Pythagorean Neutrosophic Super HyperSoft Topology

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Abstract: - In this paper, we provide Pythagorean Neutrosophic SuperHypersoft topological spaces, which are defined over a specified starting universe. Fundamental characteristics of Pythagorean Neutrosophic SuperHypersoft closed sets and Pythagorean Neutrosophic SuperHypersoft open sets are examined. The ideas of internal Pythagorean Neutrosophic SuperHypersoft border and Pythagorean Neutrosophic SuperHypersoft closure are finally introduced, and their relationship is examined.

Keywords: Pythagorean Neutrosophic SuperHypersoft sets; Pythagorean Neutrosophic SuperHypersoft topology; Pythagorean Neutrosophic SuperHypersoft open sets; Pythagorean Neutrosophic SuperHypersoft closed sets

1. Introduction

In 1965, A. Zadeh laid the groundwork for FS [17]. The fuzzy sets are determined by the level of membership values. The IVF sets notion was presented [16] to address the ambiguity surrounding membership values. We must consider membership and non-membership values in certain real-world issues in order to accurately represent an item in a dubious and ambiguous state. IFS, which are helpful in some circumstances, were initially created by Atanassov [2]. IFS, which consider both truth and falsehood values, and can be used to deal with inadequate data. Smarandache was the first to propose the idea of the Neutrosophic set [16]. The membership values for truth, doubt, and falsehood are shown by the neutrosophic set.

It was Molodstov who originally proposed the concept of a soft set [9] as a novel numerical technique for dealing with uncertain situations. He defines a soft set as a family of universal sets with parameter subsets. Soft sets are useful in many fields, AI, and basic DM problems [7]. Many scholars have studied the foundations of soft set theory in recent years. Ali at el. [1,8] offered the subset and super set, whereas Maji et al. [9] provided a theoretical analysis of soft sets.

The fuzzy topology concept, first introduced by Chang [4] and then refined by Lowen [6], is a well-known illustration of this type of generalization. Similarly, topological structures on soft sets are more broad techniques that may be applied to quantify the similarities and differences between soft-set objects in a universe.

Shabir [12] and Cagman et al. [3] have established two different variants of soft topology on soft sets. The notion of a hypersoft topological space arises as a natural extension of soft topology, built upon the framework of hypersoft sets [11]. Smarandache suggested a fresh approach for dealing with uncertainty. He expanded the $Softset_{(SS)}$ to $Hypersoft_{(HS)}$ set and Super Hyper soft set which is related to the Smarandache power set [13,14]. [5] The Pythagorean Neutrosophic Super Hypersoft set was defined by Hemalatha and Francina Shalini. This is more applicable to our everyday life.

The main aim of this paper is to introduce and investigate the fundamental concepts of Pythagorean Neutrosophic Super Hypersoft topological spaces. We define the basic operations and properties.

2. Preliminaries

Definition 2.1: [15] U represent the universe and the power set of U is $\mathfrak{W}(U)$. For t ≥ 1 let $(\mathfrak{S}_1, \mathfrak{S}_2, \mathfrak{S}_3 \ldots . \mathfrak{S}_n)_{HS}$ be t-distinct attributes, each of whose associated attributive values is the set $(\mathfrak{s}_1, \mathfrak{s}_2, \mathfrak{s}_3 \ldots . \mathfrak{s}_{t)}{}_{HS}$ with $(\mathfrak{s}_y \cap \mathfrak{s}_z)_{HS} = \emptyset$ as well as $y \neq z$ and $y, z \in \{1,2,, \ldots \ldots, t\}$, Then $(\mathfrak{F}, \mathfrak{s}_1, \mathfrak{s}_2, \mathfrak{s}_3 \ldots . \mathfrak{s}_t)_{HS}$ is a Hypersoft set over U. where $\mathfrak{F}$: $(\mathfrak{s}_1, \mathfrak{s}_2, \mathfrak{s}_3 \ldots . \mathfrak{s}_t)_{HS} \longrightarrow \mathfrak{W}$ (U).

Definition2.2: [12]

Let $\mathcal{T}_{\mathcal{H}}$ be the family of all hyper soft sets over the universe U. Then $\mathcal{T}_{\mathcal{H}}$ is said to be a Hypersoft Topology on $\mathcal{T}_{\mathcal{H}}$ if

i. $(\Phi, \mathcal{E})$ *and* $(\psi, \mathcal{E})$ belongs to $\mathcal{T}_{\mathcal{H}}$

ii. the union of any number of HS sets in $\mathcal{T}_{\mathcal{H}}$ belongs to $\mathcal{T}_{\mathcal{H}}$

iii. the intersection of finite number of HS sets in $\mathcal{T}_{\mathcal{H}}$ belongs to $\mathcal{T}_{\mathcal{H}}$.

Then $(U, \mathcal{T}_{\mathcal{H}}, \mathcal{E})$ is called a Hypersoft topological space over U.

