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Pythagorean Neutrosophic Super Hypersoft Sets and their Aggregate Operators

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Abstract:- The aim of this paper, we extend the concept of Neutrosophic Hypersoft set to Pythagorean Neutrosophic Super Hypersoft Set (PNSHSS) which includes the basic operations like Union, Intersection, AND, OR etc., of Pythagorean Neutrosophic Super Hypersoft set. Relevant examples are provided along with the presentation of the implementation validity.

Keywords: *Soft set, Neutrosophic soft set, Hypersoft set, Pythagorean Neutrosophic Super Hypersoft set, MCDM,*

1. INTRODUCTION

A. Zadeh established a foundation for fuzzy sets in 1965 [1]. The level of membership values determines the fuzzy sets. In certain instances, assigning membership values to fuzzy sets may be challenging. In order to account for the ambiguity around membership values, The interval valued fuzzy sets notion was introduced [2]. For accurate representation of an item in a dubious and uncertain state, we need take into membership and non-membership values in particular real-world problems, master frameworks, conviction frameworks, data combinations, etc. Atanassaov [3] first developed IFS, which are useful in certain situations. Insufficient data can be handled using IFS, which take into account both truth and falsehood values.

Smarandache was the first to propose the idea of the Neutrosophic set [4]. The neutrosophic set indicates the membership values for truth, uncertainty, and falsehood. Molodstov [5] introduced the concept of a soft set as a brand-new numerical tool for handling problems with ambiguous situations. According to him, a family of universal sets with parameters subsets is a soft set. Soft sets are helpful in many areas, such as artificial intelligence, game theory, and simple decision-making issues [6]. Over the past few years, various researchers have examined the fundamentals of soft set theory. A theoretical analysis of soft sets was by Maji et al.[7] addresses the subset and super set were presented by Ali at el.[8].Smarandache suggested a fresh approach for dealing with uncertainty. He expanded the $Softset_{(SS)}$ to $Hypersoft_{(HS)}$ set and Super Hyper soft set is related to the Smarandache power set [9,10,11,12,13], in TOPSIS and MCDM for different extensions of neutrosophic sets. Saqlain and colleagues presented a new algorithm.

2. PRELIMINARIES

Definition 2.1. [7] Let U represents the universe set, $\mathrm{P}(U)$ denotes the power set and Z be parameters. Consider $\text{Å} \subset Z$. Then $(F, \text{Å})$ is a soft set over U, where F is a mapping given by $F: \text{Å} \longrightarrow \mathrm{P}(U)$.

Definition 2.3. [14] U represent the universe and the power set of U is $W(U)$. For $t \geq 1$ let $(S_1, S_2, S_3 \ldots\ldots S_n)_{HS}$ be t-distinct attributes, each of whose associated attributive values is the set $(s_1, s_2, s_3 \ldots\ldots s_{t)}{}_{HS}$ with $(s_y \cap s_z)_{HS} = \emptyset$ as well as $y \neq z$ and $y, z \in \{1,2,, \ldots\ldots, t\}$, Then $(F, s_1, s_2, s_3 \ldots\ldots s_t\,)_{HS}$ is a Hypersoft set over U.

where F: $(s_1, s_2, s_3 \ldots\ldots s_t)_{HS} \longrightarrow W$ (U)
(1)
Definition 2.4: [15] Consider U be the universal set and the power set of U is $\mathrm{P}(U)$. For $t \geq 1$ let $(S_1, S_2, S_3 \ldots\ldots S_n)_{NHS}$ be t-distinct attributes, each of whose associated attributive values is the set $(s_1, s_2, s_3 \ldots\ldots s_{t)}{}_{NHS}$ with $(s_y \cap s_z)_{NHS} = \emptyset$ for $y \neq z$ and $y, z \in \{1,2,, \ldots \ldots, t\}$, The connection between these set is stated as $(s_1, s_2, s_3 \ldots\ldots s_{t)}{}_{NHS} = H$, and F: $(s_1, s_2, s_3 \ldots\ldots s_{t)}{}_{NHS} \longrightarrow P$ (U) and $(F, (s_1, s_2, s_3 \ldots\ldots s_{t)}{}_{NHS}$={(H,<x_{NHS},$T_{F(H)}(x)_{NHS}$, $I_{F(H)}(x)_{NHS}$, $F_{F(H)}(x)_{NHS}$>) : $x \in U$,}where T represents truth membership, I represents indeterminacy, and F represents falsity membership such that $T_{F(H)}(x)_{NHS}$, $I_{F(H)}(x)_{NHS}$, $F_{F(H)}(x)_{NHS} \in [0,1]$ also $0 \leq T_{F(H)}(x)_{NHS}, I_{F(H)}(x)_{NHS}, F_{F(H)}(x)_{NHS} \leq 3$. (2)

