Tuijin Jishu/Journal of Propulsion Technology
ISSN: 1001-4055
Vol. 47 No. 03 (2026)

Neutrosophic Vague Hypersoft Open and Closed Mappings

[1]Sharviya Sona S, [2]Elvina Mary L

[1]Research Scholar, PG & Research Department of Mathematics, Nirmala College for Women, Coimbatore, India.

[2]Assistant Professor, PG & Research Department of Mathematics, Nirmala College for Women, Coimbatore, India.

Abstract: In this paper, we introduce and formalize the concepts of Interval-Valued Neutrosophic Hypersoft (IVNHS) open mappings and IVNHS-closed mappings within the framework of IVNHS-topological spaces. We establish necessary and sufficient conditions for these mappings using the fundamental topological operators of interior and closure. Furthermore, we extensively investigate the behavior of these mappings under composition. Specifically, we prove that the composition of two IVNHS-open (or closed) mappings preserves their respective properties. We also derive the structural conditions under which the component mappings inherit openness or closedness from their composite forms, utilizing the auxiliary conditions of injectivity, surjectivity, and continuity. These findings enrich the structural theory of interval-valued neutrosophic hypersoft topology and pave the way for deeper geometric and algebraic mappings within multi-attribute decision-making environments.

Keywords: Neutrosophic Vague Hypersoft set (NVHSS), NVHS-topological space, NVHS-open mapping, NVHS-closed mapping, Composite mapping, Interior operator, Closure operator,Continuous mapping.

Introdution:

The generalization of neutrosophic structures into topological spaces has provided a deep geometric understanding of continuity, convergence, and separation behavior. A pioneering milestone in this direction was achieved by Salama and Alblowi (2012), who first defined neutrosophic sets and formalized the primitive axioms of neutrosophic topological spaces. This structural foundation was expanded by Mukherjee et al. (2014), who embedded interval-valued properties into soft topological spaces, exploring the interaction between interval parameterizations and topological frameworks. Following this, Öztürk and Bayramov (2017) investigated the structural behavior of mappings by introducing the topology on soft continuous function spaces, clarifying how continuity behaves under systematic functional transformations.

As structural complexity progressed towards topological boundaries, Gunduz et al. (2019) introduced and explored separation axioms within neutrosophic soft topological settings, bridging foundational set-theoretic boundaries with separation attributes. The evolution advanced from soft parameters to Cartesian sub-domains when Musa and Asaad (2022a) introduced hypersoft topological spaces, providing a dedicated space for multi-argument attribute structures. They later advanced this framework by investigating connectedness on hypersoft topological spaces (Musa and Asaad, 2022b), characterizing how spaces split or hold together under multi-attributes.

Definition 3.2.1.

Let (aη,aδ) : $(\mho_{a1}, \tau_{\mho_{a1}}, Y) \rightarrow (\mho_{a2}, \tau_{\mho_{a2}}, \dot{Y})$ be an NVHS-mapping where, $(\mho_{a1}, \tau_{\mho_{a1}}, Y)$ and $(\mho_{a1}, \tau_{\mho_{a1}}, Y)$ are NVHS-topological spaces.

1. (aη,aδ) is an NVHS-open mapping if the image of each NVHS-open set in $(\mho_1, \tau_{\mho_1}, Y)$ is an NVHS-open set in $(\mho_2, \tau_{\mho_2}, \dot{Y})$.

2. (aη,aδ) is an NVHS-closed mapping if the image under (aη,aδ) of every $\tau_{\mho_{a1}}$ NVHS-closed set is an $\tau_{\mho_{a2}}$ NVHS-closed set.

Proposition 3.2.2.

Let $(\mho_{a1}, \tau_{\mho_{a1}}, Y)$ and $(\mho_{a2}, \tau_{\mho_{a2}}, \dot{Y})$ be NVHS-topological spaces.