Definition 2.3: [10] Consider U be the universal set and the power set of U is P(U). For t ≥ 1 let $(\mathfrak{S}_1, \mathfrak{S}_2, \mathfrak{S}_3 \ldots . \mathfrak{S}_n)_{NHS}$ be t-distinct attributes, each of whose associated attributive values is the set $(\mathfrak{s}_1, \mathfrak{s}_2, \mathfrak{s}_3 \ldots . \mathfrak{s}_t)_{NHS}$ with $(\mathfrak{s}_y \cap \mathfrak{s}_z)_{NHS} = \emptyset$ for $y \neq z$ and $y, z \in \{1,2,, \ldots \ldots, t\}$, The connection between these set is stated as $(\mathfrak{s}_1, \mathfrak{s}_2, \mathfrak{s}_3 \ldots . \mathfrak{s}_{t)}{}_{NHS} = \mathcal{H}$, and $\mathfrak{F}$: $(\mathfrak{s}_1, \mathfrak{s}_2, \mathfrak{s}_3 \ldots . \mathfrak{s}_{t)}{}_{NHS} \longrightarrow \mathbb{P}$ (U) and ($\mathfrak{F}$, $(\mathfrak{s}_1, \mathfrak{s}_2, \mathfrak{s}_3 \ldots . \mathfrak{s}_{t)}{}_{NHS}$={($\mathcal{H}$,<x_{NHS},$T_{\mathfrak{F}(\mathcal{H})}(x)_{NHS}$, $I_{\mathfrak{F}(\mathcal{H})}(x)_{NHS}$, $F_{\mathfrak{F}(\mathcal{H})}(x)_{NHS}$>) : $x \in U$,}where T represents truth membership, I represents indeterminacy, and F represents falsity membership such that $T_{\mathfrak{F}(\mathcal{H})}(x)_{NHS}$, $I_{\mathfrak{F}(\mathcal{H})}(x)_{NHS}$, $F_{\mathfrak{F}(\mathcal{H})}(x)_{NHS} \in$ [0,1] also $0 \leq T_{\mathfrak{F}(\mathcal{H})}(x)_{NHS}, I_{\mathfrak{F}(\mathcal{H})}(x)_{NHS}, F_{\mathfrak{F}(\mathcal{H})}(x)_{NHS} \leq 3$.

Definition2.4: [5] Let $\mathfrak{Z}$ represent the universal set and power set of $\mathfrak{Z}$ is $\mathfrak{G}(\mathfrak{Z})$. For k≥1, let $(\mathscr{b}_1, \mathscr{b}_2, \mathscr{b}_3 \ldots . \mathscr{b}_k)_{PNSHS}$ be k-distinct attributes, each of whose associated attributive values is the set $(\mathcal{B}_1, \mathcal{B}_2, \mathcal{B}_3 \ldots . \mathcal{B}_k)_{PNSHSS}$ with $(\mathcal{B}_m \cap \mathcal{B}_n)_{PNSHSS} = \emptyset$ as well as m≠n and m, n $\in \{1,2, \ldots, k\}$. Let $\mathfrak{G}(\mathcal{B}_1)_{PNSHSS}, \mathfrak{G}(\mathcal{B}_2)_{PNSHSS}, \mathfrak{G}(\mathcal{B}_3)_{PNSHSS}, \ldots \ldots, \mathfrak{G}(\mathcal{B}_k)_{PNSHSS} = \mathfrak{X}$ be the power sets of the set $(\mathcal{B}_1, \mathcal{B}_2, \mathcal{B}_3 \ldots . \mathcal{B}_k)_{PNSHS}$ respectively.

Then ($\mathfrak{f}$, $\mathfrak{G}(\mathcal{B}_1)_{PNSHSS} \times \mathfrak{G}(\mathcal{B}_2)_{PNSHSS} \times \mathfrak{G}(\mathcal{B}_3)_{PNSHSS} \times, \ldots \ldots, \times \mathfrak{G}(\mathcal{B}_k)_{PNSHSS}$) is PNSHSS over $\mathfrak{Z}$.

Where

$\mathfrak{f}$: ($\mathfrak{G}(\mathcal{B}_1)_{PNSHSS} \times \mathfrak{G}(\mathcal{B}_2)_{PNSHSS} \times \mathfrak{G}(\mathcal{B}_3)_{PNSHSS} \times, \ldots \ldots, \times \mathfrak{G}(\mathcal{B}_k)_{PNSHSS}$) $\longrightarrow$ ($\mathfrak{P}$ $(\mathfrak{Z})_{PNSHSS}$) and

$$\mathfrak{f}(\mathfrak{G}(\mathcal{B}_1)_{PNSHSS} \times \mathfrak{G}(\mathcal{B}_2)_{PNSHSS} \times \mathfrak{G}(\mathcal{B}_3)_{PNSHSS} \times, \ldots \ldots, \times \mathfrak{G}(\mathcal{B}_k)_{PNSHSS})$$
$$= \{\mathfrak{X}, < x, T_{\mathfrak{f}(\mathfrak{X})}(x), I_{\mathfrak{f}(\mathfrak{X})}(x), F_{\mathfrak{f}(\mathfrak{X})}(x) >: x \in \mathfrak{Z}, \mathfrak{X} \in (\mathfrak{G}(\mathcal{B}_1)_{PNSHSS} \times \mathfrak{G}(\mathcal{B}_2)_{PNSHSS} \times \mathfrak{G}(\mathcal{B}_3)_{PNSHSS} \times , \ldots \ldots, \times \mathfrak{G}(\mathcal{B}_k)_{PNSHSS})\}$$

Where $T_{\mathfrak{f}(\mathfrak{X})}$ and $F_{\mathfrak{f}(\mathfrak{X})}$ are the dependent components. $I_{\mathfrak{f}(\mathfrak{X})}$ is independent component. Also,

$0 \leq (T_{\mathfrak{f}(\mathfrak{X})}(x))^2 + (I_{\mathfrak{f}(\mathfrak{X})}(x))^2 + (F_{\mathfrak{f}(\mathfrak{X})}(x))^2 \leq 2$ and $T_{\mathfrak{f}(\mathfrak{X})}(x) + F_{\mathfrak{f}(\mathfrak{X})}(x) \leq 1$.