3. Pythagorean Neutrosophic SuperHypersoft Set

Definition 3.1. Let Z represent the universal set and power set of Z is $G(Z)$. For k≥1, let $(b_1, b_2, b_3 \ldots\ldots b_k)_{PNSHS}$ be k-distinct attributes, each of whose associated attributive values is the set $(B_1, B_2, B_3 \ldots\ldots B_k)_{PNSHS}$ with $(B_m \cap B_n)_{PNSHSS} = \emptyset$ as well as m≠n and m, n $\in \{1,2, \ldots, k\}$.
Let $G(B_1)_{PNSHS}, G(B_2)_{PNSHS}, G(B_3)_{PNSHS}, \ldots \ldots, G(B_k)_{PNSHS} = T$ be the power sets of the set $(B_1, B_2, B_3 \ldots\ldots B_k)_{PNSHS}$ respectively.
Then $(f,\ G(B_1)_{PNSHS} \times G(B_2)_{PNSHS} \times G(B_3)_{PNSHS} \times, \ldots \ldots, \times G(B_k)_{PNSHS})$ is PNSHSS over Z.
where f: $(\ G(B_1)_{PNSHS} \times G(B_2)_{PNSHS} \times G(B_3)_{PNSHS} \times, \ldots \ldots, \times G(B_k)_{PNSHS}) \longrightarrow (P\ (Z)_{PNSHS})$ and

$$f(\ G(B_1)_{PNSHS} \times G(B_2)_{PNSHS} \times G(B_3)_{PNSHS} \times, \ldots \ldots, \times G(B_k)_{PNSHS}) = \{T, < x, T_{f(T)}(x), I_{f(T)}(x), F_{f(T)}(x) >:\ x \in Z,\ T \in (\ G(B_1)_{PNSHS} \times G(B_2)_{PNSHS} \times G(B_3)_{PNSHS} \times, \ldots \ldots, \times G(B_k)_{PNSHS})\} \quad (3)$$

Where $T_{f(T)}$ and $F_{f(T)}$ are the dependent components. $I_{f(T)}$ is independent component. Also,

$$0 \leq \left(T_{f(T)}(x)\right)^2 + \left(I_{f(T)}(x)\right)^2 + \left(F_{f(T)}(x)\right)^2 \leq 2 \text{ and } T_{f(T)}(x) + F_{f(T)}(x) \leq 1. \quad (4)$$

Example

$Z = \{m^1, m^2, m^3, m^4, m^5\}$, b^1 = Fridge brand, b^2 = Star rating, b^3 = Door type, b^4 = Warranty, and their attributes are Fridge brand =$\{Samsung, Haier, Panasonic\}$,Star rating = $\{3star, 5star\}$, Door type = $\{Single\ door, Double\ door\}$,Warranty = $\{10years, 15years\}$
$P(B_1) = \{\{Samsung\}, \{Haier\}, \{Panasonic\}, \{Samsung, Haier\}, \{Samsung, Panasonic\}, \{Haier, Panasonic\}, \{Samsung, Haier, Panasonic\}, \emptyset\}$

$$P(B_2) = \{\{3star\}\{\ 5star\}, \{3star, 5star\}, \emptyset\}$$

$$P(B_3) = \{\{Single\ door\}\{Double\ door\}, \{Single\ door, Double\ door\}, \emptyset\}$$

$$P(B_4) = \{\{10years\}, \{15years\}, \{10years, 15years\}, \emptyset\}$$

Let f: $(\ G(B_1)_{PNSHS} \times G(B_2)_{PNSHS} \times G(B_3)_{PNSHS} \times, \ldots \ldots, \times G(B_k)_{PNSHS}) \longrightarrow (P\ (Z)_{PNSHS})$

