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Dom-Chromatic Number of Corona Product of Path with Cycle and Path Related Graphs

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Abstract:-Graph theory has emerged as an important field of study owing to its wide range of practical applications. A proper coloring of a graph G is an assignment of different colors to adjacent vertices and the minimum number of colors required is called the chromatic number denoted by $\chi(G)$. A set $D' \subseteq V(G)$ is called a dominating set if each vertex in $V(G)\backslash D'$ has at least one neighbor in D', the smallest possible size of such a set is represented by $\gamma(G)$. By combining these two concepts, the notion of dom coloring was introduced. A dom coloring set(dc-set) is a dominating set if it includes a vertex from each color class. The minimum size of a dc-set is referred as the dom chromatic number and is symbolized by $\gamma_{dc}(G)$. In this study, we determine the dom chromatic number for the corona product of a path P_m with certain cycle and path related graphs namely wheel graph, gear graph, ladder graph, grid graphs.

Keywords: *Dominating set, Dom coloring set, Dom chromatic number, Corona product.*

1. Introduction

Graph theory has emerged as a significant branch of mathematics owing to its extensive applications in various domains. Among the numerous graph parameters graph coloring and domination have been widely studied because of their fundamental theoretical importance and practical relevance. Graph coloring is concerned with assigning colors to the vertices of a graph such that no two adjacent vertices receive the same color, thereby partitioning the vertex set into independent color classes. On the other hand, domination deals with identifying a subset of vertices that collectively dominates the graph by ensuring that every vertex either belongs to the subset or is adjacent to a vertex in it. The concept of graph coloring originated from the celebrated Four Color Problem, proposed by Francis Guthrie in 1852, which asks whether every planar graph can be properly colored using at most four colors so that adjacent regions are assigned distinct colors. Since its introduction, graph coloring has evolved into one of the most active research areas in graph theory and has inspired numerous extensions that integrate coloring with other graph invariants, including domination. In 1879, Arthur Cayley presented the first formal academic work on graph coloring, in which he examined the problem of coloring maps and the mathematical principles behind it [1]. In 1962, Ore presented the concept of domination in graph theory in his book Theory of Graphs, where he introduced conditions related to dominating sets. [13]. Recently, a number of new concepts in graph theory have been proposed by integrating both domination and coloring. One such problem is called dom coloring problem where the dom chromatic number is determined by selecting minimum dom coloring set, that satisfies certain condition called the dom coloring set (DC- condition). The dom-coloring problem was introduced in 2010 by T. N. Janakiraman and M. Poobalaranjani by defining the dom-coloring set S, such that $\chi(S) = \chi(G)$ [7]. Later in 2016, Chaluvaraju and Appajigowda introduced the dom-chromatic number of a graph, denoted by $\gamma_{dc}(G)$.defined as the minimum cardinality of a dominating set that contains at least one vertex from each color class of G [3]. Research on the dom-chromatic number of cycle-based graphs was carried out and documented in 2020 [15], and for splitting graphs of path and comb graphs in 2021 [8] for caterpillar, coconut tree, lobster, tadpole, pan and lollipop graphs, mobius ladder and circular ladder in 2022 [9,10], for wrapped butterfly & bloom graphs in 2023 [11]. In many practical applications, domination or coloring alone is inadequate for effective network analysis. The dom chromatic number combines these concepts by identifying a dominating set that contains a representative from every color class, making it useful in communication networks, optimization, and other systems requiring both coverage and conflict-free assignments.

In this paper the dom-chromatic number for the corona product of a path with cycle and path related graphs namely wheel graph, gear graph, ladder graph, grid graphs are determined.

2. Preliminaries

This section focuses on some basic definitions and results on dom chromatic number.

Definition 2.1. [2,6] A proper vertex coloring of a graph G assigns different colors to adjacent vertices. A set of vertices sharing a color is a color class.

Definition 2.2.[2,6] The chromatic number $\chi(G)$ is the minimum number of colors needed to color a graph G so that adjacent vertices receive different colors.

Definition 2.3.[2] A non-empty subset $D' \subset V(G)$ is a dominating set if every $v \in V(G) - D'$ is adjacent to some $u \in D'$. If D' has the minimum vertices, then it is the minimum dominating set, whose size is the domination number denoted by $\gamma(G)$.