3. Pythagorean Neutrosophic Super Hypersoft Topology

Definition:3.1

Let (Λ, Γ) be the family of all PNSH sets over the universe Λ and $\tau \subseteq (\Lambda, \Gamma)$. Then τ is said to be a Pythagorean Neutrosophic Super Hypersoft Topology (PNSHT in short) on Λ if

i. 0_{PNSH} *and* 1_{PNSH} belongs to τ

ii. *the union of any number of PNSH sets in* τ *belongs to* τ

iii. *the intersection of finite number of PNSH sets in* τ *belongs to* τ

then, (Λ, Γ, τ) is said to Pythagorean Neutrosophic Super Hypersoft Topological Space (PNSHTS) over Λ. Each member of τ is said to be Pythagorean Neutrosophic Super Hypersoft open set.

Definition 3.2

Let (Λ, Γ, τ) be a PNSHTS and over Λ. (ζ_p, ξ_p) is said to be pythagorean neutrosophic super Hypersoft closed set if its complement is a pythagorean neutrosophic super hypersoft open set

Example:3.3

Let $\Lambda = \{\mathfrak{x}_1, \mathfrak{x}_2, \mathfrak{x}_3\}$ be initial universe and $\mathbb{l}_1, \mathbb{l}_2, \mathbb{l}_3$ be the sets of attributes. Attributes are given as follows,

$$\mathbb{l}_1 = \{\mathbb{g}_1, \mathbb{g}_2, \mathbb{g}_3\}_{PNSH}, \qquad \mathbb{l}_2 = \{\mathbb{h}_1, \mathbb{h}_2\}_{PNSH}, \qquad \mathbb{l}_3 = \{\mathbb{i}_1, \mathbb{i}_2\}_{PNSH}$$
$$\mathbb{p}(\mathbb{l}_1) = \{\{\mathbb{g}_1\},\{\mathbb{g}_2\},\{\mathbb{g}_3\},\{\mathbb{g}_1,\mathbb{g}_2\},\{\mathbb{g}_1,\mathbb{g}_3\},\{\mathbb{g}_2,\mathbb{g}_3\},\{\mathbb{g}_1,\mathbb{g}_2,\mathbb{g}_3\},\emptyset\}_{PNSH}$$
$$\mathbb{p}(\mathbb{l}_2) = \{\{\mathbb{h}_1\},\{\mathbb{h}_2\},\{\mathbb{h}_1,\mathbb{h}_2\},\emptyset\}_{PNSH}$$
$$\mathbb{p}(\mathbb{l}_3) = \{\{\mathbb{i}_1\},\{\mathbb{i}_2\},\{\mathbb{i}_1,\mathbb{i}_2\},\emptyset\}_{PNSH}$$

Supposet that, $\mathfrak{C}_1 = \{\{\mathbb{g}_1,\mathbb{g}_3\}\}_{PNSH}$, $\mathfrak{C}_2 = \{\{\mathbb{h}_1\},\{\mathbb{h}_1,\mathbb{h}_2\}\}_{PNSH}$, $\mathfrak{C}_3 = \{\{\mathbb{i}_1\},\{\mathbb{i}_2\}\}_{PNSH}$
$\tau = \{0_{PNSH}, 1_{PNSH}, (\zeta_p,\xi_p),(\zeta_q,\xi_q)$
$\tau^* = \{0_{PNSH}, 1_{PNSH}, (\zeta_r,\xi_r),(\zeta_s,\xi_s)$
be two topologies over Λ. Here the Pythagorean Neutrosophic Super Hypersoft sets,

$$(\zeta_p,\xi_p) = \begin{cases} \langle(\{\mathbb{g}_2,\mathbb{g}_3\},\{\mathbb{h}_1\},\{\mathbb{i}_1\}),\left\{\frac{\mathfrak{x}_1}{(0.1,0.6,0.7)},\frac{\mathfrak{x}_2}{(0.3,0.5,0.7)},\frac{\mathfrak{x}_3}{(0.2,0.5,0.4)}\right\}\rangle \\ \langle(\{\mathbb{g}_2,\mathbb{g}_3\},\{\mathbb{h}_1\},\{\mathbb{i}_2\}),\left\{\frac{\mathfrak{x}_1}{(0.2,0.6,0.5)},\frac{\mathfrak{x}_2}{(0.4,0.4,0.5)},\frac{\mathfrak{x}_3}{(0.4,0.6,0.6)}\right\}\rangle \\ \langle(\{\mathbb{g}_2,\mathbb{g}_3\},\{\mathbb{h}_1,\mathbb{h}_2\},\{\mathbb{i}_1\}),\left\{\frac{\mathfrak{x}_1}{(0.4,0.5,0.6)},\frac{\mathfrak{x}_2}{(0.6,0.3,0.4)},\frac{\mathfrak{x}_3}{(0.5,0.7,0.4)}\right\}\rangle \\ \langle(\{\mathbb{g}_2,\mathbb{g}_3\},\{\mathbb{h}_1,\mathbb{h}_2\},\{\mathbb{i}_2\}),\left\{\frac{\mathfrak{x}_1}{(0.5,0.8,0.4)},\frac{\mathfrak{x}_2}{(0.2,0.8,0.7)},\frac{\mathfrak{x}_3}{(0.1,0.8,0.5)}\right\}\rangle \end{cases}$$

