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Estimation of Effective Elastic Design Parameters of Uni-directional Continuous FRP Lamina for Variable Fiber Volume Ratios Using Micromechanical Models

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Abstract: Composites made of fiber-reinforced polymers are a significant class of technical materials nowadays. They are easily made, offer exceptional mechanical qualities, and a distinctively flexible design. Compared to conventional materials, FRPs provide greater strength while reducing weight. Fiber-reinforced polymer (FRP) composites are increasingly being employed as important structural components in a variety of industries, including aerospace, automotive, bridge construction, and oil and gas pipeline construction. The sensitivity of FRPs to experimental variation makes it a lot more complicated system, yet empirical relations can readily assess it using empirical models. The mathematical relations obtained without any derivation are referred to as these models. The mechanical properties of FRP composite structures under unidirectional loading circumstances were the main focus of this investigation. Fibre volume fraction, layer geometry, fiber/matrix characteristics, matrix type, and fibre orientation are some of the variables that can be changed to change these mechanical properties. Only the variation in the fibre volume fraction from 40 to 60 percent is used to validate the results in this case. Additionally, the optimum model for assessing the composite elastic characteristics is recognized, and the differences between the models' attributes are recorded.

Keywords: Elastic Properties, FRP Composite, Micro-Mechanics, Fiber and Matrix Volume Ratios

1. Introductin

A composite material is a substance created from two or more constituent materials. It is also known as a composition material or simply composite. The constituents are mixed together to create a substance that has unique qualities, even though its chemical or physical properties may differ from its individual components. The various components stay distinct and separate inside the completed structure, setting composites apart from mixes and solid solutions. One of the constituents is discontinues and stiffer called as Reinforcement, and another, known as Matrix, which is continuous but weaker. The main factors influencing fiber-reinforced composite materials' extensive use are their low density, high strength, high stiffness-to-weight ratio, outstanding durability, and design freedom.[1].

A substance made of two or more constituent materials is called a composite material. It is sometimes referred to as a composite or a composition material. Even though the substance's physical or chemical characteristics may differ from those of its constituent parts, the mixture of these elements results in a compound with distinctive traits. Composites differ from mixes and solid solutions in that the different components remain separate and distinct inside the finished construction. One component, termed Reinforcement, is stiffer and discontinuous, while the other, named Matrix, is continuous but weaker. Fiber-reinforced composite materials' low density, high strength, high stiffness-to-weight ratio, exceptional durability, and design freedom are the primary drivers driving their widespread use.[1]. Because it is lightweight and has a high strength-to-thickness ratio, FRP is becoming more productive than any other composites available today. Composite materials known as fiber-reinforced polymers (FRP) are utilised in almost every kind of sophisticated engineering structure, including cars, sports equipment, boats, ships, offshore platforms, and civil infrastructure such as buildings and bridges. These materials are widely used and have demonstrated their effectiveness in both civil and biomedical structures. Fibre volume fraction, fibre orientation, and fiber/matrix properties can all be used to forecast these composite material parameters. These materials are usually composed of a polymer matrix reinforced with continuous or short fibres [3]. It usually incorporates polyester thermosetting plastic, epoxy, or vinyl-ester, among other elements, because of its high price and good properties [4].

Among all the composites present FRP is emerging in productivity due to its lightweight high strength-to-thickness ratio. Fiber-reinforced polymers (FRP) are composite materials used in nearly every type of advanced engineering structure, with applications in everything from boats, ships, and offshore platforms to cars, sports equipment, chemical processing machinery, and civil infrastructure like bridges and buildings. These materials are prominent and extended their proficiency in the bio-medical and civil structures. These composite material properties can be predicted by fiber volume fraction, fiber orientation, and fiber/matrix properties. Typically, a polymer matrix reinforced by brief or continuous fibers makes up these materials [3]. Due to its high cost and good qualities, it typically contains epoxy, vinyl-ester, or polyester thermosetting plastic, among other materials [4]. For a transversely isotropic UD lamina under plane stress, the independent elastic constants are replaced by four independent elastic moduli: major Poisson's ratio (υ_{12}), transverse elastic modulus (E_2), longitudinal elastic modulus (E_1), and in-plane shear modulus (G_{12}). [7].