Definition 2.4.[3,8,9] DC- condition: Let G be a graph and let $\{C_1, C_2, C_3, \dots, C_{\chi(G)}\}$ be the color classes of a proper $\chi(G)$-coloring of G. A dominating set $D' \subseteq V(G)$ is called a dom-coloring set (dc-set) if $D' \cap C_i \neq \emptyset, 1 \leq i \leq \chi(G)$, that is, D' contains at least one vertex from every color class. The minimum cardinality of a dom-coloring set is called the dom-chromatic number of G and is denoted by $\gamma_{dc}(G)$.

Definition 2.5. [5] For two graphs G' and G'' having m and n vertices, respectively, the corona product of G' & G'' is formed by taking a copy of G' together with m copies of G'', and joining the i[th] vertex of G' with the corresponding vertices of i[th] copy of G'', for $i = 1, 2, 3, \dots, m$ and it is denoted by $G' \odot G''$.

Definition 2.6. [2] A walk is a finite sequence $W = v_0 e_1 v_1 e_2 v_2 \dots e_k v_k$, whose terms are alternately vertices $v_i, 0 \leq i \leq k$ and edges $e_i, 1 \leq i \leq k$ such that the ends of e_i are v_{i-1} and v_i. If the vertices $v_0, v_1, v_2, \dots, v_k$ are distinct, then W is a path.

Definition 2.7.[2] A wheel graph $W_{1,n}$ is obtained by adding a new vertex u (called the hub) to a cycle graph C_n of order n, and joining u to every vertex of the cycle C_n.

Definition 2.8.[14] A gear graph G_n is obtained from the wheel graph $W_{1,n}$ by inserting a new vertex between every pair of adjacent vertices on the outer cycle (rim).

Definition 2.9.[12] A ladder graph L_n is a planar graph that can be viewed as two parallel path graphs P_n connected by n rungs, where each rung is an edge joining the corresponding vertices of the two paths. Equivalently, the ladder graph is the cartesian product of the path graph P_n and the path graph P_2, denoted by $P_n \text{ X } P_2 = L_n$, where X represents the cartesian product of graphs.

Definition 2.11.[4] A grid graph is the cartesian product of two path graphs, $P_m \text{ X } P_n$, where P_m has m vertices and P_n has n vertices. It consists of a grid-like structure with m rows and n columns of vertices. Equivalently, the graph has mn vertices, with two vertices adjacent if and only if they are adjacent in one factor and identical in the other.

Theorem 2.1.[3] For any graph G, $max\,\{\gamma(G), \chi(G)\} \leq \gamma_{dc}(G) \leq \gamma(G) + \chi(G) - 1$.

Theorem 2.2.[5] For any two graphs G &H, $max\{\chi(G), \chi(H)\} \leq \chi(G \odot H) \leq max\,\{\chi(G), \chi(H) + 1\}$.

3. Main Results

In this section, we determine the dom chromatic number of corona product of path graphs with wheel, gear, ladder and grid graphs.

3.1. Construction of Corona Product of a path P_m with $W_{1,n}$

Let P_m be a path graph with m vertices. The vertices of P_m are labelled as $\{v_1, v_2, v_3, \dots, v_m\}$. The corona product $P_m \odot W_{1,n}$ is obtained by taking one copy of P_m together with m copies of the wheel graph $W_{1,n}$, and joining the i[th] vertex of P_m to every vertex of the i[th] copy of $W_{1,n}$, for $i = 1,2,\ldots,m$. Let the vertices of the i[th] copy of $W_{1,n}$ be labelled as $\left\{u^{(i)}, v_1^{(i)}, v_2^{(i)}, \ldots v_n^{(i)}\right\}$, where $u^{(i)}$ denotes the hub vertex and $\left\{v_1^{(i)}, v_2^{(i)}, \ldots v_n^{(i)}\right\}$, are the vertices on the rim cycle C_n, for $i = 1,2,\ldots,m$. Hence, the resulting graph $P_m \odot W_{1,n}$ has $m + m(n+1) = m(n+2)$ vertices.