$$(\zeta_q,\xi_q) = \begin{cases} \langle(\{\mathbb{g}_2,\mathbb{g}_3\},\{\mathbb{h}_1\},\{\mathbb{i}_1\}),\left\{\frac{\mathfrak{x}_1}{(0.2,0.5,0.5)},\frac{\mathfrak{x}_2}{(0.5,0.5,0.4)},\frac{\mathfrak{x}_3}{(0.3,0.4,0.1)}\right\}\rangle \\ \langle(\{\mathbb{g}_2,\mathbb{g}_3\},\{\mathbb{h}_1\},\{\mathbb{i}_2\}),\left\{\frac{\mathfrak{x}_1}{(0.5,0.4,0.3)},\frac{\mathfrak{x}_2}{(0.6,0.2,0.4)},\frac{\mathfrak{x}_3}{(0.3,0.4,0.1)}\right\}\rangle \\ \langle(\{\mathbb{g}_2,\mathbb{g}_3\},\{\mathbb{h}_1,\mathbb{h}_2\},\{\mathbb{i}_1\}),\left\{\frac{\mathfrak{x}_1}{(0.4,0.3,0.2)},\frac{\mathfrak{x}_2}{(0.6,0.3,0.3)},\frac{\mathfrak{x}_3}{(0.6,0.7,0.3)}\right\}\rangle \\ \langle(\{\mathbb{g}_2,\mathbb{g}_3\},\{\mathbb{h}_1,\mathbb{h}_2\},\{\mathbb{i}_2\}),\left\{\frac{\mathfrak{x}_1}{(0.6,0.6,0.4)},\frac{\mathfrak{x}_2}{(0.4,0.6,0.7)},\frac{\mathfrak{x}_3}{(0.3,0.6,0.4)}\right\}\rangle \end{cases}$$

$$(\zeta_r,\xi_r) = \begin{cases} \langle(\{\mathbb{g}_2,\mathbb{g}\},\{\mathbb{h}_1\},\{\mathbb{i}_1\}),\left\{\frac{\mathfrak{x}_1}{(0.1,0.5,0.7)},\frac{\mathfrak{x}_2}{(0.2,0.6,0.7)},\frac{\mathfrak{x}_3}{(0.2,0.8,0.6)}\right\}\rangle \\ \langle(\{\mathbb{g}_2,\mathbb{g}_3\},\{\mathbb{h}_1\},\{\mathbb{i}_2\}),\left\{\frac{\mathfrak{x}_1}{(0.1,0.6,0.7)},\frac{\mathfrak{x}_2}{(0.7,0.2,0.5)},\frac{\mathfrak{x}_3}{(0.3,0.8,0.5)}\right\}\rangle \\ \langle(\{\mathbb{g}_2,\mathbb{g}_3\},\{\mathbb{h}_1,\mathbb{h}_2\},\{\mathbb{i}_1\}),\left\{\frac{\mathfrak{x}_1}{(0.6,0.4,0.5)},\frac{\mathfrak{x}_2}{(0.4,0.5,0.6)},\frac{\mathfrak{x}_3}{(0.1,0.2,0.7)}\right\}\rangle \\ \langle(\{\mathbb{g}_2,\mathbb{g}_3\},\{\mathbb{h}_1,\mathbb{h}_2\},\{\mathbb{i}_2\}),\left\{\frac{\mathfrak{x}_1}{(0.6,0.7,0.4)},\frac{\mathfrak{x}_2}{(0.3,0.6,0.7)},\frac{\mathfrak{x}_3}{(0.2,0.7,0.7)}\right\}\rangle \end{cases}$$

$$(\zeta_s,\xi_s) = \begin{cases} \langle(\{\mathbb{g}_2,\mathbb{g}_3\},\{\mathbb{h}_1\},\{\mathbb{i}_1\}),\left\{\frac{\mathfrak{x}_1}{(0.1,0.5,0.6)},\frac{\mathfrak{x}_2}{(0.3,0.5,0.4)},\frac{\mathfrak{x}_3}{(0.2,0.5,0.3)}\right\}\rangle \\ \langle(\{\mathbb{g}_2,\mathbb{g}_3\},\{\mathbb{h}_1\},\{\mathbb{i}_2\}),\left\{\frac{\mathfrak{x}_1}{(0.2,0.4,0.3)},\frac{\mathfrak{x}_2}{(0.7,0.1,0.2)},\frac{\mathfrak{x}_3}{(0.7,0.3,0.2)}\right\}\rangle \\ \langle(\{\mathbb{g}_2,\mathbb{g}_3\},\{\mathbb{h}_1,\mathbb{h}_2\},\{\mathbb{i}_1\}),\left\{\frac{\mathfrak{x}_1}{(0.7,0.3,0.2)},\frac{\mathfrak{x}_2}{(0.7,0.1,0.2)},\frac{\mathfrak{x}_3}{(0.7,0.6,0.4)}\right\}\rangle \\ \langle(\{\mathbb{g}_2,\mathbb{g}_3\},\{\mathbb{h}_1,\mathbb{h}_2\},\{\mathbb{i}_2\}),\left\{\frac{\mathfrak{x}_1}{(0.7,0.6,0.3)},\frac{\mathfrak{x}_2}{(0.6,0.3,0.2)},\frac{\mathfrak{x}_3}{(0.5,0.6,0.5)}\right\}\rangle \end{cases}$$

Since $(\zeta_p,\xi_p) \cup (\zeta_s,\xi_s) \notin \tau \cup \tau^*$, *then* $\tau \cup \tau^*$ is not a pythagorean neutrosophic super Hypersoft topology over Λ.