Assumptions made for the present analysis were [1].

- Fibres are uniformly distributed in the matrix;
- Fibres are perfectly aligned;
- There is perfect bonding between fibers and matrix;
- The composite lamina is free of voids and other irregularities; and
- The load is within the linear elastic limit.

Several models or techniques are created for assessing the qualities of FRP composite lamina in order to determine its characteristics. Because any slight alteration in the lamina must change the entire characteristics of the composite lamina, the qualities are particularly challenging to assess through experimental research. In order to ascertain the micro-mechanical behaviour of square unit cells under tensile loading conditions while taking orthotropic fibres into consideration, Syam Prasad.A1, Syed Altaf Hussain2, and Pandurangadu.V3 (2013) used three distinct FRP composite materials. Using FEA software, the longitudinal and transverse elastic modulus, shear modulus, and Poisson's ratio are calculated and compared to Halpin-Tsai's model and the Rule of Mixtures. Furthermore, the interfacial tensions generated at the fiber-matrix contact are also identified. [1]. The micro-mechanical behaviour of the FRP composites was studied by M. Sudheer1, Pradyoth K. R.1, and Shashiraj Somayaji2 (2015) using FEA software. The results were compared using analytical methodologies such as the Rule of mixtures, Halpin-Tsai's, Nielsen's, and Chamis. According to his research, when the load is applied along the fibre direction, the Epoxy/Glass composite exhibits exceptional qualities [2].The flexibility of glass, graphite, and Kevlar fibres in an epoxy matrix was developed by Syed AltafHussain, B. Siddha Reddy, and V. Nageswara Reddy (2008). Ansys and the analytical techniques, such as Halpin-Tsai's and the Theory of Formulation (MOS), are contrasted [5].In 2018, M Heidari-Rarani1, KBashandeh-Khodaei-Naeini1, and SM Mirkhalaf2 reviewed the literature on micromechanical models and contrasted the experimental data found in the literature with the analytical, semi-analytical, and material strength and elasticity data. Elastic constants are found and FEA has effective elastic and damage properties among all the approaches [7]. The primary goal of the current study is to evaluate the elastic properties analytically using four models: the Hashin, Halpin-Tsai, Chamis, and Rule of Mixtures models. Additionally, a MATLAB code is generated utilizing empirical relations to evaluate the results.

1. 1 Notations

E_1 - Longitudinal Young's Modulus
E_2 - Transverse Young's Modulus
G_{12}- Shear Modulus along
υ_{12} - Major Poisons ratio along
υ_{21} - Minor Poisons ratio along
V_f - Volume fraction of fiber
V_m - Volume fraction of matrix
E_f - Young's Modulus of fiber
E_m - Young's Modulus of matrix
η - Packing factor
G_f - Shear Modulus of fiber
G_m - Shear Modulus of matrix

2. Description of the Representative Volume Element

Fibre is distributed in FRP composites in square, rectangular, and hexagonal patterns. For this investigation, a square unit is used. For composites reinforced with continuous fibres, the ratio of the fiber's volume to the composite's total volume is equal to the ratio of the fiber's surface area to a unit cell's total surface area. The fibre volume percentage is a crucial component in determining the mechanical characteristics of composites [9].