A mapping $(a\eta, a\delta) : (\mho_{a1}, \tau_{\mho_{a1}}, Y) \to (\mho_{a2}, \tau_{\mho_{a2}}, \dot{Y})$ is an NVHS-open mapping if and only if $(a\eta, a\delta)((a\vartheta, aY)^{\circ}) \subseteq [(a\eta, a\delta)(a\vartheta, aY)]^{\circ}$, for every $(a\vartheta, aY) \in (\mho_{a1}, \tau_{\mho_{a1}}, Y)$.

Proof. Let $(a\eta, a\delta)$ be an NVHS-open mapping and let $(a\vartheta, aY) \in (\mho_{a1}, \tau_{\mho_{a1}}, Y)$. Then $(a\vartheta, aY)^{\circ}$ is an NVHS-open set. Since, $(a\eta, a\delta)$ is an NVHS-open mapping, $(a\eta, a\delta)((a\vartheta, aY)^{\circ})$ is an NVHS-open set in $(\mho_{a2}, \tau_{\mho_{a2}}, \dot{Y})$ and

$[(a\eta, a\delta)((a\vartheta, aY)^{\circ})]^{\circ} = (a\eta, a\delta)((a\vartheta, aY)^{\circ})$.

Now, $(a\vartheta, aY)^{\circ} \subseteq (a\vartheta, aY)$ and which implies

$(a\eta, a\delta)((a\vartheta, aY)^{\circ})^{\circ} \subseteq (a\eta, a\delta)(a\vartheta, aY)$.

Hence $[(a\eta, a\delta)((a\vartheta, aY)^{\circ})]^{\circ} \subseteq [(a\eta, a\delta)(a\vartheta, aY)]^{\circ}$.

Thus $(a\eta, a\delta)((a\vartheta, aY)^{\circ}) \subseteq [(a\eta, a\delta)(a\vartheta, aY)]^{\circ}$. [by (3.5)

Conversely, let $(a\eta, a\delta)((a\vartheta, aY)^{\circ}) \subseteq [(a\eta, a\delta)(a\vartheta, aY)]^{\circ}$, $\forall (a\vartheta, aY) \in (\mho_{a1}, \tau_{\mho_{a1}}, Y)$. Since $(a\vartheta, aY)$

is an NVHS-open set in $(\mho_{a1}, \tau_{\mho_{a1}}, Y)$,

we have $(a\vartheta, aY)^{\circ} = (a\vartheta, aY)$.

Hence $(a\eta, a\delta)((a\vartheta, aY)^{\circ}) = (a\eta, a\delta)(a\vartheta, aY)$.

But by hypothesis, $(a\eta, a\delta)((a\vartheta, aY)^{\circ}) \subseteq [(a\eta, a\delta)(a\vartheta, aY)]^{\circ}$.

Now (3.6) and (3.7) yields $(a\eta, a\delta)(a\vartheta, aY) \subseteq [(a\eta, a\delta)(a\vartheta, aY)]^{\circ}$.

Also $[(a\eta, a\delta)(a\vartheta, aY)]^{\circ} \subseteq (a\eta, a\delta)(a\vartheta, aY)$, $\forall (a\vartheta, aY)$.

From (3.8) and (3.9) $(a\eta, a\delta)(a\vartheta, aY) = [(a\eta, a\delta)(a\vartheta, aY)]^{\circ}$. (3.6) (3.7) (3.8) (3.9)

Thus, $(a\eta, a\delta)(a\vartheta, aY)$ is an NVHS-open set and hence $(a\eta, a\delta)$ is an NVHS-open mapping.

Proposition 3.2.3.

Let $(\mho_{a1}, \tau_{\mho_{a1}}, Y)$ and $(\mho_{a2}, \tau_{\mho_{a2}}, \dot{Y})$ be NVHS-topological spaces. A mapping $(a\eta, a\delta) : (\mho_{a1}, \tau_{\mho_{a1}}, Y) \to (\mho_{a2}, \tau_{\mho_{a2}}, \dot{Y})$ is an NVHS-closed mapping if and only if $\overline{(a\eta, \delta a)(a\vartheta, aY)} \subseteq (a\eta, a\delta)(\overline{(a\vartheta, aY)})$, for every $(a\vartheta, aY) \in (\mho_{a1}, \tau_{\mho_{a1}}, Y)$.