TABLE 1. Table representation of PNSHSS

	m^1	m^2	m^3	m^4	m^5
Samsung	(0.3,0.6,0.7)	(0.4,0.6,0.4)	(0.4,0.5,0.2)	(0.6,0.5,0.3)	(0.5,0.3,0.2)
Haier	(0.1,0.5,0.6)	(0.3,0.2,0.1)	(0.3,0.6,0.2)	(0.8,0.1,0.2)	(0.5,0.4,0.5)
Panasonic	(0.5,0.3,0.1)	(0.5,0.2,0.1)	(0.8,0.5,0.2)	(0.6,0.4,0.3)	(0.7,0.4,0.2)
$\{Samsung, Haier\}$	(0.1,0.5,0.6)	(0.1,0.2,0.5)	(0.2,0.5,0.8)	(0.3,0.4,0.6)	(0.2,0.4,0.7)

$\{Samsung, Panasonic\}$	(0.5,0.5,0.1)	(0.1,0.2,0.5)	(0.2,0.6,0.5)	(0.2,0.4,0.8)	(0.5,0.4,0.5)
$\{Haier, Panasonic\}$	(0.5,0.6,0.3)	(0.1,0.2,0.5)	(0.2,0.5,0.5)	(0.3,0.5,0.6)	(0.2,0.4,0.6)
$\{Samsung, Haier, Panaso$	(0.5,0.3,0.1)	(0.5,0.2,0.1)	(0.8,0.5,0.2)	(0.8,0.1,0.2)	(0.7,0.3,0.2)
$\emptyset$	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
3 Star	(0.6,0.4,0.3)	(0.7,0.8,0.3)	(0.4,0.4,0.3)	(0.4,0.5,0.3)	(0.9,0.2,0.1)
5 Star	(0.3,0.6,0.4)	(0.3,0.2,0.1)	(0.3,0.6,0.2)	(0.8,0.4,0.2)	(0.5,0.4,0.5)
$\{3star, 5star\}$	(0.6,0.4,0.3)	(0.7,0.2,0.1)	(0.4,0.4,0.2)	(0.8,0.4,0.2)	(0.9,0.2,0.1)
$\emptyset$	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
Single door	(0.6,0.8,0.3)	(0.7,0.6,0.2)	(0.4,0.5,0.6)	(0.8,0.2,0.1)	(0.5,0.3,0.4)
Double door	(0.3,0.6,0.4)	(0.6,0.5,0.3)	(0.5,0.7,0.2)	(0.5,0.2,0.1)	(0.8,0.5,0.2)
$\{Single\ door, Double\ door\}$	(0.6,0.6,0.3)	(0.7,0.5,0.2)	(0.5,0.5,0.2)	(0.8,0.2,0.1)	(0.8,0.3,0.2)
$\emptyset$	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
10 years	(0.6,0.4,0.3)	(0.6,0.5,0.3)	(0.5,0.4,0.3)	(0.7,0.8,0.3)	(0.8,0.2,0.1)
15 years	(0.3,0.2,0.1)	(0.4,0.8,0.6)	(0.7,0.3,0.2)	(0.3,0.6,0.4)	(0.8,0.4,0.2)
$\{10years, 15years\}$	(0.6,0.2,0.1)	(0.6,0.5,0.3)	(0.7,0.3,0.2)	(0.7,0.6,0.3)	(0.8,0.2,0.1)
$\emptyset$	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)

Pythagorean Neutrosophic Super Hyper soft set is,

$f: (\ G(B_1)_{PNSHS} \times G(B_2)_{PNSHS} \times G(B_3)_{PNSHS} \times, \dots \dots, \times G(B_k)_{PNSHS}) \longrightarrow (P\ (Z)_{PNSHS})$

Assume that,

f: ({Haier},{3star,5star},{Single door, Double door},{15years}) =$\{m^1, m^4\}$

Then the PNSHSS of above assumed relation is,

$f\ (T)$: f ({Haier},{3star,5star},{Single door, Double door},{15years})

$= \{m^1\{Haier\}(0.1,0.5,0.6), \{3star, 5star\}(0.6,0.4,0.3), \{Single\ door, Double\ door\}(0.6,0.6,0.3),$
$\{15years\}(0.3,0.2,0.1)\ m^4\{Haier\}(0.8,0.1,0.2), \{3star, 5star\}(0.8,0.4,0.2), \{Single\ door, Double\ door\}(0.8,0.2,0.1), \{15years\}(0.3,0.6,0.4)$

Therefore, the PNSHSS offers a larger variety of selection, so m^1 *and* m^4 may be
Mandatory Haier [not Samsung, Panasonic, not other possibilities]
Either 3star or 5star [not other possibilities]
Either Single door or Double door [not other possibilities]
Mandatory 15years [not other possibilities].