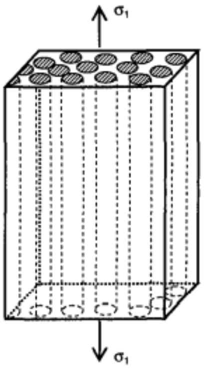

Figure 1: Unidirectional continuous fiber composite under longitudinal tension

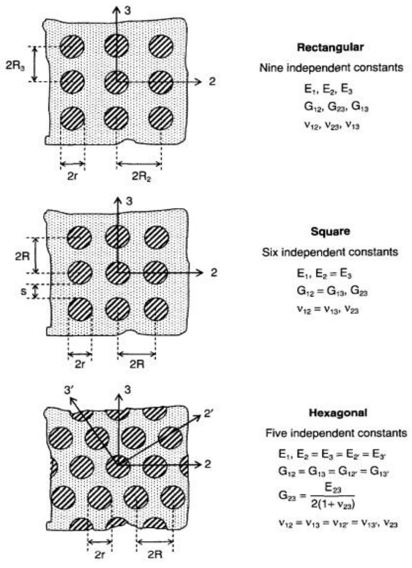

Figure 2: Idealized fibre packing geometries and elastic symmetry

2.1 Constituent Properties

Table 1. Mechanical Properties of Fiber, GPa

Fiber	Young's modulus	Shear modulus	Poisson's ratio	Bulk modulus
Glass	73.1	29.95	0.22	0.177
Carbon	22.0	12.00	0.27	0.178
Boron	23.5	48.00	0.20	0.185

Table 2. Mechanical properties of matrix, GPa

Matrix	Young's modulus	Shear modulus	Poisson's ratio	Bulk modulus
Epoxy	3.450	1.281	0.35	3.810
PTFE	2.101	1.301	0.39	4.301
PMMA	3.600	1.357	0.39	5.475

3. Formulations of Micromechanical Models

An analytical method uses various mathematical equations to evaluate the mechanical properties of composite material [1]. The methods utilized are:

3.1 Rule of mixtures

Rule of mixtures is the basic and simple method used to find out the mechanical properties for unidirectional composite lamina [1]. The properties of fibre and matrix are taken as E_f and E_m, similarly Poisson's ratio is taken as υ_f and υ_m. Longitudinal properties

$$E_1 = E_f V_f + E_m \ (1\text{-}V_f) \quad (1)$$

$$\upsilon_{12} = \upsilon_f V_f + \upsilon_m V_m \quad (2)$$

Where, V_f and V_m are volume fractions.

Transverse properties,

$$E_2 = \frac{E_m E_f}{E_m V_f + E_f V_m} \quad (3)$$

$$\upsilon_{21} = \frac{\upsilon_{12}}{E_1} E_2$$

Shear properties,

$$G_{12} = \frac{G_m G_f}{G_m V_f + G_f V_m} \quad (4)$$

3.2 Halpin T-Sai Model

In order to get more complex results on longitudinal shear modulus and transverse Young's modulus, the Halpin-Tsai equation was created as a semi-empirical model.[1] Longitudinal Young's Modulus (E1)

The strength of materials approach yields the same Halpin-Tsai equation for the longitudinal Young's modulus, that is,

$$E_1 = E_f V_f + E_m \ (1 - V_f)$$

Transverse Young's Modulus (E_2)

Given by is the transverse Young's modulus, E_2

$$\frac{E_2}{E_m} = \frac{1+\zeta \eta V_f}{1-\eta V_f} \tag{5}$$

Where, $\eta = \frac{\left(\frac{E_f}{E_m}\right)-1}{\left(\frac{E_f}{E_m}\right)+\zeta}$

The following factors affect the term "ζ," which is known as the reinforcing factor: Fiber geometry Packing geometry Loading conditions

Major Poisson's Ratio(υ_{12})

The strength of materials technique yields the same results as the Halpin and Tsai equation for the Major Poisson's ratio, that is,

$$\upsilon_{12} = \upsilon_f V_f + \upsilon_m V_m$$

In-Plane Shear Modulus (G12)

The in-plane shear modulus G12 Halpin and Tsai equation is

$$\frac{G_{12}}{G_m} = \frac{1+\zeta \eta V_f}{1-\eta V_f} \tag{6}$$

Where, $\eta = \frac{\left(\frac{G_f}{G_m}\right)-1}{\left(\frac{G}{G_m}\right)+\zeta}$

