the thermal Rayleigh number, Lewis number, solutal di usitivity, and solutal Rayleigh number destabilize the ow in it. Weakly non-linear analysis results in the amplitude equation

Keywords: *Chemical Reaction, Linear stability analysis.*

1. Introduction

Due to its numerous applications in science, engineering, and technology, including plastic processing, oil recovery, and food engineering, Rayleigh-Benard convection in rotating fluids has drawn a lot attention in the modern era [1,2]. It has been observed that the turbulent convection motion in planetary atmosphere can take on a helical shape [3,4].

Levina and hee colleagues discussed the investigations on the hydrodynamic alpha effect [5-8]. An averaged equation describing the formation of large-scale structures with the influence of the Coriolis force in an anisotropic helical force on the commencement of conversation were investigated by Essoun and Chabi Orou [10]. They demonstrated how the helical force influences the Pr and Ta parameter space, which is divide into zones of Oscillatory and steady convection.

Later, Essoun and Chabi Orou {11] examined the impact of helical force on the begging of Kuppers-Lortz instability. In the presence of a magnetic field, Pomalni et al. [12] investigated the non-monotonic impact of helical force on the begging of conversation. Hounsoua et al. [13] investigated how helical force affected a rotating ferrofluid's begging convection. Research on the begging of convection phenomena with chemical reactions is widely available. In contrast, there aren't many studies in the literature discussing how chemical reactions affect Darcy-Benard convective events. Teinberg and Brand [14], Gatica et al. [15] examined binary mixtures and reactive porous media to investigate their thermoconvective instabilities for the first time.

Subsequently, Gatica et al. [15] used a variational technique to examine both monotonic and oscillatory instabilities in a porous media while taking chemical reactions into account. Pritchard and Richardson [16] investigated the thermosolutal convection of a binary uid in a porous layer by taking chemical equilibrium on boundary surfaces into account. Using a normal mode technique, the impact of the solute's dissolution on the start of con vection is thoroughly investigated. The purpose of this article is to investigate the impact of magnetic, Cori olis, and chemical reaction forces on the onset of convection in a porous medium. The linear stability theory of Thermal Rayleigh number for the onset of magneto-rotating convection in a porous medium with rst-order chemical reaction has not, as far as we are aware, been studied. This article's outline is as follows: Section 2 describes the mathematical formulation under consideration; Section 3 presents the linear stability analysis; Section 4 dis cusses the weakly non-linear analysis; Section 5 discusses heat transport by convection; and Section 6 presents the ndings and discussion. The paper's conclusion is given in Section 7

2. Mathematical formulation

Examining a horizontal layer conned between z ϵ (0, d) with constant temperature T_0 and T_0 + ΔT (ΔT > 0) and solutal concentration S_0 and S_0 +ΔS (ΔS >0) are upheld at the top and lower limits, correspondingly. The system's dimensional governing equations can be expressed as

$$\nabla \cdot V = 0, \tag{1}$$

$$\rho_0\left(\frac{\partial \boldsymbol{V}}{\partial t} + (\boldsymbol{V}\cdot\nabla)\boldsymbol{V}\right) = -\nabla P + \mu\nabla^2\boldsymbol{V} + \rho_0\, a\Omega d\,\boldsymbol{f} + \rho g + 2\rho_0\Omega(\boldsymbol{V}\times e_z), \tag{2}$$

$$\frac{\partial T}{\partial t} + (V\cdot\nabla)T = \kappa\nabla^2 T \quad (3),$$

$$\frac{\partial S}{\partial t} + (V\cdot\nabla)S = D_v\nabla^2 S + \chi\left(S_{eq}(T) - S\right), \tag{4}$$

$$\rho = \rho_0\left(1 - \beta_T(T - T_0) + \beta_S(S - S_0)\right). \tag{5}$$

where

$$f = e_z\left(curl(V)\right)_z - \frac{\partial(e_z\times V)}{\partial z}, \qquad e_z = (0,0,1). \tag{6}$$

We consider that the solute concentration at equilibrium is a linear function of temperature. So

that $S_{eq}(T) = S_0 + \pi\,(T - T_0)$, and at the boundaries, when we assume chemical equilibrium, we obtain. $\phi = \frac{\Delta S}{\delta T}$ .The conduction state is characterized by

$$T_b = \left(1 - \frac{z}{d}\right)\Delta T + T_0 \quad (7)$$

$$S_b = \left(1 - \frac{z}{d}\right)\Delta S + S_0 \quad (8)$$

On the basic state, we superpose small perturbations in the form

$\mathbf{V} = \boldsymbol{V}_0 + V'$,

$T = T_b\,(z) + T'$,

$S = S_b\,(z) + S'$,

$P = P_b\,(z) + P'$,

$\rho = \rho_b\,(z) + \rho'\,(x, y, z, t)$. (9)

After presenting the subsequent non-dimensional parameters

$(x', y', z') = d\,(x^*, y^*, z^*)$,

$t' = \frac{d^2}{\kappa} t^*$,

$P' = \frac{\rho_0 \kappa^2}{d^2} P^*$,

$S' = (\Delta S)\, S^*$,

$T' = (\Delta T)\, \theta^*$,

$V' = \frac{\kappa}{d} V^*$

The following non-dimensional equations are obtaines (on dropping the asterisks),

$$\nabla . \boldsymbol{V} = 0 \quad (10)$$

$$\frac{1}{Pr}\left(\frac{\partial \boldsymbol{V}}{\partial t} + (\boldsymbol{V}\cdot\nabla)\boldsymbol{V}\right) = -\frac{\nabla P}{Pr} + \nabla^2 \mathbf{V} + S_h \mathbf{f} + (R_T\theta - R_S S)\, e_z + Ta^{\frac{1}{2}}(\mathbf{V} \times e_z), \quad (11)$$

$$\frac{\partial \theta}{\partial t} + (\boldsymbol{V}\cdot\nabla)\theta = \omega + \nabla^2\theta \quad (12)$$

$$\frac{\partial S}{\partial t} + (\boldsymbol{V}\cdot\nabla)S = \omega + \frac{1}{Le}\nabla^2 S + D_v(\theta - S), \quad (13)$$

Where

$$R_T = \frac{\beta_T g \Delta T d^3}{\kappa \nu} \quad R_S = \frac{\beta_S \Delta S d^3}{\kappa \nu}, \; C_S = \frac{\mu_1}{\mu d^2},$$

$$Pr = \frac{\mu}{\rho_0 \kappa}, \qquad Le = \frac{\kappa}{D_v}, \qquad Dm = \frac{d^2 \chi}{\kappa}, \qquad Ta = \frac{2\rho_0 \Omega d^2}{\mu}.$$

All quantities which are used in the above equations have explained in nomenclature.