Proposition:3.4

Let (Λ, Γ, τ) be a PNSHTS over Λ. Then

i. 0_{PNSH} *and* 1_{PNSH} are PNSH closed sets over *Λ*
ii. the intersection of any number of PNSH closed sets is a PNSH closed set over *Λ*.
iii. the union of finite number of PNSH closed sets is a PNSH closed set over *Λ*.

Proof.

It is clear for the definition of pythagorean neutrosophic super Hypersoft topological space.

Definition:3.5

Let (Λ, Γ) be the family of all PNSH sets over the universe Λ.

i. if $\tau = \{0_{PNSH}, 1_{PNSH}\}$, then τ is said to be the Pythagorean Neutrosophic Super Hypersoft indiscrete topology and (Λ, Γ, τ) is said to be a PNSH indiscrete topological space over Λ.

ii. if $\tau = \{\Lambda, \Gamma\}$, then τ is said to be the Pythagorean Neutrosophic Super Hypersoft discrete topology and (Λ, Γ, τ) is said to be a PNSH discrete topological space over Λ.

Definition:3.6

Let $(\Lambda, \Gamma, \tau_1)$ and $(\Lambda, \Gamma, \tau_2)$ be two pythagorean neutrosophic super Hypersoft topological spaces on Λ. If $\tau_1 \subseteq \tau_2$, then τ_2 is said to be finer than τ_1. $if\ \tau_1 \subseteq \tau_2\ or\ \tau_2 \subseteq \tau_1$, then $\tau_1\ and\ \tau_2$ are said to be comparable pythagorean neutrosophic super Hypersoft topologies over Λ.

Proposition:3.7

Let $(\Lambda, \Gamma, \tau_1)$ and $(\Lambda, \Gamma, \tau_2)$ be two PNSHTS over the same universe Λ. Then $(\Lambda, \Gamma, \tau_1 \cap \tau_2)$ is a PNSHTS over Λ.

Proof.

1. Since $0_{PNSH}, 1_{PNSH} \in \tau_1 and\ 0_{PNSH}, 1_{PNSH} \in \tau_2, then\ 0_{PNSH}, 1_{PNSH} \in \tau_1 \cap \tau_2)$.

2. Suppose that $\{(\zeta_i, \xi_i) |\ i \in I\ \}$ be a family of pythagorean neutrosophic super Hypersoft sets in $\tau_1 \cap \tau_2$. Then $(\zeta_i, \xi_i) \in \tau_1\ and\ (\zeta_i, \xi_i) \in \tau_2\ for\ all\ i \in I$ so, $\cup_{i\in I} (\zeta_i, \xi_i) \in \tau_1$ and $\cup_{i\in I} (\zeta_i, \xi_i) \in \tau_2. thus\ \cup_{i\in I} (\zeta_i, \xi_i) \in \tau_1 \cap \tau_2$.

3. Let $\{(\zeta_i, \xi_i) |\ i \in p, q, \dots, n\}$ be the family of the finite number of pythagorean neutrosophic super Hypersoft sets in $\tau_1 \cap \tau_2$.

Then $(\zeta_i, \xi_i) \in \tau_1\ and\ (\zeta_i, \xi_i) \in \tau_2\ for\ i = p, q, \dots, n$ so, $\cap_{i\in I} (\zeta_i, \xi_i) \in \tau_1$ and $\cap_{i\in I} (\zeta_i, \xi_i) \in \tau_2$. Thus $\cap_{i\in I} (\zeta_i, \xi_i) \in \tau_1 \cap \tau_2$.

Remark:3.8

The union of two pythagorean neutrosophic super Hypersoft topologies over Λ may not be a pythagorean neutrosophic super Hypersoft topology on Λ.

Definition:3.9

Let (Λ, Γ, τ) be a PNSHTS over Λ and $(\zeta_p, \xi_p) \in (\Lambda, \Gamma)$ be a pythagorean neutrosophic super Hypersoft set.

Then, the pythagorean neutrosophic super Hypersoft interior of (ζ_p, ξ_p) denoted $PNSH\ int(\zeta_p, \xi_p)$ is the union of all PNSH open subsets of (ζ_p, ξ_p).

Clearly $PNSH\ int(\zeta_p, \xi_p)$ is the biggest PNSH open set over Λ which contained by (ζ_p, ξ_p).

Proposition:3.10

Let (Λ, Γ, τ) be a PNSHTS over Λ and $(\zeta_p, \xi_p) \in (\Lambda, \Gamma)$ be a pythagorean neutrosophic super Hypersoft set.