3.3 Chamis Model

Chamis is a method that modifies the rule of mixture by substituting the square root of the fiber volume percent. This approach, which provides five formulas to assess elastic characteristics, is the most used.[2]

Longitudinal Young's Modulus (E1)

$$E_1 = E_f V_f + E_m \ (1 - V_f)$$

Transverse Young's Modulus (E_2)

$$E_2 = \frac{E_m}{1- \ \{\sqrt{V_f} + [1 - (\frac{E_m}{E_f}) \ ]\}} \tag{7}$$

Major Poisson's Ratio(υ_{12})

$$\upsilon_{12} = \upsilon_f V_f + \upsilon_m V_m$$

In-Plane Shear Modulus (G12)

$$G_{12} = \frac{G_m}{1- \ \{\sqrt{V_f} + [1 - (\frac{G_m}{G_f}) \ ]\}} \tag{8}$$

3.4 Hashin's Model

Hashin provided a different formula for computing the elastic moduli of fiber reinforced composites in the main and shear directions..[12]

Longitudinal Young's Modulus (E1)

$$E_1 = E_f V_f + E_m \ (1\text{-}V_f) + \frac{4(\upsilon_f-\upsilon_m)\text{^}2\, V_f V_m}{\left(\frac{V_m}{k_f}\right)+\left(\frac{V_f}{k_m}\right)+\left(\frac{1}{G_m}\right)} \tag{9}$$

Transverse Young's Modulus (E_2)

$$E_2 = G_m + \frac{V_f}{\left[\left(\frac{1}{G_f}\right) - G_m\right]+\left(\frac{V_m}{2G_m}\right)} \tag{10}$$

Major Poisson's Ratio(υ_{12})

$$\upsilon_{12} = \upsilon_f V_f + \upsilon_{12} \ (1\text{-}V_f) + \frac{(\upsilon_f-\upsilon_m)\left(\left(\frac{1}{k_m}\right)-\left(\frac{1}{k_f}\right)\right)V_f V_m}{\left(\frac{V_m}{k_f}\right)+\left(\frac{V_f}{k_m}\right)+\left(\frac{1}{G_m}\right)} \tag{11}$$

In-Plane Shear Modulus (G12)

$$G_{12} = G_m + \frac{V_f}{\left[\left(\frac{1}{G_f}\right) - G_m\right]+(k_m+2G_m)/2G_m(k_m+2G_m)V_m} \tag{12}$$

4. Results and Discussions

In the current work, various models for unidirectional fiber reinforced composite materials are used to estimate the mechanical properties of FRP composite lamina. A MATLAB code is engendered for various FRP composites. For example, a glass-fibre reinforced polymer composite is considered; epoxy resin is used as a matrix material, etc.