Taking the curl of (11) and the third component of curl of (11) yields,

$$\frac{1}{Pr}\left(\frac{\partial}{\partial t}-\nabla^2\right)w_z - S_h\frac{\partial^2 w}{\partial z^2} - Ta^{\frac{1}{2}}\frac{\partial w}{\partial z} = 0 \qquad (14)$$

$$\frac{1}{Pr}\left(\frac{\partial}{\partial t}-\nabla^2\right)\nabla^2 w - S_h(\nabla_h^2 - \frac{\partial^2}{\partial z^2})w_z + (R_S S - R_T\theta)\nabla_h^2 + Ta^{\frac{1}{2}}\frac{\partial w_z}{\partial z} = 0 \qquad (15)$$

Where $w_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$

By Removing w_z from equation (14) and (15), we obtain,

$$\left(\frac{1}{Pr}\left(\frac{\partial}{\partial t}-\nabla^2\right)\right)^2 \nabla^2 w - S_h^2\nabla_h^2\frac{\partial^2 w}{\partial z^2} - S_h\nabla_h^2 Ta^{\frac{1}{2}}\frac{\partial w_z}{\partial z} + S_h^2\frac{\partial^4 w}{\partial z^4} + 2S_h Ta^{\frac{1}{2}}\frac{\partial^3 w}{\partial z^3}$$

$$+(R_S S - R_T\theta)\ \frac{1}{Pr}\left(\frac{\partial}{\partial t}-\nabla^2\right)\nabla_h^2 + Ta^{\frac{1}{2}}\frac{\partial^2 w}{\partial z^2} = 0 \qquad (16)$$

$$\frac{\partial\theta}{\partial t} = \omega + \nabla^2\theta \qquad (17)$$

$$\frac{\partial S}{\partial t} = \omega + \frac{1}{Le}\nabla^2 S + D_v(\theta - S) \qquad (18)$$

Where

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2};$$

& $\nabla_h^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$:

3. Linear stability analysis

Let's write the perturbation to introduce the normal modes in the form of

$$(\omega,\theta,S) = (w,\theta,S)Sin(n\pi z)e^{i(lx+my)+\sigma t}. \qquad (19)$$

Substituting the above normal solution into Eqs, (16) – (18), we get

$$\left(-\left(\frac{\sigma}{Pr}+\delta^2\right)^2\delta^2 - S_h^2\pi^2(\pi^2 - q^2) - Ta\pi^2\right)\omega + \left(R_T\left(\frac{\sigma}{Pr}+\delta^2\right)q^2\right)\theta$$

$$-\left(R_S\left(\frac{\sigma}{pr}+\delta^2\right)q^2\right)S = 0 \qquad (20)$$

$$\omega - (\delta^2+\sigma)\theta = 0 \qquad (21)$$

$$\omega + D_v\theta - \left(\frac{\delta^2}{Le} + \sigma + D_v\, S\right) = 0 \qquad (22)$$

Where

$q^2 = l^2 + m^2$ is the wave number,
$\sigma = \iota\omega$, $\delta^2 = \pi^2 + q^2$.

Requiring zero determinant of the above system, one obtains,

$$R_T = \frac{LeR_S(D_v+\delta^2+\iota\omega)}{D_v Le+\delta^2+\iota Le\omega} + (\delta^2+\iota\omega)\frac{(-\pi^4 Pr^2 S_h^2 + \pi^2 Pr^2(q^2 S_h^2 + Ta^2) + (Pr\delta^3 + \iota\delta\omega)^2}{Prq^2\,(Pr\delta^2+\iota\omega)} \qquad (23)$$

3.1 . Stationary convection

Substituting $\omega = 0$ in Eq. (23), then we get

$$R_{T_{sc}} = \frac{D_v\, Le(-\pi^4 S_h^2 + q^2(R_S + \pi^2 S_h^2) + \pi^2 Ta + \delta^6) + \delta^2(Leq^2 R_S - \pi^4 S_h^2 + \pi^2(q^2 S_h^2 + Ta) + \delta^6)}{q^2(\, D_v Le + \delta^2)} \quad (24)$$

3.2. Oscillatory convection

To find theR_T for oscillatory convection we nd the roots of imaginary part of Rayleigh number. On substituting roots into the real part of Rayleigh number, we get the $R_T$for oscillatory convection.

$$R_{T_{oc}} = \frac{A_1 * A_2 * \omega^2 + A_3 * \omega^2 * \omega^2 + A_4 * \omega^2 * \omega^2 * \omega^2}{Prq2\,(Pr\delta^4 + \omega^2)((\, D_v Le + \delta^2)^2 + Le^2 * \omega^2)} \quad (25)$$

Where $$\omega^2 = \frac{-B_2 + \sqrt{B_2^2 - 4 * B_1 * B_3}}{2 * B_1} \quad (26)$$

Where

$$A_1 = Pr^3\delta^4(D_v Le + \delta^2)\,(D_v Le(-\pi^4 S_h^2 + q^2(R_S + \pi^2 S_h^2) + \pi^2 Ta + \delta^6) + \delta^2(Leq^2 R_S + \pi^2((-\pi^2 + q^2)S_h^2 + Ta) + \delta^6)\,;$$

$$A_2 = \mathrm{Pr}\,(D_v^2 Le^2(q^2(R_S + \pi^2 Pr S_h^2) + \delta^6 - \mathrm{Pr}\,(\pi^4 S_h^2 - \pi^2 Ta + \delta^6)) + \delta^4(Leq^2 R_S + \delta^6 + Pr\left(\pi^2\left((-\pi^2 + q^2)S_h^2 + Ta\right) - \delta^6\right) + Le^2 Pr^2(-\pi^4 S_h^2 + q^2(R_S + \pi^2 S_h^2) + \pi^2 Ta + \delta^6) + D_v Le\delta^2\left(q^2(R_S + LeR_S + 2\pi^2 Pr S_h^2) + 2\left(\delta^6 - Pr(\pi^4 S_h^2 - \pi^2 Ta + \delta^6)\right)\right))\,;$$

$$A_3 = -2D_v Le\delta^4 - \delta^6 + Le^2\left(-D_v^2\delta^2 + Pr^2\left(\pi^2\left((-\pi^2 + q^2)S_h^2 + Ta\right) - \delta^6\right) + \mathrm{Pr}(q^2 R_S + \delta^6)\right);$$

$$A_4 = -Le^2\delta^2;$$

$$B_1 = Pr^2\delta^2\,((D_v^2 Le^2 + 2D_v Le\delta^2)\left(-\pi^2(-1 + Pr)\left((\pi^2 - q^2)S_h^2 - Ta\right) + (1 + Pr)\delta^6\right) + \delta^4\left(LePrq^2 R_S - Le^2 Pr R_S - \pi^2(-1 + Pr)\left((\pi^2 - q^2)S_h^2 - Ta\right) + (1 + Pr)\delta^6\right))\,;$$

$$B_2 = \delta^2\,(LePr(q^2(R_S - LeR_S + Le\pi^2(-1 + Pr)Pr S_h^2) - Le\pi^2(-1 + Pr)\,\mathrm{Pr}(\pi^2 S_h^2 - Ta)) + D_v^2 Le^2(1 + Pr)\delta^2 + 2D_v Le(1 + Pr)\delta^4 + (1 + Pr)(1 + Le^2 Pr^2)\delta^6);$$

$$B_3 = Le^2(1 + Pr)\delta^4;$$

4. Weakly non-linear analysis

To investigate the type of convective motion, non-linear theory is needed. We consider the non-dimensional equations with non linear terms, which are,

$$\frac{1}{Pr}\left(\frac{\partial \boldsymbol{V}}{\partial t}+(\boldsymbol{V}\cdot\nabla)\boldsymbol{V}\right) = -\frac{\nabla P}{Pr}+\nabla^2\mathbf{V}+S_h\mathbf{f} + (R_T\theta - R_S S)\,e_z + Ta^{\frac{1}{2}}(\mathbf{V}\times e_z), \quad (27)$$

$$\frac{\partial\theta}{\partial t}+(V\cdot\nabla)\theta = \omega+\nabla^2\theta \quad (28)$$

$$\frac{\partial S}{\partial t}+(V\cdot\nabla)S = \omega+\frac{1}{Le}\nabla^2 S + D_v(\theta - S), \quad (29)$$