Then

1. $PNSH\ int(\zeta_p, \xi_p)$ is the largest PNSH open set contained in (ζ_p, ξ_p)
2. (ζ_p, ξ_p) is a PNSH open set if and only if $(\zeta_p, \xi_p) = PNSH\ int(\zeta_p, \xi_p)$

Proof.

1. Follows from previous definition.

2. Let (ζ_p, ξ_p) be PNSH open set. Then (ζ_p, ξ_p) is surely identical with the largest PNSH open subset of (ζ_p, ξ_p). But by (1.), $PNSH\ int(\zeta_p, \xi_p)$ is the largest Hypersoft open subset of (ζ_p, ξ_p). Hence, $(\zeta_p, \xi_p) = PNSH\ int(\zeta_p, \xi_p)$.

Conversely,

Let $(\zeta_p, \xi_p) = PNSH\ int(\zeta_p, \xi_p)$. By (1.), $PNSH\ int(\zeta_p, \xi_p)$ is a PNSH open set and therefore (ζ_p, ξ_p) is also PNSH open set.

Theorem:3.11

Let (Λ, Γ, τ) be a PNSHTS over Λ and $(\zeta_p, \xi_p), (\zeta_q, \xi_q) \in (\Lambda, \Gamma)$. Then

1. $PNSH\ int(0_{PNSH}) = 0_{PNSH}\ and\ PNSH\ int(1_{PNSH}) = 1_{PNSH}$
2. $PNSH\ int(\zeta_p, \xi_p) \subseteq (\zeta_p, \xi_p)$
3. (ζ_p, ξ_p) is PNSH open set if and only if $(\zeta_p, \xi_p) = PNSH\ int(\zeta_p, \xi_p)$

4. $PNSH\ int(PNSH\ int(\zeta_p,\xi_p)) = PNSH\ int(\zeta_p,\xi_p)$
5. $(\zeta_p,\xi_p) \subseteq (\zeta_q,\xi_q)$ implies $PNSH\ int(\zeta_p,\xi_p) \subseteq PNSH\ int\ (\zeta_q,\xi_q)$
6. $PNSH\ int(\zeta_p,\xi_p) \cup PNSH\ int\ (\zeta_q,\xi_q) \subseteq PNSH\ int((\zeta_p,\xi_p) \cup (\zeta_q,\xi_q))$
7. $PNSH\ int((\zeta_p,\xi_p) \cap (\zeta_q,\xi_q)) = PNSH\ int(\zeta_p,\xi_p) \cap PNSH\ int\ (\zeta_q,\xi_q)$

Proof.

1. Since, 0_{PNSH} *and* 1_{PNSH} are PNSH open sets, then by the previous proposition (2.) $PNSH\ int(0_{PNSH}) = (0_{PNSH})$ and $PNSH\ int(1_{PNSH}) = 1_{PNSH}$.

2.let $v \in PNSH\ int\ (\zeta_p,\xi_p)$ then v is a PNSH interior point of (ζ_p,ξ_p). This implies that (ζ_p,ξ_p) is a PNSH neighbourhood of v. Then $v \in (\zeta_p,\xi_p)$. Hence $PNSH\ int\ (\zeta_p,\xi_p) \subseteq (\zeta_p,\xi_p)$.

3.The proof is obvious.

4.By previous Proposition (1.) $PNSH\ int(\zeta_p,\xi_p)$ is the PNSH open set. Hence by (2.) of the same proposition $PNSH\ int(PNSH\ int(\zeta_p,\xi_p)) = PNSH\ int(\zeta_p,\xi_p)$

5.Let $v \in PNSH\ int(\zeta_p,\xi_p)$. Then v is a PNSH interior point of (ζ_p,ξ_p) and so (ζ_p,ξ_p) is a PNSH neighbourhood of v. Since $(\zeta_p,\xi_p) \subseteq (\zeta_q,\xi_q)$, (ζ_q,ξ_q) is also a PNSH neighbourhood of v. This implies that $v \in PNSH\ int(\zeta_q,\xi_q)$. Thus $PNSH\ int(\zeta_p,\xi_p) \subseteq PNSH\ int(\zeta_q,\xi_q)$.

6. $(\zeta_p,\xi_p) \subseteq (\zeta_p,\xi_p) \cup (\zeta_q,\xi_q)$ implies $PNSH\ int(\zeta_p,\xi_p) \subseteq PNSH\ int((\zeta_p,\xi_p) \cup (\zeta_q,\xi_q))$ and $(\zeta_q,\xi_q) \subseteq (\zeta_p,\xi_p) \cup (\zeta_q,\xi_q)$ implies that $PNSH\ int(\zeta_q,\xi_q) \subseteq PNSH\ int((\zeta_p,\xi_p) \cup (\zeta_q,\xi_q))$. Hence, $PNSH\ int(\zeta_p,\xi_p) \cup PNSH\ int(\zeta_q,\xi_q) \subseteq PNSH\ int((\zeta_p,\xi_p) \cup (\zeta_q,\xi_q))$