Theorem 3.1

Let P_m and $W_{1,n}$ be the path and wheel graph respectively with $m = 2,3,4,\ n \geq 3,\ n$ is odd. Then the dom chromatic number of corona product of P_m and $W_{1,n}$ is $\gamma_{dc}(P_m \odot W_{1,n}) = 5$.

Proof. Let $G = P_m \odot W_{1,n}, m = 2,3,4,\ n \geq 3,\ n$ is odd. Let $V(P_m) = \{v_1, v_2, v_3, \ldots, v_m\}$ and the vertices of the i[th] copy of $W_{1,n}$ be $\left\{u^{(i)}, v_1^{(i)}, v_2^{(i)}, \ldots, v_n^{(i)}\right\}$. It is clear that $\gamma(G) = m$, since the vertices of path P_m is enough to dominate the vertices of $W_{1,n}$. Also, $\chi(P_m) = 2$ as in [6],

$$\chi(W_{1,n}) = \begin{cases} 3, & if\ n\ is\ even \\ 4, & if\ n\ is\ odd \end{cases}$$

Since each vertex of P_m is adjacent to every vertex of the corresponding copy of $W_{1,n}$, we require one more color for the proper coloring of G. Hence,

$$\chi(G) = \begin{cases} 4, & if\ n\ is\ even \\ 5, & if\ n\ is\ odd \end{cases}$$

By Theorem 2.1, $\gamma_{dc} \geq max\{m, 5\}$. Hence, $\gamma_{dc}(G) \geq 5$.

We color the vertices of G as follows. Assign the color i (taken modulo 5) to $v_i, 1 \leq i \leq m$ and assign the colors $i+1, i+2$ (taken modulo 4) to the vertices of the i[th] copy of $W_{1,n}$ adjacent to v_i in that order in the clockwise direction starting from $v_1^{(i)}$ to $v_{n-1}^{(i)}$. Assign the color $i+3$ (taken modulo 4) to $v_n^{(i)}$. Assign the color $i+4$ (taken modulo 4) to $u^{(i)}$. Clearly, the above coloring is a proper coloring of G Refer Figure 1.

$$\text{Let } D = \begin{cases} V(P_m) \cup \left\{v_{n-1}^{(i)}, v_n^{(i)}, u^{(i)}\right\}, & if\ m = 2 \\ V(P_m) \cup \left\{\ v_n^{(i)}, u^{(i)}\ \right\}, & if\ m = 3 \\ V(P_m) \cup \left\{\ u^{(i)}\ \right\}, & if\ m = 4 \end{cases}$$

Clearly, D is a dc-set of G, which implies that $\gamma_{dc}(G) \leq 5$. Thus, $\gamma_{dc}(G) = 5$.

Theorem 3.2

Let P_m and $W_{1,n}$ be the path and wheel graph respectively with $m = 2,3,4, n \geq 3,\ n$ is even. Then the dom chromatic number of corona product of P_m and $W_{1,n}$ is $\gamma_{dc}(P_m \odot W_{1,n}) = 4$.

Proof. Let $G = P_m \odot W_{1,n}, m = 2,3,4,\ n \geq 3,\ n$ is even. Let $V(P_m) = \{v_1, v_2, v_3, \ldots, v_m\}$ and the vertices of the i[th] copy of $W_{1,n}$ be $\left\{u^{(i)}, v_1^{(i)}, v_2^{(i)}, \ldots v_n^{(i)}\right\}$. It is clear that $\gamma(G) = m$, since the vertices of path P_m is enough to dominate the vertices of $W_{1,n}$. Also, $\chi(P_m) = 2$ as in [6],

$$\chi(W_{1,n}) = \begin{cases} 3, & if\ n\ is\ even \\ 4, & if\ n\ is\ odd \end{cases}$$

Since each vertex of P_m is adjacent to every vertex of the corresponding copy of $W_{1,n}$, we require one more color for the proper coloring of G. Hence,

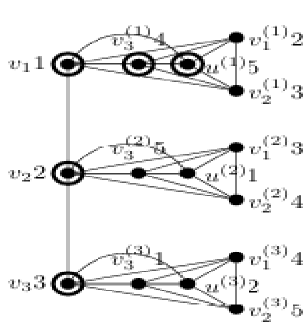
$$\chi(G) = \begin{cases} 4, & if\ n\ is\ even \\ 5, & if\ n\ is\ odd \end{cases}$$

By Theorem 2.1, $\gamma_{dc} \geq max\{m, 4\}$. Hence, $\gamma_{dc}(G) \geq 4$.