To Study the weakly non-linear theor, we follow the multiple scale analysis. We write the governing equations as follows (after eliminating θ and S),

$$\mathcal{L}w = \mathcal{N} \quad (30)$$

Where,

$$\mathcal{L} = A_1 * A_2 * A_3 + R_S * A_1 * B_1 * \nabla_h^2 + A_3$$

$$\mathcal{N} = \left(B_1 * B_2 * B_3 * S_h - B_1 * B_2 * Ta^{\frac{1}{2}}\right) * N_1 + A_1 * B_1 * B_2 * N_2 + A_3 * N_3 + R_S * B_1 * A_1 * \nabla_h^2 * N_4$$

Where;

$A_1 = \frac{1}{Pr}\frac{\partial}{\partial t} - \nabla^2$; $\quad A_2 = A_1^2\nabla^2 - S_h^2\nabla_h^2\frac{\partial^2}{\partial z^2} + S_h^2\frac{\partial^4}{\partial z^4} + Ta\frac{\partial^2}{\partial z^2}$;

$A_3 = D_v R_S A_1 \nabla_h^2 - R_T A_1 B_2 \nabla_h^2$; $\quad B_1 = \frac{\partial}{\partial t} - \nabla^2$

$B_2 = \frac{\partial}{\partial t} - \frac{\nabla^2}{Le} + D_v$; $\quad B_3 = \nabla_h^2 - \frac{\partial^2}{\partial z^2}$

$N_1 = -\frac{1}{Pr}\nabla\times(\boldsymbol{V}\cdot\nabla)\boldsymbol{V}\cdot e_z$; $\quad N_2 = -\frac{1}{Pr}\nabla\times\nabla\times(\boldsymbol{V}\cdot\nabla)\boldsymbol{V}\cdot e_z$

$N_3 = (\boldsymbol{V}\cdot\nabla)\theta$; $\quad N_4 = (\boldsymbol{V}\cdot\nabla)S$

We write *u, v, w,* θ and S in the series of expansion interms of ϵ,

$$u = \epsilon u_0 + \epsilon^2 u_1 + \epsilon^3 u_3 + \cdots,$$

$$v = \epsilon v_0 + \epsilon^2 v_1 + \epsilon^3 v_3 + \ldots,$$

$$w = \epsilon w_0 + \epsilon^2 w_1 + \epsilon^3 w_3 + \cdots,$$

$$\theta = \epsilon\theta_0 + \epsilon^2\theta_1 + \epsilon^3\theta_3 + \cdots,$$

$$S = \epsilon S_0 + \epsilon^2 S_1 + \epsilon^3 S_3 + \cdots,$$

Where,

$$\epsilon^2 = \frac{Ra - Ra_{sc}}{Ra_{sc}} \ll 1$$

The first approximations are,

$$u_0 = \frac{i\pi}{q_{sc}}[Ae^{iq_{sc}x}cos\pi z - c.c] \qquad (31)$$

$$w_0 = [Ae^{iq_{sc}x}sin\pi z - c.c] \qquad (32)$$

$$\theta_0 = \frac{1}{\delta^2}[Ae^{iq_{sc}x}sin\pi z - c.c] \qquad (33)$$

$$S_0 = \frac{Le(D_v+\delta^2)}{(D_vLe+\delta^2)\delta^2}\left[Ae^{iq_{sc}x}sin\pi z - c.c\right] \quad (34)$$

Where,

c.c – Complex conjugate,

A= A (X, Y, Z, T) – Amplitude

Now the scale variables (X, Y, Z, T) are as follows,

$X = \epsilon x$, $\quad Y = \epsilon^{\frac{1}{2}}y$, $\quad Z= z$, $\quad T = \epsilon^2 t$,

The differential operations can be expressed in the following way bu using these scallings:

$$\frac{\partial}{\partial x} \to \frac{\partial}{\partial x} + \epsilon\frac{\partial}{\partial X},$$

$$\frac{\partial}{\partial y} \to \frac{\partial}{\partial y} + \epsilon^{\frac{1}{2}}\frac{\partial}{\partial Y},$$

$$\frac{\partial}{\partial z} \to \frac{\partial}{\partial Z}$$

$$\frac{\partial}{\partial t} \to \epsilon^2\frac{\partial}{\partial T} \qquad (35)$$

By using Eq. (35), the operators *L* and *N of Eq. (31)* can be written as,

$$\mathcal{L} = \mathcal{L}_0 + \epsilon\mathcal{L}_1 + \epsilon^2\mathcal{L}_3 + \cdots,$$

$$\mathcal{N} = \mathcal{N}_0 + \epsilon\mathcal{N}_1 + \epsilon^2\mathcal{N}_3 + \cdots, \qquad (36)$$

On substituting Eq. (36) into (31), and comparing the coefficients of ϵ, ϵ^2 and ϵ^3, one obtains,

$$\mathcal{L}_0 w_0 = 0, \qquad (37)$$

$$\mathcal{L}_0 w_1 + \mathcal{L}_1 w_0 = \mathcal{N}_0, \qquad (38)$$

$$\mathcal{L}_0 w_2 + \mathcal{L}_1 w_2 + \mathcal{L}_2 w_0 = \mathcal{N}_1. \qquad (39)$$

Where,

$$\mathcal{L}_0 = -\mathrm{D}_v Ta \frac{\partial^2}{\partial Z^2}\nabla^2 - \mathrm{D}_v S_h^2 \frac{\partial^4}{\partial Z^4}\nabla^2 + \frac{1}{Le} Ta \frac{\partial^2}{\partial Z^2}\nabla^4 + \frac{1}{Le} S_h^2 \frac{\partial^4}{\partial Z^4}(\nabla^2)^2 - \mathrm{D}_v(\nabla^2)^4 + \frac{1}{Le}(\nabla^2)^5 - \mathrm{D}_v \mathrm{R}_S \nabla^2\nabla_h^2 + \mathrm{D}_v \mathrm{R}_T \nabla^2\nabla_h^2 + \mathrm{D}_v S_h^2 \frac{\partial^2}{\partial Z^2}\nabla^2\nabla_h^2 + \mathrm{R}_S(\nabla^2)^2\nabla_h^2 - \frac{1}{Le}\mathrm{R}_T(\nabla^2)^2\nabla_h^2 - \frac{1}{Le} S_h^2 \frac{\partial^2}{\partial Z^2}\nabla^2\nabla_h^2.$$