Definition 3.2. PNSHS Subset

Let $f(T_{PNSHSS})_1$, $f(T_{PNSHSS})_2$ be two PNSHSS in the universe Z. For k ≥1, let $(b_1, b_2, b_3 \dots .. b_k)_{PNSHS}$ be k-distinct attributes, each of whose associated attributive values is the set $(B_1, B_2, B_3 \dots .. B_k)_{PNSHS}$ with $(B_m \cap B_n)_{PNSHSS} = \emptyset$ as well as m≠n, m, n $\in \{1,2, \dots, k\}$. Let $G(B_1)_{PNSHS}, G(B_2)_{PNSHS}, G(B_3)_{PNSHS}, \dots \dots, G(B_k)_{PNSHS} = T$ be the power sets of the set $(B_1, B_2, B_3 \dots .. B_k)_{PNSHS}$ respectively.

Then $f(T_{PNSHSS})_1$ is the PNSHS subset of $f(T_{PNSHSS})_2$ if

$T(f(T_{PNSHSS})_1) \leq T(f(T_{PNSHSS})_2), I(f(T_{PNSHSS})_1) \leq I(f(T_{PNSHSS})_2), F(f(T_{PNSHSS})_1) \leq F(f(T_{PNSHSS})_2)$ (5)

Example

Let $f(T_{PNSHSS})_1$ and $f(T_{PNSHSS})_2$ be two PNSHSS over the same universe $Z = \{m^1, m^2, m^3, m^4, m^5\}$. Then PNSHSS $f(T_{PNSHSS})_1 = f((Haier, Panasonic), 5Star, (Single\ door, Double\ door)) = \{m^2, m^3\}$

$$f(T_{PNSHSS})_2 = f((Haier, Panasonic), 5Star) = \{m^2\}$$

TABLE 2. PNSHSS $f(T_{PNSHSS})_1$

$f(T_{PNSHSS})_1$	m^2	m^3
$(Haier, Panasonic)$	(0.1,0.2,0.5)	(0.2,0.5,0.5)
5 Star	(0.3,0.2,0.1)	(0.3,0.6,0.2)
$(Single\ door, Double\ door)$	(0.7,0.5,0.2)	(0.5,0.5,0.2)

TABLE 3. PNSHSS $f(T_{PNSHSS})_2$

$f(T_{PNSHSS})_2$	m^2
$(Haier, Panasonic)$	(0.6,0.3,0.4)
5 Star	(0.4,0.5,0.1)

$$f(T_{PNSHSS})_1 \subseteq f(T_{PNSHSS})_2 = f\big((Haier, Panasonic), 5Star, (Single\ door, Double\ door)\big)$$
$$\subseteq f((Haier, Panasonic), 5Star)$$
$= \{m^2(Haier, Panasonic)(0.1,0.2,0.5), 5Star(0.3,0.2,0.1), (Single\ door, Double\ door)(0.7,0.5,0.2)\ m^3(Haier, Panasonic)($
$$\subseteq \{m_2(Haier, Panasonic)(0.1,0.2,0.5), 5Star(0.3,0.2,0.1)$$

From the above we can see that membership values of (Haier, Panasonic) for m_2 in both sets $(0.1,0.2,0.5), (0.6,0.3,0.4)$ which implies PNSHS Subset as $0.1 < 0.6, 0.2 < 0.3$ *and* $0.5 > 0.4$. This implies that $(0.1,0.2,0.5) \subset (0.6,0.3,0.4)$ the same applied to the remaining $f(T_1)_{PNSHSS}$ *and* $f(T_1)_{PNSHSS}$ properties.

Definition 3.3. PNSHS Null Set

Let $f(T_{PNSHSS})_1$, *be* PNSHSS in the universe Z. For k≥1, let $(b_1, b_2, b_3 \ldots.. b_k)_{PNSHS}$ be k-distinct attributes, each of whose associated attributive values is the set $(B_1, B_2, B_3 \ldots.. B_k)_{PNSHS}$ with $(B_m \cap B_n)_{PNSHSS} = \emptyset$ as well as m≠n and m, n ∈ $\{1,2, \ldots, k\}$. $G(B_1)_{PNSHS}, G(B_2)_{PNSHS}, G(B_3)_{PNSHS}, \ldots \ldots, G(B_k)_{PNSHS} = T$ be the power sets of the set $(B_1, B_2, B_3 \ldots.. B_k)_{PNSHS}$ respectively.