7.since $(\zeta_p,\xi_p) \cap (\zeta_q,\xi_q) \subseteq (\zeta_p,\xi_p)$ and $(\zeta_p,\xi_p) \cap (\zeta_q,\xi_q) \subseteq (\zeta_q,\xi_q)$ we have $PNSH\ int\ ((\zeta_p,\xi_p) \cap (\zeta_q,\xi_q)) \subseteq PNSH\ int\ (\zeta_p,\xi_p)$ and $PNSH\ int\ ((\zeta_p,\xi_p) \cap (\zeta_q,\xi_q)) \subseteq PNSH\ int\ (\zeta_q,\xi_q)$. This implies that $PNSH\ int\ ((\zeta_p,\xi_p) \cap (\zeta_q,\xi_q)) \subseteq PNSH\ int\ (\zeta_p,\xi_p) \cap PNSH\ int\ (\zeta_q,\xi_q)$. again let $v \in PNSH\ int\ (\zeta_p,\xi_p) \cap PNSH\ int\ (\zeta_q,\xi_q)$. Then $v \in PNSH\ int\ (\zeta_p,\xi_p)$ and $v \in PNSH\ int\ (\zeta_q,\xi_q)$. Hence v is a PNSH interior point of each of the PNSH sets (ζ_p,ξ_p) *and* (ζ_q,ξ_q). It follows that $(\zeta_p,\xi_p)$and (ζ_q,ξ_q) are PNSH neighbourhoods of v so that their intersection $(\zeta_p,\xi_p) \cap (\zeta_q,\xi_q)$ is also a PNSH neighbourhood of v. Hence, $v \in PNSH\ int\ ((\zeta_p,\xi_p) \cap (\zeta_q,\xi_q))$. Thus $PNSH\ int\ (\zeta_p,\xi_p) \cap PNSH\ int\ (\zeta_q,\xi_q) \subseteq PNSH\ int\ ((\zeta_p,\xi_p) \cap (\zeta_q,\xi_q))$. Therefore $PNSH\ int\ (\zeta_p,\xi_p) \cap PNSH\ int\ (\zeta_q,\xi_q) = PNSH\ int\ ((\zeta_p,\xi_p) \cap (\zeta_q,\xi_q))$

Definition:3.12

Let (Λ,Γ,τ) be a PNSHTS over Λ and $(\zeta_p,\xi_p) \in (\Lambda,\Gamma)$ be a pythagorean neutrosophic super Hypersoft set. Then, the pythagorean neutrosophic super Hypersoft closure of (ζ_p,ξ_p) denoted by $PNSH\ cl(\zeta_p,\xi_p)$ is the intersection of all pythagorean neutrosophic Super Hypersoft closed supersets of (ζ_p,ξ_p).

Clearly $PNSH\ cl(\zeta_p,\xi_p)$ is the smallest PNSH closed over Λ which containing (ζ_p,ξ_p).

Proposition:3.13

Let (Λ,Γ,τ) be a PNSHTS over Λ and $(\zeta_p,\xi_p) \in (\Lambda,\Gamma)$ be a pythagorean neutrosophic super Hypersoft set. Then

1. $PNSH\ cl(\zeta_p,\xi_p)$ is the smallest PNSH closed set containing in (ζ_p,ξ_p)
2. (ζ_p,ξ_p) is a PNSH closed set if and only if $(\zeta_p,\xi_p) = PNSH\ cl(\zeta_p,\xi_p)$

Proof.

1. Follows from previous definition.
2. Let (ζ_p,ξ_p) be PNSH closed set. So, (ζ_p,ξ_p) itself is the smallest PNSH closed set over Λ Containing (ζ_p,ξ_p) and hence $(\zeta_p,\xi_p) = PNSH\ cl(\zeta_p,\xi_p)$. Conversely, suppose that $(\zeta_p,\xi_p) = PNSH\ cl(\zeta_p,\xi_p)$. By (1.) $PNSH\ cl(\zeta_p,\xi_p)$ is a pythagorean neutrosophic super Hypersoft closed set over Λ.

Proposition:3.14

Let (Λ,Γ,τ) be a PNSHTS over Λ and $(\zeta_p,\xi_p), (\zeta_p,\xi_p) \in (\Lambda,\Gamma)$. Then

1. $PNSH\ cl(0_{PNSH}) = 0_{PNSH}\ and\ PNSH\ cl(1_{PNSH}) = 1_{PNSH}$
2. $(\zeta_p,\xi_p) \subseteq PNSH\ cl(\zeta_p,\xi_p)$

3. (ζ_p, ξ_p) is PNSH open set if and only if $(\zeta_p, \xi_p) = PNSH\ cl(\zeta_p, \xi_p)$
4. $PNSH\ cl(PNSH\ cl(\zeta_p, \xi_p)) = PNSH\ cl(\zeta_p, \xi_p)$
5. $(\zeta_p, \xi_p) \subseteq (\zeta_q, \xi_q)$ implies $PNSH\ cl(\zeta_p, \xi_p) \subseteq PNSH\ cl\ (\zeta_q, \xi_q)$
6. $PNSH\ cl((\zeta_p, \xi_p) \cup (\zeta_q, \xi_q)) = PNSH\ cl(\zeta_p, \xi_p) \cup PNSH\ cl\ (\zeta_q, \xi_q)$
7. $PNSH\ cl((\zeta_p, \xi_p) \cap (\zeta_q, \xi_q)) \subseteq PNSH\ cl(\zeta_p, \xi_p) \cap PNSH\ cl\ (\zeta_q, \xi_q)$

Proof.