We color the vertices of G as follows. Assign the color i (taken modulo 4) to $v_i, 1 \leq i \leq m$ and assign the colors $i+1, i+2$ (taken modulo 4) to the vertices of the i$^{\text{th}}$ copy of $W_{1,n}$ adjacent to v_i in that order in the clockwise direction starting from $v_1^{(i)}$ to $v_n^{(i)}$. Assign the color $i+4$ (taken modulo 4) to $u^{(i)}$. Clearly, the above coloring is a proper coloring of G. Refer Figure 2.

$$\text{Let } D = \begin{cases} V(P_m) \cup \left\{v_n^{(i)}, u^{(i)}\right\}, & if\ m = 2 \\ V(P_m) \cup \{\, u^{(i)} \,\}, & if\ m = 3 \\ V(P_m), & if\ m = 4 \end{cases}$$

Clearly, D is a dc-set of G, which implies that $\gamma_{dc}(G) \leq 4$. Thus, $\gamma_{dc}(G) = 4$.



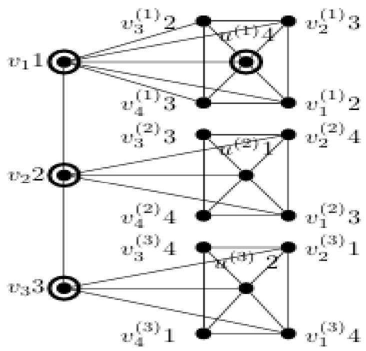


Figure 1: Encircled vertices form the dom coloring set $\{v_1, v_2, v_3, v_3^{(1)}, u^{(1)}\}, \gamma_{dc}(P_3 \odot W_{1,3}) = 5$.
Figure 2: Encircled vertices form the dom coloring set $\{v_1, v_2, v_3, u^{(1)}\}, \gamma_{dc}(P_3 \odot W_{1,4}) = 4$.

Theorem 3.3

Let P_m and $W_{1,n}$ be the path and wheel graph respectively with $m \geq 5, n \geq 3$. Then the dom chromatic number of corona product of P_m and $W_{1,n}$ is $\gamma_{dc}(P_m \odot W_{1,n}) = m$.

Proof. The proof is similar to that of Theorem 3.1 and Theorem 3.2 for n is even and odd respectively. Refer Figure 3.

By Theorem 2.1, $\gamma_{dc} \geq max\{m, 4\}$. Hence, $\gamma_{dc}(G) \geq m$.

Let $D = V(P_m)$.

Clearly, D is a dc-set of G, which implies that $\gamma_{dc}(G) \leq m$. Thus, $\gamma_{dc}(G) = m$.

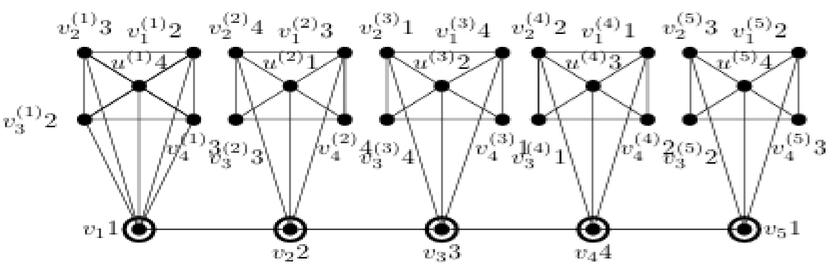


Figure 3: Encircled vertices form the dom coloring set $\{v_1, v_2, v_{3,} v_4, v_5\}, \gamma_{dc}(P_5 \odot W_{1,4}) = 5$