Then $f(T_{PNSHSS})_1$ is Null PNSHSS if $T(f(T_{PNSHSS})_1) = 0, I(f(T_{PNSHSS})_1) = 0, F(f(T_{PNSHSS})_1) = 0$. (6)

Example

Consider the PNSHSS $f(T_{PNSHSS})_1$ over the universe $Z = \{m^1, m^2, m^3, m^4, m^5\}$. Then PNSHSS $f(T_{PNSHSS})_1 = F(Samsung, (5Star, 3Star), Double\ door) = \{m^2, m^3\}$ be Null PNSHSS if its PNSHSS values are 0.

TABLE 4. PNSHSS $f(T_{PNSHSS})_1$

$f(T_{PNSHSS})_1$	m^3	m^4
Haier	(0.0,0.0,0.0)	(0.0,0.0,0.0)
(5star,3Star)	(0.0,0.0,0.0)	(0.0,0.0,0.0)
Double door	(0.0,0.0,0.0)	(0.0,0.0,0.0)

$$f(T_{PNSHSS})_1 = f(Samsung, (5Star, 3Star), Double\ door)$$

$= \{< m^3, (\{Haier\}(0.0,0.0,0.0), \{5Star, 3Star\}(0.0,0.0,0.0), \{Double\ door\}(0.0,0.0,0.0)) >,$
$< m^4, (\{Haier\}(0.0,0.0,0.0), \{5Star, 3Star\}(0.0,0.0,0.0), \{Double\ door\}(0.0,0.0,0.0)) >$

Definition 3.4. PNSHS Equal Set

Let $f(T_{PNSHSS})_1, f(T_{PNSHSS})_2$ be two PNSHSS in the universe Z. For k≥1, let $(b_1, b_2, b_3 \ldots\ldots b_k)_{PNSHS}$ be k-distinct attributes, each of whose associated attributive values is the set $(B_1, B_2, B_3 \ldots\ldots B_k)_{PNSHS}$ with $(B_m \cap B_n)_{PNSHSS} = \emptyset$ as well as m≠n and m, n ∈ $\{1,2, \ldots, k\}$.

Let $G(B_1)_{PNSHS}, G(B_2)_{PNSHS}, G(B_3)_{PNSHS}, \ldots\ldots, G(B_k)_{PNSHS} = T$ be the power sets of the set $(B_1, B_2, B_3 \ldots\ldots B_k)_{PNSHS}$ respectively. Then $f(T_{PNSHSS})_1$ is the PNSHS equal set of $f(T_{PNSHSS})_2$ if,

$T(f(T_{PNSHSS})_1) = T(f(T_{PNSHSS})_2),\ I(f(T_{PNSHSS})_1) = I(f(T_{PNSHSS})_2),\ F(f(T_{PNSHSS})_1) = F(f(T_{PNSHSS})_2)$ (7)

Example

Let two PNSHSS $f(T_{PNSHSS})_1$, $f(T_{PNSHSS})_2$ in the same universe $Z = \{m^1, m^2, m^3, m^4, m^5\}$. Then PNSHSS

$$f(T_{PNSHSS})_1 = f(Samsung, 3Star, Single\ door) = \{m^2, m^3\}$$
$$f(T_{PNSHSS})_2 = f(Samsung, 3Star) = \{m^2\}$$

TABLE 5. PNSHSS $f(T_{PNSHSS})_1$

$f(T_{PNSHSS})_1$	m^1	m^3
Panasonic	(0.5,0.3,0.1)	(0.4,0.5,0.2)
3Star	(0.6,0.4,0.3)	(0.4,0.4,0.3)
Single door	(0.6,0.8,0.3)	(0.4,0.5,0.6)

TABLE 6. PNSHSS $f(T_{PNSHSS})_2$

$f(T_{PNSHSS})_2$	m^1
Panasonic	(0.5,0.3,0.1)
3Star	(0.6,0.4,0.3)

$$(f(T_{PNSHSS})_1 = f(T_{PNSHSS})_2) = (f(Samsung, 3Star, Single\ door) = f(Samsung, 3Star))$$
$= \{< m^1, (\{Panasonic\}(0.5,0.3,0.1), \{3Star\}(0.6,0.4,0.3), \{Single\ door\}(0.6,0.8,0.3) >)$
$< m^3, (\{Panasonic\}(0.4,0.5,0.2), \{3Star\}(0.4,0.4,0.3), \{Single\ door\}(0.4,0.5,0.6) >)$
$= \{< m^1, (\{Panasonic\}(0.5,0.3,0.1), \{3Star\}(0.6,0.4,0.3) >)$