Proof.

Let $(a\eta, a\delta) : (\mho_{a1}, \tau_{\mho_{a1}}, Y) \to (\mho_{a2}, \tau_{\mho_{a2}}, \dot{Y})$ be an NVHS-closed mapping and $(a\vartheta, aY) \in (\mho_{a1}, \tau_{\mho_{a1}}, Y)$. Since, $\overline{(a\vartheta, aY)}$ is an NVHS-closed set, we have $(a\eta, a\delta)(\overline{(a\vartheta, aY)})$ is an NVHS-closed set.

This implies that $(a\eta, a\delta)(\overline{(a\vartheta, aY)}) = \overline{(a\eta, a\delta)\overline{(a\vartheta, aY)}}$.

As, $(a\vartheta, aY) \subseteq \overline{(a\vartheta, aY)}$ always,

we have $(a\eta, a\delta)(a\vartheta, aY) \subseteq (a\eta, a\delta)(\overline{(a\vartheta, aY)})$.

Now, $\overline{(a\eta, a\delta)(a\vartheta, aY)} \subseteq \overline{(a\eta, a\delta)\overline{(a\vartheta, aY)}} = (a\eta, a\delta)(\overline{(a\vartheta, aY)})$.

Hence, $\overline{(a\eta, a\delta)(a\vartheta, aY)} \subseteq (a\eta, a\delta)(\overline{(a\vartheta, aY)})$, for every $(a\vartheta, aY) \in (\mho_{a1}, \tau_{\mho_{a1}}, Y)$.

Conversely, let $\overline{(a\eta, a\delta)(a\vartheta, aY)} \subseteq (a\eta, a\delta)(\overline{(a\vartheta, aY)})$, for every $(a\vartheta, aY) \in (\mho_{a1}, \tau_{\mho_{a1}}, Y)$

Let $(a\vartheta, aY)$ be any NVHS-closed set in $(\mho_{a1}, Y)$ so that,

$\overline{(a\vartheta, aY)}$ = (aϑ, aY)

Now, (aη, aδ)($\overline{(a\vartheta, aY)}$) = (aη, aδ)(aϑ, aY) (3.10)

Also, by hypothesis $\overline{(a\eta, a\delta)(a\vartheta, aY)}$ ⊆ (aη, aδ)($\overline{(a\vartheta, aY)}$) (3.11)

Hence from (3.10) and (3.11) we get $\overline{(a\eta, a\delta)(a\vartheta, aY)}$ ⊆ (aη, aδ)(aϑ, aY) (3.12)

But, we know that (aϑ, aY) ⊆ $\overline{(a\vartheta, aY)}$ always.

Hence (aη, aδ)(aϑ, aY) ⊆ $\overline{(a\eta, a\delta)(a\vartheta, aY)}$. (3.13)

Hence, from (3.12) and (3.13) we get (aη, aδ)(aϑ, aY) = $\overline{(a\eta, \delta)(a\vartheta, aY)}$.

Thus (aη, aδ) (aϑ, aY) is an NVHS-closed set. Hence, (aη, aδ) is an NVHS-closed mapping.

Proposition 3.2.4.

Let ($\mho_{a1}$, $\tau_{\mho_{a1}}$,Y), ($\mho_{a2}$, $\tau_{\mho_{a2}}$, $\dot{Y}$) and ($\mho_{a3}$, $\tau_{\mho_{a3}}$,$\ddot{Y}$) be NVHS topological spaces. Let (aη,aδ) : ($\mho_{a1}$, $\tau_{\mho_{a1}}$,Y) → ($\mho_{a2}$, $\tau_{\mho_{a2}}$, $\dot{Y}$) and (aψ, a$\dot{\delta}$) : ($\mho_{a2}$, $\tau_{\mho_{a2}}$, $\dot{Y}$) → ($\mho_{a3}$, $\tau_{\mho_{a3}}$,$\ddot{Y}$) be two NVHS-maps.