$$\mathcal{L}_1 = \left(-\mathrm{D}_v Ta \frac{\partial^2}{\partial Z^2} - \mathrm{D}_v S_h^2 \frac{\partial^4}{\partial Z^4}\right) 2\frac{\partial^2}{\partial x \partial X} + \left(-\mathrm{D}_v RS + \mathrm{D}_v RT + \mathrm{D}_v S_h^2 \frac{\partial^2}{\partial Z^2}\right) 2\frac{\partial^2}{\partial x \partial X}\nabla^2 + 2\,LeTa \frac{\partial^2}{\partial Z^2}\left(2\frac{\partial^2}{\partial x \partial X}\right)\nabla^2 + 2\,LeS_h^2 \frac{\partial^2}{\partial Z^2}\left(2\frac{\partial^2}{\partial x \partial X}\right)\nabla^2 + (\mathrm{R}_S - \frac{1}{Le}\mathrm{R}_T - \frac{1}{Le} S_h^2 \frac{\partial^2}{\partial Z^2}) 2\frac{\partial^2}{\partial x \partial X}(\nabla^2)^2 - 8\,\mathrm{D}_v \frac{\partial^2}{\partial x \partial X}(\nabla^2)^3 + \frac{10}{Le}\frac{\partial^2}{\partial x \partial X}(\nabla^2)^4 - \mathrm{D}_v \mathrm{R}_S (2\frac{\partial^2}{\partial x \partial X})\nabla_h^2 + (\mathrm{D}_v \mathrm{R}_T + \mathrm{D}_v S_h^2 \frac{\partial^2}{\partial Z^2}) 2\frac{\partial^2}{\partial x \partial X}\nabla_h^2 + (2\mathrm{R}_S - 2\,Le\mathrm{R}_T - \frac{2}{Le} S_h^2 \frac{\partial^2}{\partial Z^2})\left(2\frac{\partial^2}{\partial x \partial X}\right)\nabla^2\nabla_h^2.$$

$$\mathcal{L}_2 = \mathrm{D}_v Ta \frac{\partial}{\partial T}\frac{\partial^2}{\partial Z^2} + \mathrm{D}_v S_h^2 \frac{\partial}{\partial T}\frac{\partial^4}{\partial Z^4} - \mathrm{D}_v Ta \frac{\partial^2}{\partial Z^2}\frac{\partial^2}{\partial X^2} - \mathrm{D}_v S_h^2 \frac{\partial^4}{\partial Z^4}\frac{\partial^2}{\partial X^2} - \mathrm{D}_v \mathrm{R}_S \left(2\frac{\partial^2}{\partial x \partial X}\right)^2 + \left(\mathrm{D}_v \mathrm{R}_T + \mathrm{D}_v S_h^2 \frac{\partial^2}{\partial Z^2} + \frac{1}{Le} Ta \frac{\partial^2}{\partial Z^2} + \frac{1}{Le} S_h^2 \frac{\partial^4}{\partial Z^4}\right)\left(2\frac{\partial^2}{\partial x \partial X}\right)^2 - \frac{\partial}{\partial T}\left(Ta \frac{\partial^2}{\partial Z^2} + \frac{1}{Le} Ta \frac{\partial^2}{\partial Z^2} + S_h^2 \frac{\partial^4}{\partial Z^4} + \frac{1}{Le} S_h^2 \frac{\partial^4}{\partial Z^4}\right)\nabla^2 - \mathrm{D}_v \mathrm{R}_S \frac{\partial^2}{\partial X^2}\nabla^2 + \left(\mathrm{D}_v \mathrm{R}_T + \mathrm{D}_v S_h^2 \frac{\partial^2}{\partial Z^2} + \frac{2}{Le} Ta \frac{\partial^2}{\partial Z^2} + \frac{2}{Le} S_h^2 \frac{\partial^4}{\partial Z^4}\right)\frac{\partial^2}{\partial X^2}\nabla^2 + 2\mathrm{R}_S \left(2\frac{\partial^2}{\partial x \partial X}\right)^2 \nabla^2 + \left(-\frac{2}{Le}\mathrm{R}_T - \frac{2}{Le} S_h^2 \frac{\partial^2}{\partial Z^2}\right)\left(2\frac{\partial^2}{\partial x \partial X}\right)^2 \nabla^2 + \left(\mathrm{R}_S - \frac{1}{Le}\mathrm{R}_T - \frac{1}{Le} S_h^2 \frac{\partial^2}{\partial Z^2}\right)\frac{\partial^2}{\partial X^2}(\nabla^2)^2 - 6\mathrm{D}_v \left(2\frac{\partial^2}{\partial x \partial X}\right)^2 (\nabla^2)^2 + (\mathrm{D}_v \frac{\partial}{\partial T} + \frac{2}{Pr}\mathrm{D}_v \frac{\partial}{\partial T} - 4\mathrm{D}_v \frac{\partial^2}{\partial X^2} + \frac{10}{Le}\left(2\frac{\partial^2}{\partial x \partial X}\right)^2 (\nabla^2)^3 - \frac{\partial}{\partial T}\left(1 + \frac{1}{Le} + \frac{2}{LePr}\right)(\nabla^2)^4 - \frac{5}{Le}(\nabla^2)^4 + \frac{\partial}{\partial T}(\frac{1}{Pr}\mathrm{D}_v \mathrm{R}_S - \frac{1}{Pr}\mathrm{D}_v \mathrm{R}_T - \mathrm{D}_v S_h^2 \frac{\partial^2}{\partial Z^2}\nabla_h^2 + \left(-\mathrm{D}_v \mathrm{R}_S + \mathrm{D}_v \mathrm{R}_T + \mathrm{D}_v S_h^2 \frac{\partial^2}{\partial Z^2}\right)\frac{\partial^2}{\partial X^2}\nabla_h^2 + \left(\mathrm{R}_S - \frac{1}{Le}\mathrm{R}_T - \frac{1}{Le} S_h^2 \frac{\partial^2}{\partial Z^2}\right)\left(2\frac{\partial^2}{\partial x \partial X}\right)^2 \nabla_h^2 + \frac{\partial}{\partial T}(-\mathrm{R}_S - \frac{1}{Pr}\mathrm{R}_S + \mathrm{R}_T + \frac{1}{LePr}\mathrm{R}_T + S_h^2 \frac{\partial^2}{\partial Z^2} + \frac{1}{Le} S_h^2 \frac{\partial^2}{\partial Z^2})\nabla^2\nabla_h^2 + (2\mathrm{R}_S - \frac{2}{Le}\mathrm{R}_T - \frac{2}{Le} S_h^2 \frac{\partial^2}{\partial Z^2})\frac{\partial^2}{\partial X^2}\nabla^2\nabla_h^2$$

Let us substitute the solution $\mathcal{L}_0 w_0 = 0$ and one obtains,

$$R_{T_{sc}} = \frac{D_v\, Le(-\pi^4 S_h^2 + q^2(R_S + \pi^2 S_h^2) + \pi^2 Ta + \delta^6) + \delta^2(Leq^2 R_S - \pi^4 S_h^2 + \pi^2(q^2 S_h^2 + Ta) + \delta^6)}{q^2(D_v Le + \delta^2)} \quad (40)$$

On substitute Eqs. (31) to (34) in $\mathcal{L}_0 w_1 + \mathcal{L}_1 w_0 = \mathcal{N}_0$ we get $\mathcal{N}_0 = 0$ and $\mathcal{L}_0 w_0 = 0$.