3.2 Construction of Corona Product of a path P_m with G_n

Let P_m be a path graph with m vertices. The vertices of P_m are labelled as $\{v_1, v_2, v_{3,}, \ldots, v_m\}$,. The corona product $P_m \odot G_n$ is obtained by taking one copy of P_m together with m copies of the gear graph G_n, and joining the i$^{\text{th}}$

vertex of P_m to every vertex of the i[th] copy of G_n, for $i = 1,2,3,\ldots,m$. Let the vertices of the i[th] copy of G_n be labelled as $\left\{u^{(i)}, v_1^{(i)}, v_2^{(i)}, \ldots v_{2n}^{(i)}\right\}$, where $u^{(i)}$ denotes the hub vertex and $\left\{v_1^{(i)}, v_2^{(i)}, \ldots v_{2n}^{(i)}\right\}$ are the vertices on the rim cycle C_{2n}, for $i = 1,2,3,\ldots,m$. Hence, the resulting graph $P_m \odot G_n$ as $m + m(2n+1)$ vertices.

Theorem 3.4

Let P_m and G_n be the path and gear graph respectively with $m \geq 2$ and $n \geq 3$. Then the dom chromatic number of the corona product $P_m \odot G_n$ is given by,

$$\gamma_{dc}(P_m \odot G_n) = \begin{cases} 3, & if\ m = 2, n \geq 3 \\ m, & if\ m \geq 3, n \geq 3 \end{cases}$$

Proof. Let $G = P_m \odot G_n, m \geq 2$ and $n \geq 3$. Let $V(P_m) = \{v_1, v_2, v_3, \ldots, v_m\}$ and the vertices of the i[th] copy of G_n be $\left\{u^{(i)}, v_1^{(i)}, v_2^{(i)}, \ldots v_{2n}^{(i)}\right\}$. It is clear that $\gamma(G) = m$, since the vertices of path P_m is enough to dominate the vertices of G_n. Also, $\chi(P_m) = 2$. Since the gear graph G_n is bipartite, its chromatic number is $\chi(G_n) = 2$.

Since each vertex of P_m is adjacent to every vertex of the corresponding copy of G_n, we require one more color for the proper coloring of G. Hence, $\chi(G) = 3$.

Case (i) When $m = 3, n \geq 3$

By Theorem 2.1, $\gamma_{dc} \geq max\{m, 3\}$. Hence, $\gamma_{dc}(G) \geq 3$.

We color the vertices of G as follows. Assign the color i (taken modulo 3) to $v_i, 1 \leq i \leq m$ and assign the colors $i+1, i+2$ (taken modulo 3) to the vertices of the i[th] copy of G_n adjacent to v_i in that order in the clockwise direction starting from $v_1^{(i)}$ to $v_{2n}^{(i)}$. Assign the color $i+2$ (taken modulo 3) to $u^{(i)}$. Clearly, the above coloring is a proper coloring of G. Refer Figure 4.

Let $D = V(P_m) \cup \{u^{(i)}\}$.

Clearly, D is a dc-set of G, which implies that $\gamma_{dc}(G) \leq 3$. Thus, $\gamma_{dc}(G) = 3$.

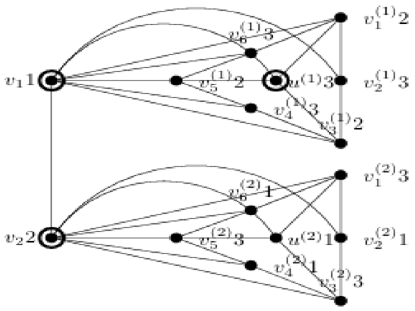
Case (ii) When $m \geq 3, n \geq 3$

By Theorem 2.1, $\gamma_{dc} \geq max\{m, 3\}$. Hence, $\gamma_{dc}(G) \geq m$.

We color the vertices of G as follows. Assign the color i (taken modulo 3) to $v_i, 1 \leq i \leq m$ and assign the colors $i+1, i+2$ (taken modulo 3) to the vertices of the i[th] copy of G_n adjacent to v_i in that order in the clockwise direction starting from $v_1^{(i)}$ to $v_{2n}^{(i)}$. Assign the color $i+2$ (taken modulo 3) to $u^{(i)}$. Clearly, the above coloring is a proper coloring of G. Refer Figure 5.

Let $D = V(P_m)$.

Clearly, D is a dc-set of G, which implies that $\gamma_{dc}(G) \leq m$. Thus, $\gamma_{dc}(G) = m$.