1. Since $0_{PNSH}, and\ 1_{PNSH}$ are pythagorean neutrosophic super Hypersoft closed sets. Then the previous proposition (2.) we have $PNSH\ cl(0_{PNSH}) = 0_{PNSH}\ and\ PNSH\ cl(1_{PNSH}) = 1_{PNSH}$

2. By previous proposition (1.), $PNSH\ cl(\zeta_p, \xi_p)$ is the smallest PNSH closed set containing (ζ_p, ξ_p) and also $(\zeta_p, \xi_p) \subseteq PNSH\ cl(\zeta_p, \xi_p)$

3. The Proof is obvious.

4.Since $PNSH\ cl(\zeta_p, \xi_p)$ is a PNSH closed set, therefore by previous proposition (2.) we have $PNSH\ cl(PNSH\ cl(\zeta_p, \xi_p)) = PNSH\ cl(\zeta_p, \xi_p)$.

5. $(\zeta_q, \xi_q) \subseteq PNSH\ cl(\zeta_q, \xi_q)$. Since $(\zeta_p, \xi_p) \subseteq (\zeta_q, \xi_q)$, we have $(\zeta_p, \xi_p) \subseteq PNSH\ cl(\zeta_q, \xi_q)$. But $PNSH\ cl(\zeta_q, \xi_q)$ is a PNSH closed set. Thus, $PNSH\ cl(\zeta_q, \xi_q)$ is a PNSH closed set containing (ζ_p, ξ_p). Since $PNSH\ cl(\zeta_p, \xi_p)$ is the smallest PNSH closed set over Λ containing (ζ_p, ξ_p), sp we have $PNSH\ cl(\zeta_p, \xi_p) \subseteq PNSH\ cl(\zeta_q, \xi_q)$

6. since $(\zeta_p, \xi_p) \subseteq (\zeta_p, \xi_p) \cup (\zeta_q, \xi_q)$ and $(\zeta_q, \xi_q) \subseteq (\zeta_p, \xi_p) \cup (\zeta_q, \xi_q)$ we have $PNSH\ cl(\zeta_p, \xi_p) \subseteq PNSH\ cl((\zeta_p, \xi_p) \cup (\zeta_q, \xi_q))$ and $PNSH\ cl(\zeta_q, \xi_q) \subseteq PNSH\ cl((\zeta_p, \xi_p) \cup (\zeta_q, \xi_q))$. Hence, $PNSH\ cl(\zeta_p, \xi_p) \cup PNSH\ cl(\zeta_q, \xi_q) \subseteq PNSH\ cl((\zeta_p, \xi_p) \cup (\zeta_q, \xi_q))$. Now since, $PNSH\ cl(\zeta_p, \xi_p)\ and\ PNSH\ cl(\zeta_q, \xi_q)$ are PNSH closed sets, $PNSH\ cl(\zeta_p, \xi_p) \cup PNSH\ cl(\zeta_q, \xi_q)$ is also PNSH closed. Also $(\zeta_p, \xi_p) \subseteq PNSH\ cl(\zeta_p, \xi_p)\ and\ (\zeta_q, \xi_q) \subseteq PNSH\ cl(\zeta_q, \xi_q)$ implies that $(\zeta_p, \xi_p) \cup (\zeta_q, \xi_q) \subseteq PNSH\ cl(\zeta_p, \xi_p) \cup PNSH\ cl(\zeta_q, \xi_q)$. Thus $PNSH\ cl(\zeta_p, \xi_p) \cup PNSH\ cl(\zeta_q, \xi_q)$ is a PNSH closed containing $(\zeta_p, \xi_p) \cup (\zeta_q, \xi_q)$. Since $PNSH\ cl((\zeta_p, \xi_p) \cup (\zeta_q, \xi_q))$ is the smallest PNSH closed set containing $(\zeta_p, \xi_p) \cup (\zeta_q, \xi_q)$, we have $PNSH\ cl((\zeta_p, \xi_p) \cup (\zeta_q, \xi_q)) \subseteq PNSH\ cl(\zeta_p, \xi_p) \cup PNSH\ cl(\zeta_q, \xi_q)$. Hence $PNSH\ cl((\zeta_p, \xi_p) \cup (\zeta_q, \xi_q)) = PNSH\ cl(\zeta_p, \xi_p) \cup\ PNSH\ cl(\zeta_q, \xi_q)$.

7. Since $(\zeta_p, \xi_p) \cap (\zeta_q, \xi_q) \subseteq (\zeta_p, \xi_p)\ and\ (\zeta_p, \xi_p) \cap (\zeta_q, \xi_q) \subseteq (\zeta_q, \xi_q), then\ PNSH\ cl((\zeta_p, \xi_p) \cap (\zeta_q, \xi_q)) \subseteq PNSH\ cl(\zeta_p, \xi_p)\ and\ PNSH\ cl((\zeta_p, \xi_p) \cap (\zeta_q, \xi_q)) \subseteq PNSH\ cl(\zeta_q, \xi_q)$. Therefore, $PNSH\ cl((\zeta_p, \xi_p) \cap (\zeta_q, \xi_q)) \subseteq PNSH\ cl(\zeta_p, \xi_p) \cap PNSH\ cl(\zeta_q, \xi_q)$.

4. Conclusion

Pythagorean neutrosophic super Hypersoft topological spaces, which are defined over the initial universe with a specified set of parameters, were introduced in this study. Concepts based on our definition, such as PNSH closure and PNSH interior, were presented, examined, and their connections were analyzed.

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