The equation reduces to,

$$w_1 = 0, \quad (41)$$

$$u_1 = 0, \quad (42)$$

$$\theta_1 = \frac{1}{4\delta^2\pi}|A|^2 Sin2\pi z, \quad (43)$$

$$S_1 = -\frac{\pi Le(\mathrm{D}_v + \delta^2)}{\delta^2(\delta^2 + \mathrm{D}_v Le)(4\pi^2 + Le\mathrm{D}_v)}|A|^2 Sin2\pi z \quad (44)$$

On substituting these solutions in Eq. (39), we obtain the Newell-Whitehead equation in the form of,

$$\lambda_0\frac{\partial A}{\partial T} - \lambda_1\left(\frac{\partial}{\partial X} - \frac{i}{2q}\frac{\partial^2}{\partial Y^2}\right)^2 A - \lambda_2 A + \lambda_3|A|^2 A = 0. \quad (45)$$

Where,

$$\lambda_0 = -\mathrm{D}_v Ta\pi^2 + \mathrm{D}_v S_h^2\pi^4 - \left(Ta + \frac{Ta}{Le} - S_h^2\pi^2 + \frac{S_h^2}{Le}\pi^2\right)\pi^2\delta^2 - \left(\mathrm{D}_v + \frac{2}{Pr}Dv\right)\delta^6 + \left(1 + \frac{1}{Le} + \frac{2}{LePr}\right)\delta^8 - \left(\frac{\mathrm{D}_v\mathrm{R}_S}{Pr} - \frac{\mathrm{D}_v\mathrm{R}_T}{Pr} + \mathrm{D}_v S_h^2\pi^2\right)q^2 + \left(-\mathrm{R}_S - \frac{\mathrm{R}_S}{Pr} + \mathrm{R}_T + \frac{\mathrm{R}_T}{LePr} - S_h^2\pi^2 - \frac{S_h^2}{Le}\right)\delta^2 q^2, \quad (46)$$

$$\lambda_1 = -\mathrm{D}_v\mathrm{R}_S + \mathrm{D}_v\mathrm{R}_T + \left(\mathrm{D}_v S_h^2 + \frac{Ta}{Le} - \frac{S_h^2}{Le}\pi^2\right) - 2\left(\mathrm{R}_S - \frac{\mathrm{R}_T}{Le} - \frac{S_h^2}{Le}\right)\delta^2 - 6Dv\delta^4 - \frac{10}{Le}\delta^6 - \left(\mathrm{R}_S - \frac{\mathrm{R}_T}{Le} + \frac{S_h^2}{Le}\pi^2\right)q^2, \quad (47)$$

$$\lambda_2 = \mathrm{R}_T\delta^2 q^2\left(\mathrm{D}_v + \frac{1}{Le}\delta^2\right), \quad (48)$$

$$\lambda_3 = \frac{\delta^2 q^2\pi^2\mathrm{R}_S Le^2(\mathrm{D}_v + \delta^2)}{(\delta^2 + Le\mathrm{D}_v)(4\pi^2 + Le\mathrm{D}_v)} - \frac{1}{4}\mathrm{D}_v\mathrm{R}_S q^2 + \mathrm{R}_T q^2\left(\delta^2 + \frac{\mathrm{D}_v Le}{4Le}\right) \quad (49)$$

Considering only the x-dependence terms in Eq. (46), one obtains,

$$\frac{d^2 A}{dX^2} + \frac{\lambda_2}{\lambda_1}\left(1 - \frac{\lambda_3}{\lambda_1}|A|^2\right)A = 0, \quad (50)$$

$$\therefore A(X) = A_0 tanh\left(\frac{X}{\Lambda_0}\right). \quad (51)$$

Where,

$$A_0 = \left(\frac{\lambda_2}{\lambda_1}\right)^{\frac{1}{2}}; \Lambda_0 = \left(\frac{2\lambda_1}{\lambda_2}\right)^{\frac{1}{2}}$$

5. Heat transport by convection

We obtain the maximum of sdteady amplitude ($|A_{max}|$) from Eq. (51) as

$$(|A_{max}|) = \left(\frac{\epsilon^2\lambda_2}{\lambda_3}\right)^{\frac{1}{2}};$$

Nusselt number is defined as follows in relation to amplitude *A;*

$$Nu = 1 + \frac{\epsilon^2}{|\delta_{sc}^2|}|A_{max}|^2,$$

From this, we obtain convection for R > $R_{T_{sc}}$ and conduction for R ≤ $R_{T_{sc}}$. From Eq. (45), is valid for > 0 which is possible when R > $R_{T_{sc}}$, this we get,

1) *Nu > 1,* for convection
2) *Nu* ≤ 1, for conduction (see in Fig. 12).

6. Results & Discussion

Numerous theoretical and experimental research have attempted to represent the convective instability problem in the study of geographical fluid dynamics: the numerical results and cinclusions are discussed in this part. For the beginning of stationary and oscillatory instability, the Critical Rayleigh numbers have been found using the one term Galerkin weighted residual approach.

Figs. 1-6 shows the neutral stability curves at the onset of Oscillatory convection for the differen values of the physical parameters.

Figure 1 illustrates the neutral stability curves for the parametric plane's (q, $R_{T_{oc}}$) exchange of stabilities with distict values of Lewis numberLe (Le=0.5,0.4,0.3,0.2,0.1) and for the values Rs = 10, Pr = 6, Sh = 3, Dv = 4, Ta = 15. For each assignment the Lewis number, Solutal Rayleigh number R_s, Solutal Diffusivity Dv, Prandtle number Pr, Helical force parameter Sh and Taylor number Ta a neutral stability curve has been displayed. This figure shows that there is instability in the system as the neutral stability curves trend monotonically downward as *Le* decreases. Stated differently, a decrease in Le has a destabilizing effect. This figure shows that there is instability in the system as the neutral stability curves trend monotonically downward as Le decreases. Stated differently, a decrease in *Le* has a destabilizing effect.

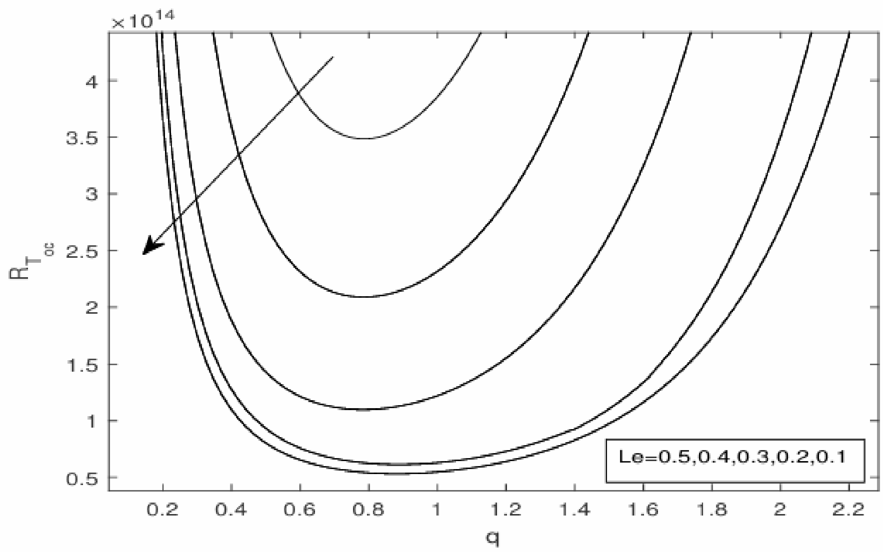

Figure 1: Neutral curves at the onset of Oscillatory convection for the various value of Le and fir the fixed values of Rs = 10, Pr = 6, Sh = 3, Dv = 4 and Ta =15.