Definition 3.5.Complement of PNSHSS

Let $f(T_{PNSHSS})_1$ be PNSHSS in the universe Z. For k≥1, let $(b_1, b_2, b_3 \ldots\ldots b_k)_{PNSHS}$ be k-distinct attributes, each of whose associated attributive values is the set $(B_1, B_2, B_3 \ldots\ldots B_k)_{PNSHS}$ with $(B_m \cap B_n)_{PNSHSS} = \emptyset$ as well as m≠n and m, n ∈ $\{1,2, \ldots, k\}$. Let $G(B_1)_{PNSHS}, G(B_2)_{PNSHS}, G(B_3)_{PNSHS}, \ldots\ldots, G(B_k)_{PNSHS} = T$ be the power sets of the set $(B_1, B_2, B_3 \ldots\ldots B_k)_{PNSHS}$ respectively.

Then $f(T_{PNSHSS})_1$ is Compliment of PNSHSS of $f(T_1)_{PNSHSS}$ if

$T^c(f(T_{PNSHSS})_1) = F(f(T_{PNSHSS})_1),\ I^c(f(T_{PNSHSS})_1) = I(f(T_{PNSHSS})_1).F^c(f(T_{PNSHSS})_1) = T(f(T_{PNSHSS})_1)$ (8)

Example

Consider the PNSHSS $f(T_{PNSHSS})_1$ over the universe $Z = \{m^1, m^2, m^3, m^4, m^5\}$. Then PNSHSS

$f(T_1)_{PNSHSS} = F(panasonic, 3star, (Single\ door, Double\ door)) = \{m^1, m^4\}$ is $T^c(f(T_{PNSHSS})_1) = F(f(T_{PNSHSS})_1), I^c(f(T_{PNSHSS})_1) = I(f(T_{PNSHSS})_1), F^c(f(T_{PNSHSS})_1) = T(f(T_{PNSHSS})_1)$.

TABLE 7. PNSHSS $f(T_{PNSHSS})_1$

$f(T_{PNSHSS})_1$	m^1	m^4
Not Panasonic	(0.1,0.3,0.5)	(0.3,0.4,0.6)
Not 3Star	(0.4,0.6,0.3)	(0.2,0.4,0.8)
Not (Single door, Double door)	(0.3,0.6,0.6)	(0.1,0.2,0.8)

$$f^c((T_{PNSHSS})_1) = f(Not\ Panasonic, Not\ 3\ Star, Not(Single\ door, Double\ door))$$
$= \{$
$< m^1, (\{Not\ Panasonic\}(0.1,0.3,0.5), \{Not\ 3\ Star\}(0.4,0.6,0.3), \{(Not\ Single\ door, Double\ door)\}(0.3,0.6,0.6))$
$>$,
$< m^4, (\{Not\ Panasonic\}(0.3,0.4,0.6), \{Not\ 3\ Star\}(0.2,0.4,0.8), \{(Not\ Single\ door, Double\ door)\}(0.1,0.2,0.8))$
$>$,

In $f(T_{PNSHSS})_1$, we can observe that the membership values of Panasonic for m^1 are (0.5,0.3,0.1) and its complement is (0.1,0.3,0.5), which meet the requirements for PNSHSS complement. This indicates that the complement of (0.5,0.3,0.1) is (0.1,0.3,0.5). This also held true for the remaining terms.

Definition 3.6. PNSHSS for Union

Let $f(T_{PNSHSS})_1, f(T_{PNSHSS})_2$ be two PNSHSS in the universe Z. For k≥1, let $(b_1, b_2, b_3 \ldots\ldots b_k)_{PNSHS}$ be k-distinct attributes, each of whose associated attributive values is the set $(B_1, B_2, B_3 \ldots\ldots B_k)_{PNSHS}$ with $(B_m \cap B_n)_{PNSHSS} = \emptyset$ as well as m≠n and m,n $\in \{1,2,\ldots,k\}$. Let $G(B_1)_{PNSHS}, G(B_2)_{PNSHS}, G(B_3)_{PNSHS}, \ldots\ldots, G(B_k)_{PNSHS} = T$ be the power sets of the set $(B_1, B_2, B_3 \ldots\ldots B_k)_{PNSHS}$ respectively.