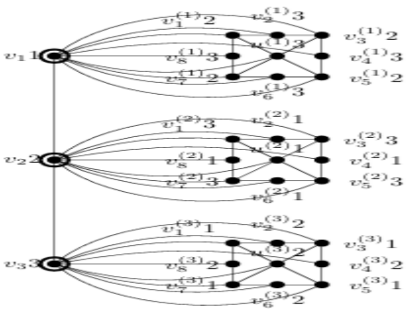


Figure 4: Encircled vertices form the dom coloring set $\{v_1, v_2, u^{(1)}\}$, $\gamma_{dc}(P_2 \odot G_3) = 3$

Figure 5: Encircled vertices form the dom coloring set $\{v_1, v_2, v_3\}$, $\gamma_{dc}(P_3 \odot G_4) = 3$

3.3 Construction of Corona Product of a path P_m with L_n

Let P_m be a path graph with m vertices. The vertices of P_m are labelled as $\{v_1, v_2, v_3, \ldots, v_m\}$. The corona product $P_m \odot L_n$ is obtained by taking one copy of P_m together with m copies of the ladder graph L_n, and joining the i[th] vertex of P_m to every vertex of the i[th] copy of L_n, for $i = 1,2,3,\ldots,m$. Let the vertices of the i[th] copy of L_n be labelled as $\left\{v_1^{(i)}, v_2^{(i)}, v_3^{(i)}, \ldots v_n^{(i)}\right\}$, for the first column & $\left\{u_1^{(i)}, u_2^{(i)}, u_3^{(i)}, \ldots u_n^{(i)}\right\}$ for the second column. Hence, the resulting graph $P_m \odot L_n$ has $m + 2mn$ vertices.

Theorem 3.5

Let P_m and L_n be a path and a ladder graph respectively with $m \geq 2$ and $n \geq 2$. Then the dom chromatic number of the corona product $P_m \odot L_n$ is $\gamma_{dc}(P_m \odot L_n) = \begin{cases} 3, & if\ m = 2, n \geq 2 \\ m, & if\ m \geq 3, n \geq 2 \end{cases}$

Proof. Let $G = P_m \odot L_n, m \geq 2$ and $n \geq 2$. Let $V(P_m) = \{v_1, v_2, v_3, \ldots, v_m\}$ and the vertices of the i[th] copy of L_n be $\left\{v_1^{(i)}, v_2^{(i)}, \ldots v_n^{(i)}, u_1^{(i)}, u_2^{(i)}, \ldots u_n^{(i)}\right\}$. It is clear that $\gamma(G) = m$, since the vertices of path P_m is enough to dominate the vertices of L_n. Also, $\chi(P_m) = 2$. Since the ladder graph L_n is bipartite, it admits a $2-$coloring. Therefore, $\chi(L_n) = 2$.

Since each vertex of P_m is adjacent to every vertex of the corresponding copy of L_n, we require one more color for the proper coloring of G. Hence, $\chi(G) = 3$.

Case (i) When $m = 3, n \geq 2$

By Theorem 2.1, $\gamma_{dc} \geq max\{m, 3\}$. Hence, $\gamma_{dc}(G) \geq 3$.

We color the vertices of G as follows. Assign the color i (taken modulo 3) to $v_i, 1 \leq i \leq m$ and assign the colors $i + 1, i + 2$ (taken modulo 3) to the vertices of the i[th] copy of L_n adjacent to v_i in that order in the clockwise direction starting from $v_1^{(i)}$ to $v_n^{(i)}$. Assign the colors $i + 2, i + 1$ (taken modulo 3) to the vertices of the i[th] copy of L_n adjacent to v_i in that order in the clockwise direction starting from $u_1^{(i)}$ to $u_n^{(i)}$. Clearly, the above coloring is a proper coloring of G. Refer Figure 6.

Let $D = V(P_m) \cup \left\{ u_1^{(1)} \right\}$.

Clearly, D is a dc-set of G, which implies that $\gamma_{dc}(G) \leq 3$. Thus, $\gamma_{dc}(G) = 3$.

Case (ii) When $m \geq 3, n \geq 2$

By Theorem 2.1, $\gamma_{dc} \geq max\{m, 3\}$. Hence, $\gamma_{dc}(G) \geq m$. Proceed as similar as above case for coloring. Refer Figure 7

Let $D = V(P_m)$.