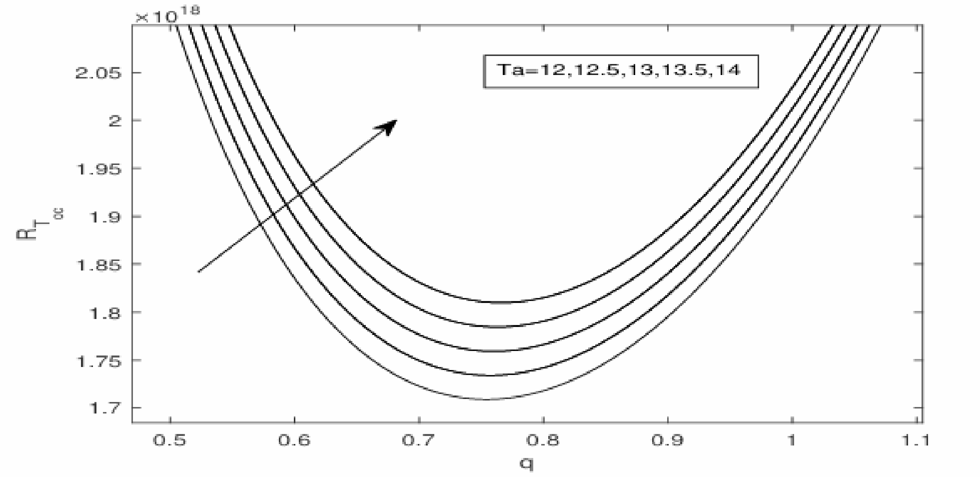

Figure 2: Neutral curves at the onset of Oscillatory convection for the various value of Ta and fir the fixed values of Rs = 10, Pr = 10, Sh = 3, Dv = 4 and Le = 5.

The Critical Rayleigh number variation with Ta for fixexed values of Rs = 10, Le = 6, Sh = 3, Dv = 4, Pr = 10 is depicted in Figure 2. Various frames with Ta = 12, 12.5, 13, 13.5, 14 are taken into consideration. As the values of Ta increase, it also raises the Critical Rayleigh number. Consequently, a rise in Ta values has the effect of bringing the system to equilibrium.

Figure 3 refers to the exchange of stabilities and shows neutral curves in the plane (q, $R_{T_{oc}}$) for fixed values of Rs = 10, Le = 5, Sh= 3, Dv = 4, Ta = 14 and for various values of Solutal diffusivity Dv. We consider several frames with Dv = 4, 5,6,7,8. This picture shows that as D_v increases, the neutral stability curves rise upward monotonically, indicating that a significant stabilization of the system.

Figure 4 shows how the fixed values of Rs = 10, Le = 6, Sh = 3, Ta = 15, Dv = 4 and effect the various values of Pr. Various frames are taken into consideration using Pr = 0.1, 0.15, 0.2, 0.25, 0.3. The critical Rayleigh number rises as the Prandtle number grows, according to the effect of Prandtle number on the neutral stability curves for the onset of Oscillatory convection. That is, the system becomes stable when the values of Pr increase. The Critical Rayleigh number change with wave number for distinct values of Solutal Rayleigh number R_S and fixed values of Pr = 10, Le = 5, Sh = 3, Dv = 4, Ta = 15 is depicted in Figure 5. We explore various frames with Rs = 10,11,12,13,14. The Solutal Rayleigh number's impact on the neutral stability curves at the onset of oscillatory convection. It demonstrates that as the Solutal Rayleigh number increases, the critical Rayleigh number falls. That is, a rise in R_S values causes the system to become unstable.

The Critical Rayleigh number variation with wave number for various values S_h for fixed values of Rs = 10, Le = 5, Ta = 15, Dv = 4, Pr = 10 is depicted in Figure 6. Various frames are taken into consideration using S_h = 1, 1.25, 1.5, 1.75, 2. When Oscillatory convection begins, the Critical Rayleigh number rises in parallel with increases in Sh values. Consequently, a rise in Sh values has the effect of bringing the system into equilibrium.

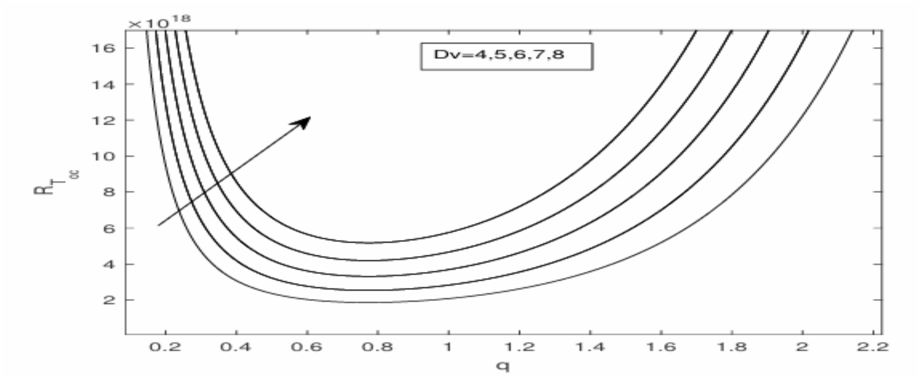

Figure 3: Neutral curves at the onset of Oscillatory convection for the various values of Dv and for the fixed values of RS = 10, Le = 5, Sh = 3, Pr = 10, Ta = 15.

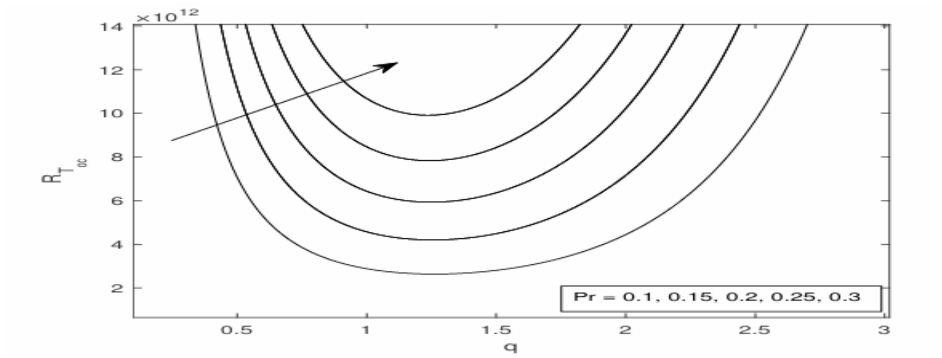

Figure 4: Neutral curves at the onset of Oscillatory convection for the various values of Pr and for the fixed values of RS = 10, Le = 5, Sh = 3, Ta = 15, Dv = 4.