Then $f(T_{PNSHSS})_1 \cup f(T_{PNSHSS})_2$ is given as

$\cup (T(f(T_{PNSHSS})_1, f(T_{PNSHSS})_2)) = \{T(f(T_{PNSHSS})_1)$ $if\ x_{PNSHS} \in T_1$ $T(f(T_{PNSHSS})_2)$ $if\ x_{PNSHS} \in T_2$ $max(T(f(T_{PNSHSS})_1), T(f(T_{PNSHSS})_2))$ $if\ x_{PNSHS} \in \cap (T_1, T_2)$ (9)

$\cup (I(f(T_{PNSHSS})_1, f(T_{PNSHSS})_2)) = \{I(f(T_{PNSHSS})_1)$ $if\ x_{PNSHS} \in T_1$ $I(f(T_{PNSHSS})_2)$ $if\ x_{PNSHS} \in T_2$ $min(I(f(T_{PNSHSS})_1), I(f(T_{PNSHSS})_2))$ $if\ x_{PNSHS} \in \cap (T_1, T_2)$ (10)

$\cup (F(f(T_{PNSHSS})_1, f(T_{PNSHSS})_2)) = \{F(f(T_{PNSHSS})_1)$ $if\ x_{PNSHS} \in T_1$ $F(f(T_{PNSHSS})_2)$ $if\ x_{PNSHS} \in T_2$ $min(F(f(T_{PNSHSS})_1), F(f(T_{PNSHSS})_2))$ $if\ x_{PNSHS} \in \cap (T_1, T_2)$ (11)

Example

Let two PNSHSS $f(T_{PNSHSS})_1$, $f(T_{PNSHSS})_2$ in the same universe $Z = \{m^1, m^2, m^3, m^4, m^5\}$.

$f(T_{PNSHSS})_1 = f((Samsung, Haier), 3Star, Single\ door, 15Years) = \{m^2, m^5\}$

$f(T_{PNSHSS})_2 = f((Samsung, Haier), Single\ door, 15Years) = \{m^2\}$

TABLE 8. PNSHSS $f(T_{PNSHSS})_1$

$f(T_{PNSHSS})_1$	m^2	m^5
$(Samsung, Haier)$	(0.1,0.2,0.5)	(0.2,0.4,0.7)
3Star	(0.7,0.8,0.3)	(0.9,0.2,0.1)

Single door	(0.7,0.6,0.2)	(0.5,0.3,0.4)
15 Years	(0.4,0.8,0.6)	(0.8,0.4,0.2)

TABLE 9. PNSHSS $f(T_{PNSHSS})_2$

$f(T_{PNSHSS})_2$	m^2
$(Samsung, Haier)$	(0.4,0.3,0.2)
Single door	(0.3,0.4,0.6)
15 Years	(0.8,0.6,0.1)

TABLE 10. Union of PNSHSS $f(T_{PNSHSS})_1$ and $f(T_{PNSHSS})_2$

$f(T_{PNSHSS})_1 \cup f(T_{PNSHSS})_2)$	m^2	m^5
$(Samsung, Haier)$	(0.4,0.2,0.4)	(0.2,0.4,0.7)
3Star	(0.7,0.0,0.0)	(0.9,0.2,0.1)
Single door	(0.7,0.4,0.2)	(0.5,0.3,0.4)
15 Years	(0.8,0.6,0.1)	(0.8,0.4,0.2)

$$(f(T_{PNSHSS})_1 \cup f(T_{PNSHSS})_2) = F\big((Samsung, Haier), 3Star, Single\ door, 15Years\big) \cup F\big((Samsung, Haier), Single\ door, 15Years\big)$$

$= \{< m^2, (Samsung, Haier)\{0.4,0.2,0.4\}, 3Star\ \{0.7,0.0,0.0\}, Single\ door\{0.7,0.4,0.2\}, 15years\{(0.8,0.6,0.1)\} < m^5, (Samsung, Haier)\{0.2,0.4,0.7\}, 3Star\ \{0.9,0.2,0.1\}, Single\ door\{0.5,0.3,0.4\}, 15years\{(0.8,0.4,0.2)\}$

Definition 3.7. PNSHSS for Intersection

Let $f(T_{PNSHSS})_1$, $f(T_{PNSHSS})_2$ be two PNSHSS in the universe Z. For k≥1, let $(b_1, b_2, b_3 \ldots\ldots b_k)_{PNSHS}$ be k-distinct attributes, each of whose associated attributive values is the set $(B_1, B_2, B_3 \ldots\ldots B_k)_{PNSHS}$ with $(B_m \cap B_n)_{PNSHSS} = \emptyset$ as well as m≠n and m, n $\in \{1,2, \ldots, k\}$. $G(B_1)_{PNSHS}, G(B_2)_{PNSHS}, G(B_3)_{PNSHS}, \ldots\ldots, G(B_k)_{PNSHS} = T$ be the power sets of the set $(B_1, B_2, B_3 \ldots\ldots B_k)_{PNSHS}$ respectively.