Clearly, D is a dc-set of G, which implies that $\gamma_{dc}(G) \leq m$. Thus, $\gamma_{dc}(G) = m$.

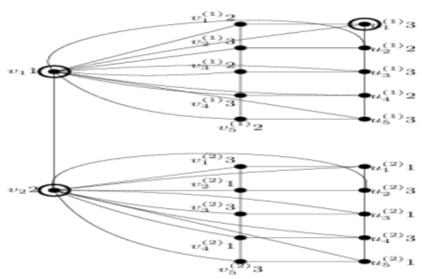
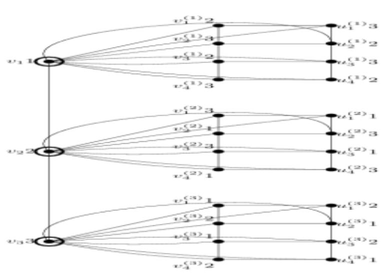



Figure 6: Encircled vertices form the dom coloring set $\{v_1, v_2, v_1^{(1)}\}$, $\gamma_{dc}(P_2 \odot L_3) = 3$

Figure 7: Encircled vertices form the dom coloring set $\{v_1, v_2, v_3\}$, $\gamma_{dc}(P_3 \odot L_4) = 3$

3.4 Construction of Corona Product of a path P_m with $G_{t,n}$

Let P_m be a path graph with m vertices. The vertices of P_m are labelled as $\{v_1, v_2, v_{3,}, \dots, v_m\}$. The corona product $P_m \odot G_{t,n}$ is obtained by taking one copy of P_m together with m copies of the grid graph $G_{t,n}$ and joining the i$^{\text{th}}$ vertex of P_m to every vertex of the i$^{\text{th}}$ copy of $G_{t,n}$, for $i = 1,2,3,\dots,m$. Let the vertices of the i$^{\text{th}}$ copy of $G_{t,,n}$ be labelled as $\left\{v_{jk}^{(i)}, 1 \le i \le m, 1 \le j \le t, 1 \le k \le n\right\}$, where $v_{jk}^{(i)}$denote the vertices in j$^{\text{th}}$ row and k$^{\text{th}}$ column in the i$^{\text{th}}$ copy of $G_{t,n}$. Hence, the resulting graph $P_m \odot G_{t,n}$ has $m + mtn$ vertices.

Theorem 3.6

Let P_m and $G_{t,n}$ be a path and a grid graph respectively with $m, t, n \ge 2$. Then the dom chromatic number of the corona product $P_m \odot G_{t,n}$ is $\gamma_{dc}(P_m \odot G_{t,n}) = \begin{cases} 3, & if\ m = 2, t, n, \ge 2 \\ m, & if\ m \ge 3, t, n \ge 2 \end{cases}$

Proof. Let $G = P_m \odot G_{t,n}, m, t, n \ge 2$. Let $V(P_m) = \{v_1, v_2, v_3, \dots, v_m\}$ and the vertices of the i$^{\text{th}}$ copy of $G_{t,n}$ be $\left\{v_{jk}^{(i)}, 1 \le i \le m, 1 \le j \le t, 1 \le k \le n\right\}$. It is clear that $\gamma(G) = m$, since the vertices of path P_m is enough to dominate the vertices of $G_{t,n}$ Also, $\chi(P_m) = 2$.. Since $G_{t,n}$, is the cartesian product of two bipartite graphs P_t and P_n, it admits a proper 2 −coloring. Hence $\chi\left(G_{t,n}\right) = 2$.

Since each vertex of P_m is adjacent to every vertex of the corresponding copy of $G_{t,n}$, we require one more color for the proper coloring of G. Hence, $\chi(G) = 3$.