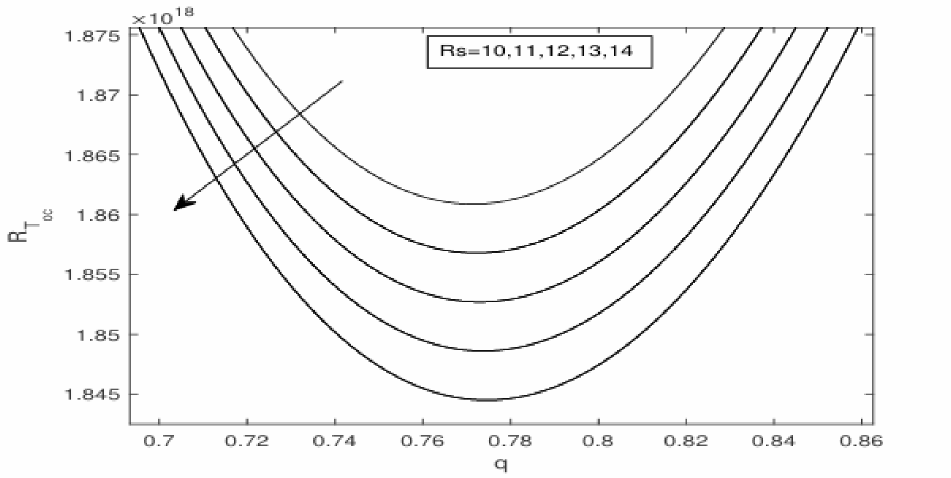

Figure 5: Neutral curves at the onset of Oscillatory convection for the various values of RS and for the xed values of Pr = 10, Le = 5, Sh = 3, Pr = 10, Dv = 4.

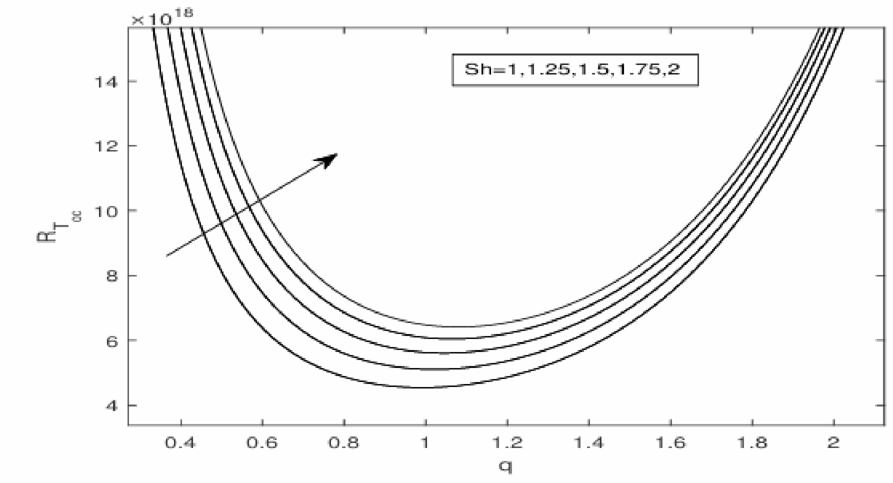

Figure 6: Neutral curves at the onset of Oscillatory convection for the various values of Sh and for the xed values of RS = 10, Le = 5, Ta = 15, Pr = 10, Dv = 4.

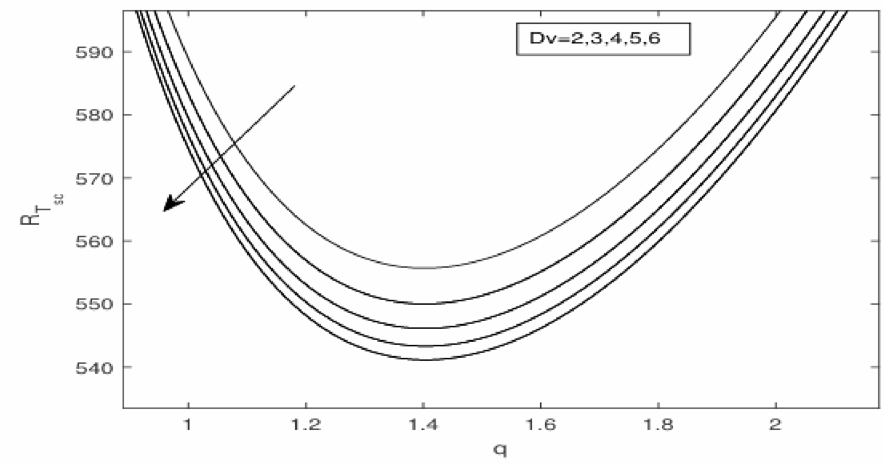

Figure 7: Neutral curves at the onset of Oscillatory convection for the various values of Dv and for the xed values of RS = 14, Le = 5, Sh = 3, Pr = 10, Ta = 5.

The neutral stability curves at the onset of stationary convection for the di erent physical parameter values are displayed in Figs. 7-11.

Figure 7 illustrates how the solutal di usivity parameter D_v effects the stationary convection inset. It can be seen from the graph that, for rising values, the solutal diffusivity parameter pushes back the start of convection for varying D_v values and constant Rs = 14, Le = 5, Sh = 3, Ta = 5, Pr = 10 values. We investigate various frames with Dv = 2,3,4,5,6. As the values of D_v grow, the Critical Rayleigh number falls. Consequently, a rise in Dv values has the consequence of making the system unstable.

Figure 8 shows how the critical Rayleigh number effects when convection starts. It is evident from the figure that stationary convection is stabilized by the Lewis number. Rs, Dv, Sh, Ta, Pr fixed values are shown in the graph, and Several frames are taken into consideration when the Lewis number is increased for $Le \in (20,60)$. The effect of the Lewis number on the neutral stability curves at the onset of stationary convection shows that the critical Rayleigh number increases in parallel with the Solutal Rayleigh number. In other words, when the values of Le rise, the system becomes stable.

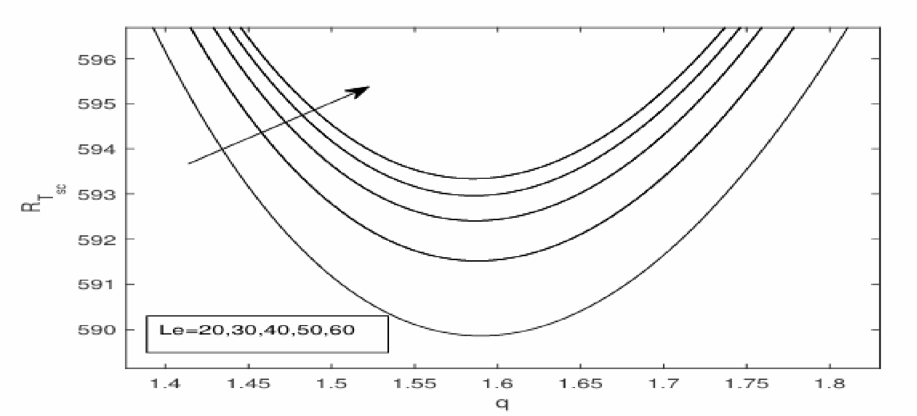

Figure 8: Neutral curves at the onset of stationary convection for the various values of Le and for the xed values of RS = 14, Dv = 4, Sh = 3, Pr = 10, Ta = 6.