1. If (aη, aδ) and (aψ, $\dot{a\delta}$) are NVHS-open mappings, then (aψ, a$\dot{\delta}$)∘(aη, aδ) is also an NVHS-open mapping.

2. If (aψ, a$\dot{\delta}$)∘(aη, aδ) is an NVHS-open mapping and if (aη, aδ) is a surjective NVHS continuous mapping, then (aψ, a$\dot{\delta}$) is an NVHS-open mapping.

3. If (aψ, a$\dot{\delta}$)∘(aη, aδ) is an NVHS-open mapping and if (aψ, a$\dot{\delta}$) is a injective NVHS continuous mapping, then (aη, aδ) is an NVHS-open mapping.

Proof.

1. Let (aϑ, aY) be an NVHS-open set in ($\mho_{a1}$, $\tau_{\mho_{a1}}$,Y). Since (aη, aδ) is an NVHS open mapping, we have (aη, aδ)(aϑ, aY) is an NVHS-open set in ($\mho_{a2}$, $\tau_{\mho_{a2}}$, $\dot{Y}$). Again, since (aψ, a$\dot{\delta}$) is an NVHS-open map, we have (aψ, a$\dot{\delta}$)((aη, aδ)(aϑ, aY)) = ((aψ, a$\dot{\delta}$)∘(aη,aδ))(aϑ,aY) is an NVHS-open set in ($\mho_{a3}$, $\tau_{\mho_{a3}}$,$\ddot{Y}$). Hence, (aψ, a$\dot{\delta}$)∘(aη, aδ) is an NVHS-open mapping.

2. Let (aλ, a$\dot{Y}$) be an NVHS-open set in($\mho_{a2}$, $\tau_{\mho_{a2}}$, $\dot{Y}$). Since (aη, aδ) is an NVHS continuous mapping, we have $(a\eta, a\delta)^{-1}$(aλ, a$\dot{Y}$) is an NVHS-open set in ($\mho_{a1}$, $\tau_{\mho_{a1}}$,Y). Again, since (aψ, a$\dot{\delta}$)∘(aη, aδ) is an NVHS-open mapping, we have ((aψ, a$\dot{\delta}$)∘(aη, aδ))[$(a\eta, a\delta)^{-1}$ (aλ, a$\dot{Y}$)] is an NVHS-open set in ($\mho_{a3}$, $\tau_{\mho_{a3}}$,$\ddot{Y}$).

As, (aη, aδ) is surjective,

((aψ, a$\dot{\delta}$)∘(aη, aδ))[$(a\eta, a\delta)^{-1}$(aλ, a$\dot{Y}$)] = (aψ, a$\dot{\delta}$)((aη, aδ)[$(a\eta, a\delta)^{-1}$ (aλ,a $\dot{Y}$)]) =(aψ, a$\dot{\delta}$)(aλ, a$\dot{Y}$) is an NVHS-open set in ($\mho_{a3}$, $\tau_{\mho_{a3}}$,$\ddot{Y}$).

Therefore, (aψ, a$\dot{\delta}$) is an NVHS-open mapping.

3. Let (aϑ, aY) be an NVHS-open set in ($\mho_{a1}$, $\tau_{\mho_{a1}}$,Y). Since ((aψ, a$\dot{\delta}$)∘(aη, aδ)) is an NVHS-open mapping, we have ((aψ, a$\dot{\delta}$)∘(aη, aδ))(aϑ, aY) is an NVHS-open set in ($\mho_{a3}$, $\tau_{\mho_{a3}}$,$\ddot{Y}$).