Case (i) When $m = 3, t,\ n \ge 2$

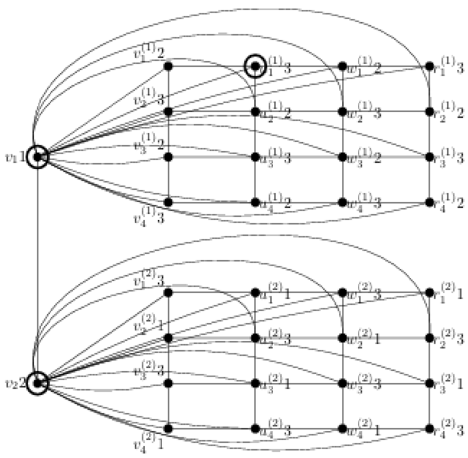
By Theorem 2.1, $\gamma_{dc} \ge max\{m, 3\}$. Hence, $\gamma_{dc}(G) \ge 3$.

We color the vertices of G as follows. Assign the color i (taken modulo 3) to $v_i, 1 \le i \le m$. More precisely, for $1 \le j \le t, 1 \le k \le n$, define

$$c\left(v_{jk}^{(i)}\right) = \begin{cases} i + 1\ (mod\ 3), & j + k\ is\ even \\ i + 2\ (mod\ 3), & j + k\ is\ odd \end{cases}$$

Clearly, the above coloring is a proper coloring of G. Refer Figure 8. Now, let $D = V(P_m) \cup \left\{\ v_{12}^{(1)}\right\}$.

Clearly, D is a dc-set of G, which implies that $\gamma_{dc}(G) \le 3$. Thus, $\gamma_{dc}(G) = 3$.



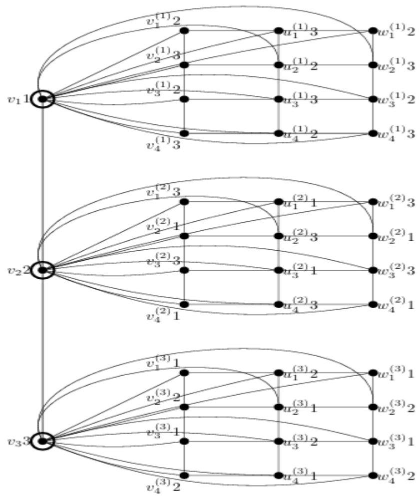


Figure 8: Encircled vertices form the dom coloring set $\{v_1, v_2, v_{12}^{(1)}\}$, $\gamma_{dc}(P_2 \odot G_{4,4}) = 3$

Figure 9: Encircled vertices form the dom coloring set $\{v_1, v_2, v_3\}$, $\gamma_{dc}(P_3 \odot G_{4,3}) = 3$

Case (ii) When $m \geq 3, t,\ n \geq 2$

By Theorem 2.1, $\gamma_{dc} \geq max\{m, 3\}$. Hence, $\gamma_{dc}(G) \geq m$. Proceed as similar as above case for coloring. Refer Figure 9. Now, let $D = V(P_m)$.

Clearly, D is a dc-set of G, which implies that $\gamma_{dc}(G) \leq m$. Thus, $\gamma_{dc}(G) = m$.

4. Application

The corona product $P_m \odot G_{t,n}$ provides a suitable graph-theoretic model for hierarchical sensor and Internet of Things (IoT) networks. Here, the path P_m can represent a sequence of backbone monitoring stations or control units, while each copy of the grid graph $G_{t,n}$ represents a collection of sensors arranged in rows and columns within the corresponding monitoring region. The edges of the grid describe communication links between neighboring sensors, whereas the corona operation connects each backbone station to every sensor in its associated region. In this setting, the dom-chromatic number represents the minimum number of selected monitoring or control devices required to dominate the entire network while satisfying the dom-coloring condition, that is, the selected set must contain at least one vertex from every color class. Thus, the parameter provides a combined measure of network coverage and resource assignment, which can be useful in designing efficient sensor placement, monitoring, communication channel allocation and interference free control strategies in distributed IoT networks.

5. Conclusion

In this paper, we have determined the dom-chromatic number for the corona product of a path with cycle and path related graphs namely wheel graph, gear graph, ladder graph, and grid graphs. The corona product of two graphs give rise to a complex network together with the structural property. For each of the considered graph families, we analyzed the interplay between domination and proper vertex coloring, and determined the dom-chromatic number by constructing the dominating set satisfying the dom-coloring condition. As future work, the investigation can be extended to the dom-chromatic number of corona products involving other graph families and graph operations.

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