Figure 9 illustrates how the Critical Rayleigh number varies with wave number and how the fixed values of Dv = 4, Le = 20, Sh = 3, Ta = 15, Pr = 6 change for varying values of Rs. Various frames are examined with Rs = 10,11,12,13, 14. As the values of Rs increase, correspondingly increases the Critical Rayleigh number. Consequently, a rise in Rs values has the effect of bringing the system into balance. Figure 10 gives a representation of $R_{T_{sc}}$versus q with distinct values of Sh while for Sh ∈ (1,2) the fixed values of Dv = 4, Le = 5, Rs = 14, Ta = 15, Pr = 10. This figure illustrates how the enhancement of $R_{T_{sc}}$ is accompanied by a rise in the value of q. On the other hand, stationary Rayleigh falls as Sh increases. In other words, the system becomes unstable when the Sh numbers increase.

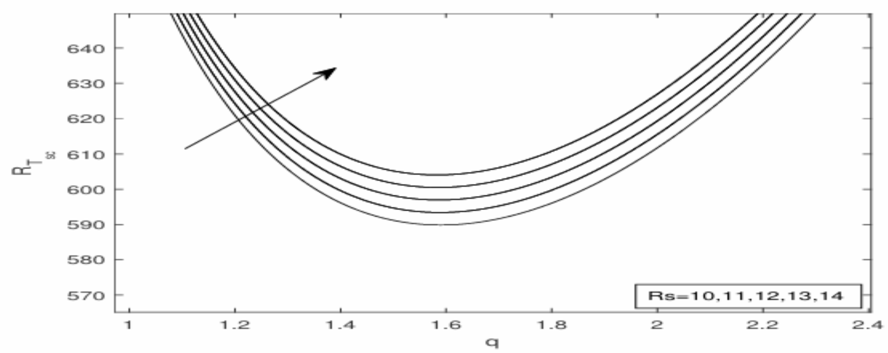

Figure 9: Neutral curves at the onset of stationary convection for the various values of Rs and for the xed values of Dv = 4, Le = 20, Sh = 3, Pr = 6, Ta = 15.

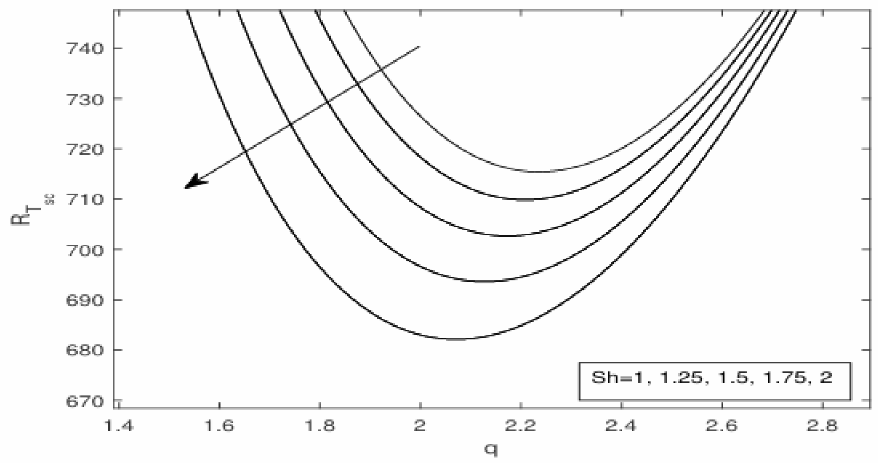

Figure 10: Neutral curves at the onset of stationary convection for the various values of Sh and for the xed values of RS = 14, Le = 5, Dv = 4, Pr = 10, Ta = 15.

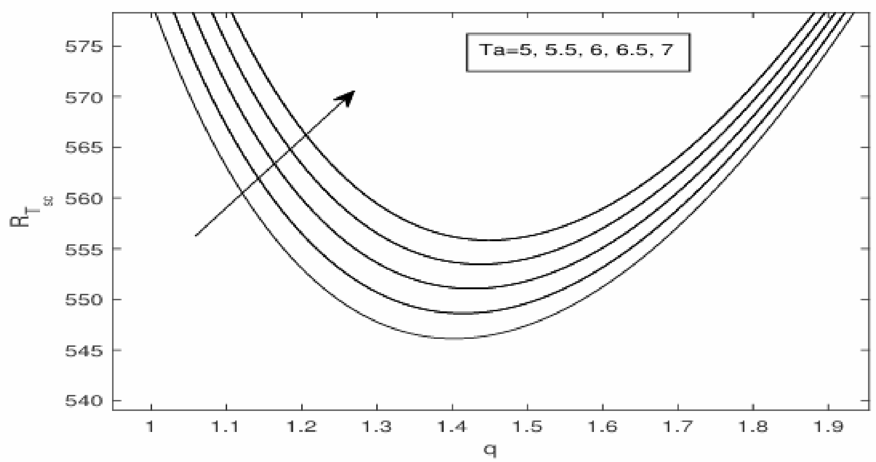

Figure 11: Neutral curves at the onset of stationary convection for the various values of Ta and for the xed values of RS = 14, Le = 5, Sh = 3, Pr = 10, Dv = 4.

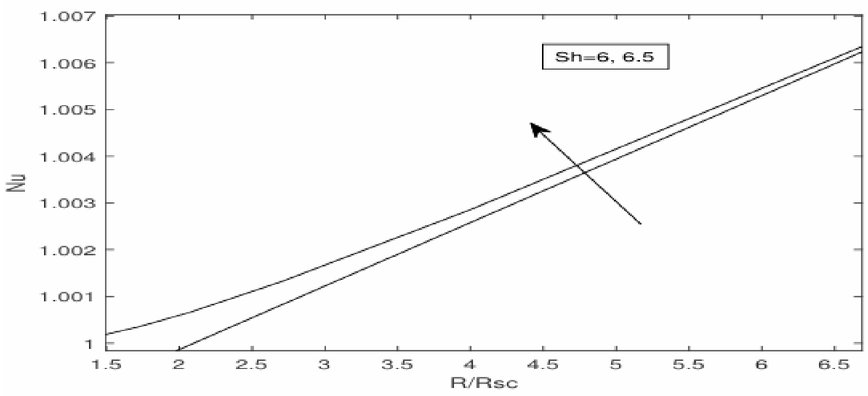

Figure 12: The Figure is plotted for RS = 1.5, Pr = 20, Dv = 1.5, Ta = 5, and Le = 15

With diverse values of Ta, $R_{T_{sc}}$ vs q is visually shown in Fig. 11. For Ta, the fixed values of Rs = 14, Le = 5, Sh = 3, Dv = 4, Pr = 10 are present. This graphic shows how the value of q increases along with the enhancement of $R_{T_{sc}}$. However, when Ta increases, stationary Rayleigh increases. That is, the system becomes stable when the values of Ta increase.

7. Conclussions

This work considers the convective instability problem of a general viscous fluid with helical force effect. Analyses the both weakly nonlinear and linear. To study the linear and weakly nonlinear behaviors, two different approaches are used. The linear theory is investigated using the Galerkin approach, while the weakly nonlinear theory is explored through multiple scale analysis. Surprisingly, the Prandtl number exhibit no discernible impact on stationary convection. But in its Lewis number, Solutal Rayleigh number, Taylor number parameters are stabilizing the flow and Solutal diffusivity, helical force parameters are destabilizing the flow. While, in oscillatory convection it is found that Solutal Rayleigh number, Lewis number parameters are destabilizing the flow and helical force, Prandtl number, Taylor number, Solutal diffusivity parameters are stabilizing effect on the flow. Amplitude equation is derived in weakly nonlinear analysis.

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