Again, since (aψ, a$\dot{\delta}$) is an NVHS-continuous mapping, we have $(a\psi, a\,\dot{\delta})^{-1}$ (((aψ, a$\dot{\delta}$)∘(aη, aδ))(aϑ, aY)) is an NVHS-open set in ($\mho_{a2}$, $\tau_{\mho_{a2}}$, $\dot{Y}$). As, (aψ, a$\dot{\delta}$) is injective, $(a\psi, a\dot{\delta})^{-1}$ (((aψ, a$\dot{\delta}$)∘(aη, aδ))(aϑ, aY)) = $(a\psi, a\dot{\delta})^{-1}$ ((aψ, a$\dot{\delta}$)((aη, aδ)(aϑ, aY))) = (aη, aδ)(aϑ, aY) is an NVHS-open set in ($\mho_{a2}$, $\tau_{\mho_{a2}}$, Ẏ). Hence, (aη, aδ) is an NVHS-open mapping.

Proposition 3.2.5.

Let $(\mho_{a1}, \tau_{\mho_{a1}}, Y)$, $(\mho_{a2}, \tau_{\mho_{a2}}, \dot{Y})$ and $(\mho_{a3}, \tau_{\mho_{a3}}, \ddot{Y})$ be NVHS topological spaces. Let $(a\eta, a\delta) : (\mho_{a1}, \tau_{\mho_{a1}}, Y) \rightarrow (\mho_{a2}, \tau_{\mho_{a2}}, \dot{Y})$ and $(a\psi, ' a\delta) : (\mho_{a2}, \tau_{\mho_{a2}}, \dot{Y}) \rightarrow (\mho_{a3}, \tau_{\mho_{a3}}, \ddot{Y})$ be two NVHS-maps.

1. If $(a\eta, a\delta)$ and $(a\psi, a\dot{\delta})$ are NVHS-closed mappings, then $(a\psi, a\dot{\delta})\circ(a\eta, a\delta)$ is also an NVHS-closed map.

2. If $(a\psi, a\dot{\delta})\circ(a\eta, a\delta)$ is an NVHS-closed mapping and if $(a\eta, a\delta)$ is a surjective NVHS-continuous mapping, then $(a\psi, a\dot{\delta})$ is an NVHS-closed mapping.

3. If $(a\psi, a\dot{\delta})\circ(a\eta, a\delta)$ is an NVHS-closed mapping and if $(a\psi, a\dot{\delta})$ is an injective NVHS-continuous mapping, then $(a\eta, a\delta)$ is an NVHS-closed mapping.

Proof. The proof is similar to Proposition 3.2.4 following Definition 3.2.1.

References:

[1] A. A. Salama and S. A. Alblowi, Neutrosophic set and neutrosophic topological spaces, IOSR Journal of Mathematics (IOSR-JM), 3(4), September-October 2012, 31 − 35, doi: 10.9790/5728-0343135.

[2] Anjan Mukherjee, Mithun Datta and Florentin Smarandache, Interval valued neutrosophic soft topological spaces, Neutrosophic Sets and Systems, 6, 2014, 18 − 27.

[3] Cigdem Gunduz, Taha Yasin Ozturk and Sadi Bayramov, Separation axioms on neutrosophic soft topological spaces, Turkish journal of mathematics, 43(1), 2019, 498 − 510, doi: 10.3906/mat-1805-110.

[4] Sagvan Y. Musa and Baravan A. Asaad, Hypersoft topological spaces, Neutrosophic Sets and Systems, 49, 2022, 397 − 415.

[5] Sagvan Y. Musa and Baravan A. Asaad, Connectedness on hypersoft topological spaces, Neutrosophic Sets and Systems, 51, 2022, 666 – 680

[6] Taha Yasin Ozt¨urk and Sadi Bayramov, ¨ Topology on soft continuous function spaces, Mathematical and Computational Applications, 22(2), 2017.

[7] Rana Muhammad Zulqarnain, Xiao Long Xin, Muhammad Saqlain, Muhammad Saeed, Florentin Smarandache and Muhammad Irfan Ahamad, Some fun222 damental operations on interval valued neutrosophic hypersoft set with their properties, Neutrosophic Sets and Systems, 40, 2021, 134 − 148.