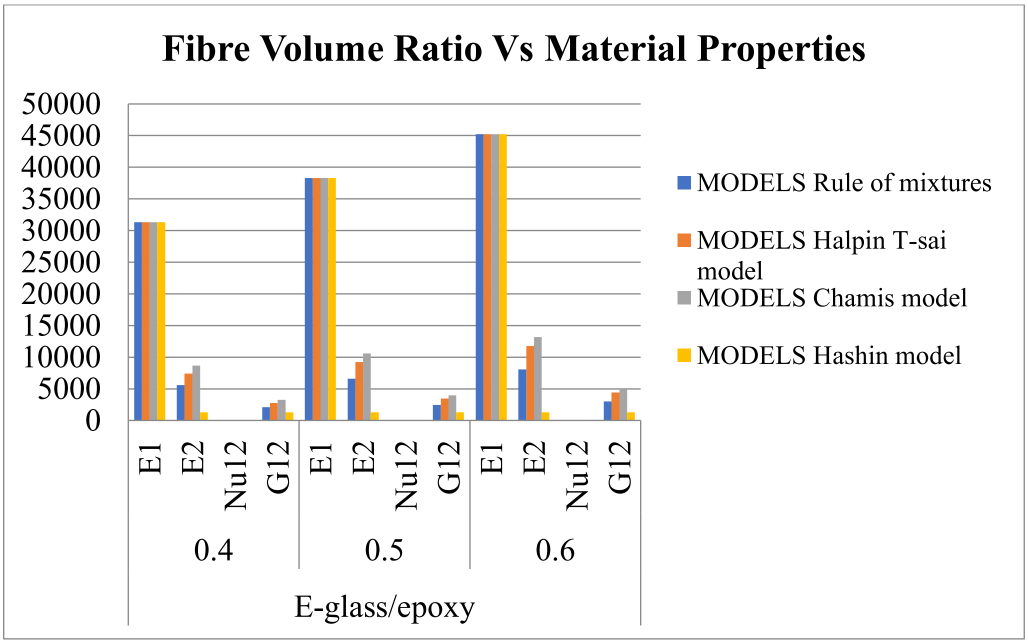

Figure 3: E-glass / Epoxy

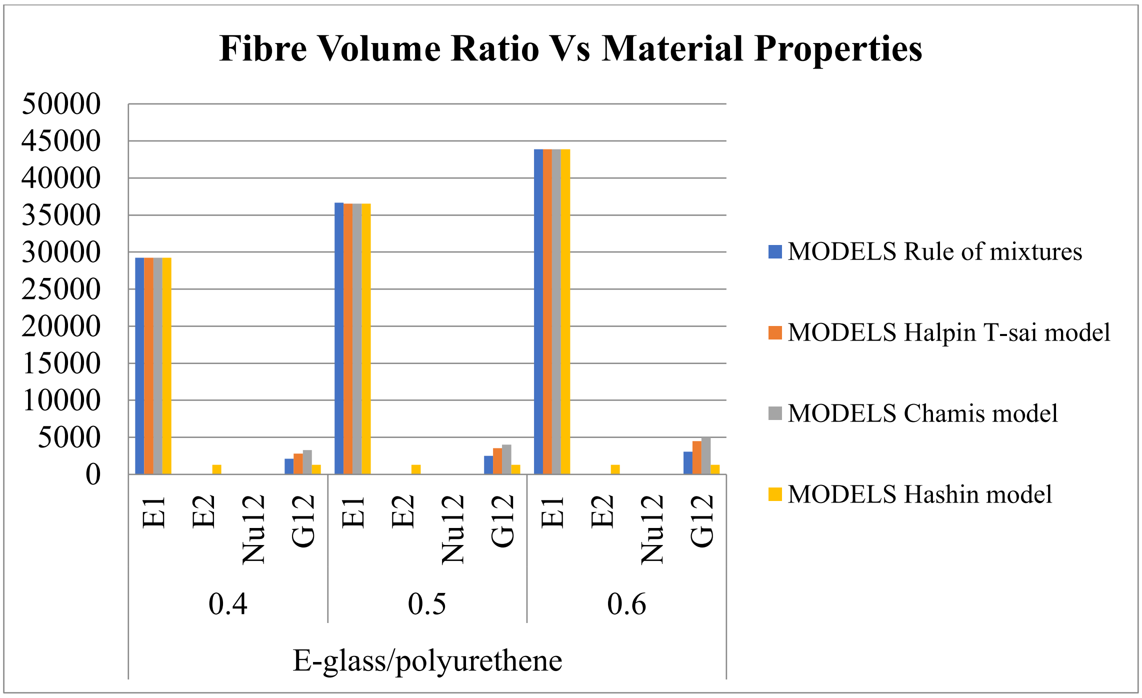

Figure 4: E-glass/ Polyurethane

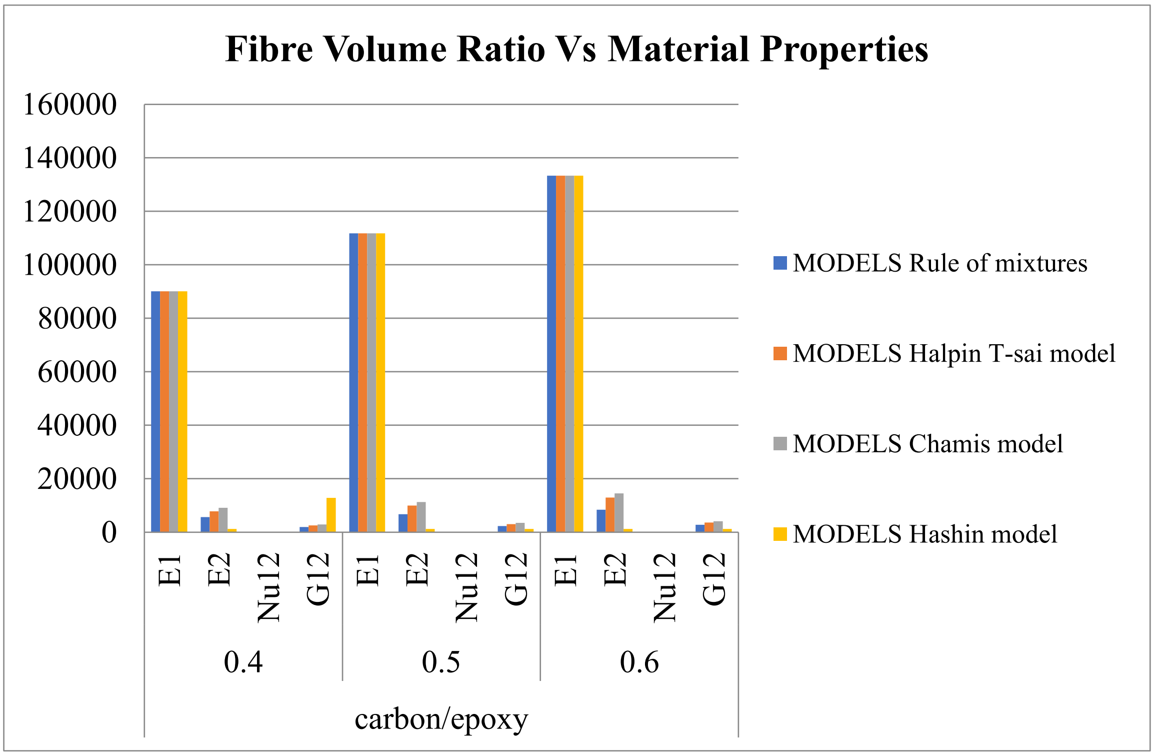

Figure 5: Carbon/ Epoxy

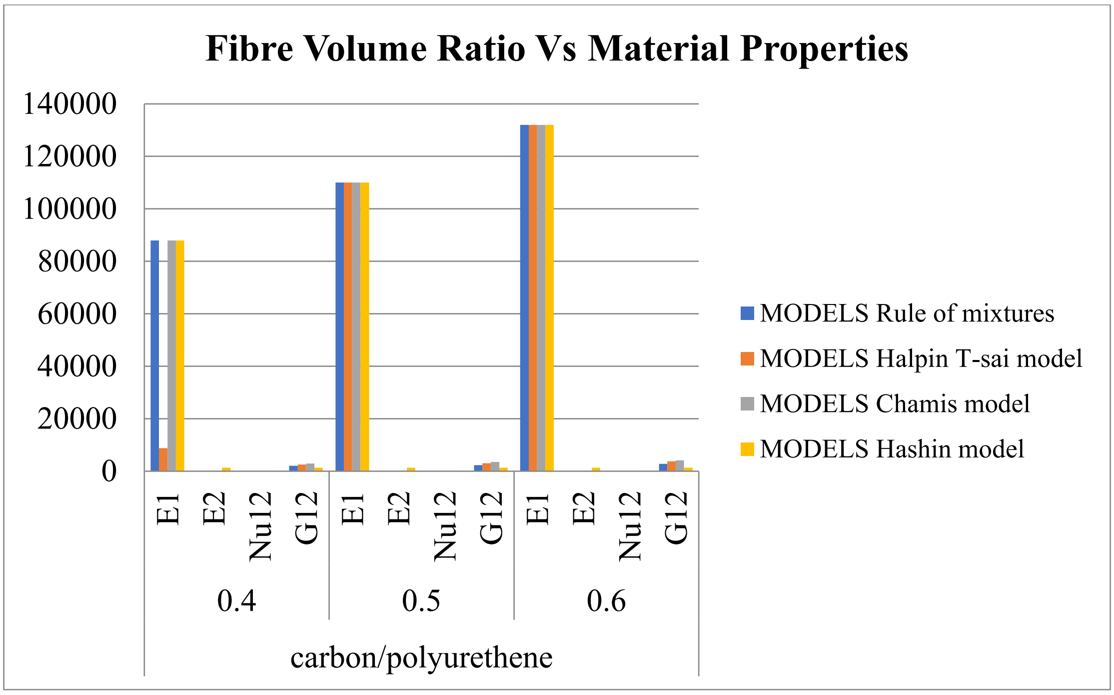

Figure 6: Carbon/ Polyurethane

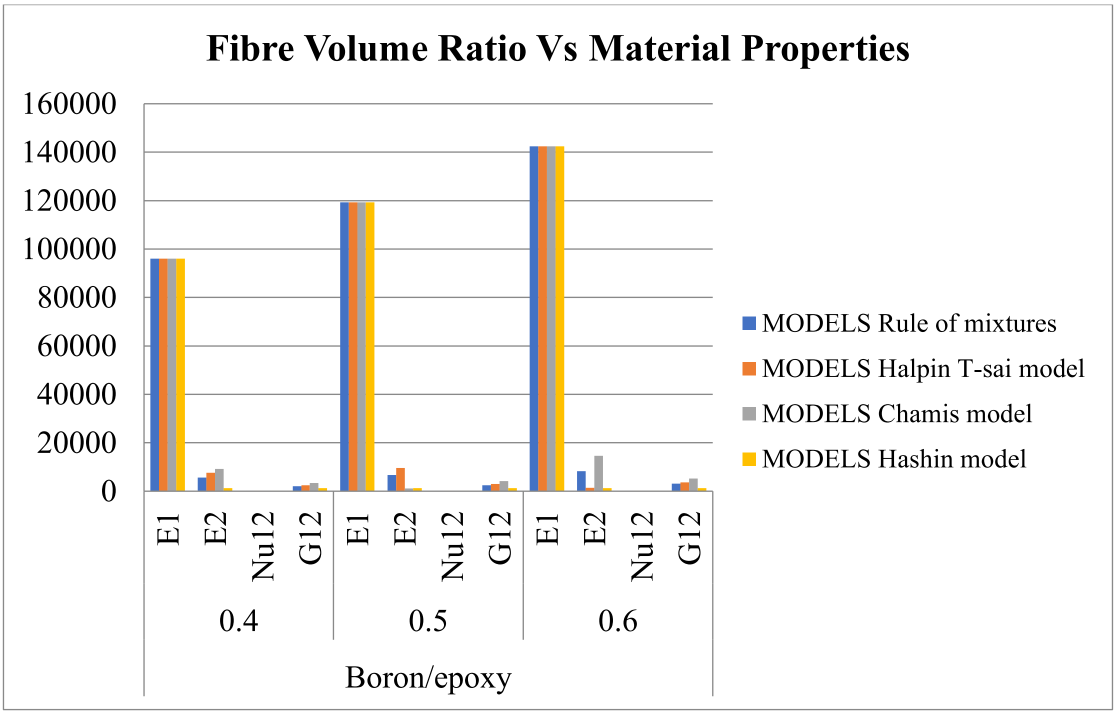

Figure 7**:** Boron/Epoxy

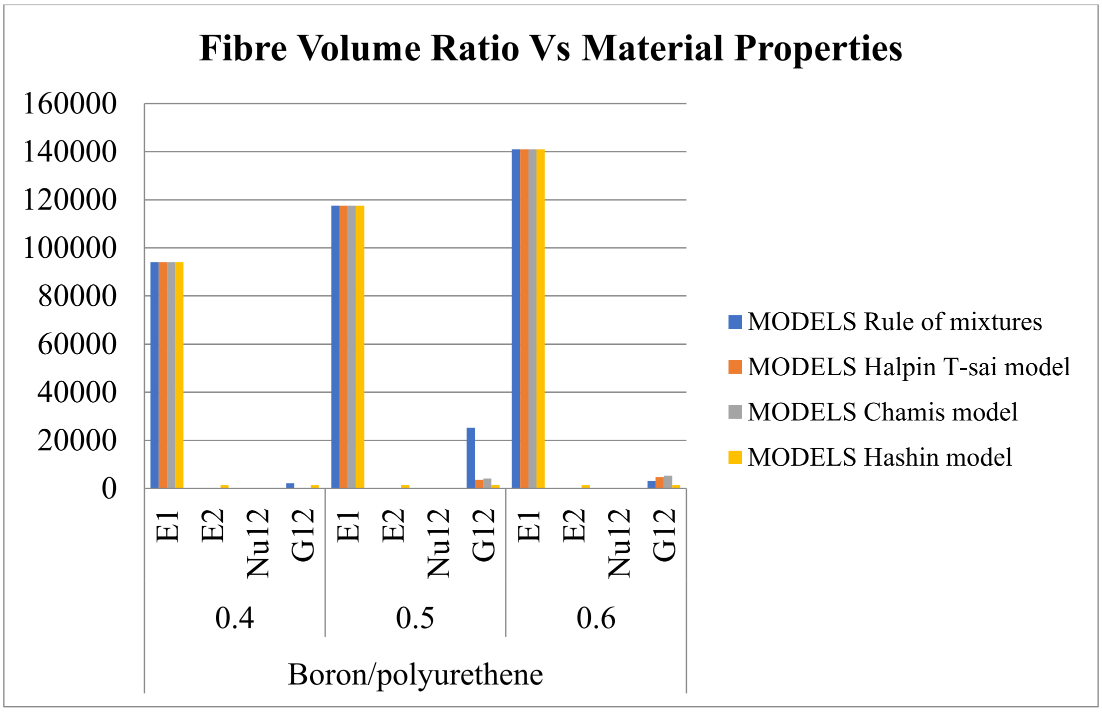

Figure 8: Boron/Polyurethane

5. Conclusion

The present study on the evaluation of mechanical properties is conducted using the MATLAB software and the elastic constants vary among the four models. The variation in the material properties of the composite are as follows:

The longitudinal modulus E1 is constant or almost same in the four models as it rises along with the Fiber Volume Ratio. The transverse modulus E2 possess higher value in Chamis model and the least in Hashin model. It gradually increases with increase in fibre volume ratio.

Shear modulus steadily rises as the fiber volume ratio rises. i.e. it acquires higher value at 60%.Based on the strength criteria we take the highest longitudinal modulus into consideration. For this the Halpin-Tsai's and Hashin model is the best. And these methods give the elastic properties nearly similar to the experimental investigation.

The MATLAB software is used in this work to evaluate mechanical properties, and the four models have different elastic constants. The following are the differences in the composite's material properties: As the Fibre Volume Ratio increases, the longitudinal modulus E1 remains constant or almost the same in all four models. The Hashin model has the lowest transverse modulus E2, whereas the Chamis model has the highest value. With an increase in the fibre volume ratio, it progressively rises.

As the fibre volume ratio increases, the shear modulus increases continuously, reaching a greater value at 60%. It is to be considering the highest longitudinal modulus based on the strength criterion. The best model for this is that of Halpin-Tsai and Hashin

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