Then $f(T_{PNSHSS})_1 \cap f(T_{PNSHSS})_2$ is

$\cap\,(T(f(T_{PNSHSS})_1,\ f(T_{PNSHSS})_2)) = \{T(f(T_{PNSHSS})_1)\quad if\ x_{PNSHS} \in T_1\ T(f(T_{PNSHSS})_2)\quad if\ x_{PNSHS} \in T_2\ min\big(T(f(T_{PNSHSS})_1), T(f(T_{PNSHSS})_2)\big)\ if\ x_{PNSHS} \in \cap\,(T_1, T_2)$ (12)

$\cap\,(I(f(T_{PNSHSS})_1,\ f(T_{PNSHSS})_2)) = \{I(f(T_{PNSHSS})_1)\quad if\ x_{PNSHS} \in T_1\ I(f(T_{PNSHSS})_2)\quad if\ x_{PNSHS} \in T_2\ max\big(I(f(T_{PNSHSS})_1), I(f(T_{PNSHSS})_2)\big)\ if\ x_{PNSHS} \in \cap\,(T_1, T_2)$ (13)

$\cap\,(F(f(T_{PNSHSS})_1,\ f(T_{PNSHSS})_2)) = \{F(f(T_{PNSHSS})_1)\quad if\ x_{PNSHS} \in T_1\ F(f(T_{PNSHSS})_2)\quad if\ x_{PNSHS} \in T_2\ max\big(F(f(T_{PNSHSS})_1), F(f(T_{PNSHSS})_2)\big)\ if\ x_{PNSHS} \in \cap\,(T_1, T_2)$ (14)

Example

Let $f(T_{PNSHSS})_1$ and $f(T_{PNSHSS})_2$ be two PNSHSS in the same universe Z.

$Z = \{m^1, m^2, m^3, m^4, m^5\}$.

$f(T_{PNSHSS})_1 = f\big((Samsung, Haier), 3Star, Single\ door, 15Years\big) = \{m^2, m^5\}$ and

$f(T_{PNSHSS})_2 = f\big((Samsung, Haier), Single\ door, 15Years\big) = \{m^2\}$ given below

TABLE 11. PNSHSS $f(T_{PNSHSS})_1$

$f(T_{PNSHSS})_1$	m^2	m^5
$(Samsung, Haier)$	(0.1,0.2,0.5)	(0.2,0.4,0.7)
3Star	(0.7,0.8,0.3)	(0.9,0.2,0.1)
Single door	(0.7,0.6,0.2)	(0.5,0.3,0.4)
15 Years	(0.4,0.8,0.6)	(0.8,0.4,0.2)

TABLE 12. PNSHSS $f(T_{PNSHSS})_2$

$f(T_{PNSHSS})_2$	m^2
$(Samsung, Haier)$	(0.4,0.3,0.2)
Single door	(0.3,0.4,0.6)
15 Years	(0.8,0.6,0.1)

TABLE 13. Intersection of PNSHSS $f(T_{PNSHSS})_1$ and $f(T_{PNSHSS})_2$

$(f(T_{PNSHSS})_1 \cap f(T_{PNSHSS})_2)$	m^2	m^5
$(Samsung, Haier)$	(0.1,0.3,0.5)	(0.2,0.4,0.7)
3Star	(0.0,0.8,0.3)	(0.9,0.2,0.1)
Single door	(0.3,0.6,0.6)	(0.5,0.3,0.4)
15 Years	(0.4,0.8,0.6)	(0.8,0.4,0.2)

$$f(T_{PNSHSS})_1 \cap f(T_{PNSHSS})_2 = f\big((Samsung, Haier), 3Star, Single\ door, 15Years\big) \cap f\big((Samsung, Haier), Single\ door, 15Years\big)$$

$= \{<$
$m^2, (Samsung, Haier)\{0.1,0.3,0.5\}, 3Star\ \{0.0,0.8,0.3\}, Single\ door\{0.3,0.6,0.6\}, 15years\{(0.4,0.8,0.6)\} <$
$m^5, (Samsung, Haier)\{0.2,0.4,0.7\}, 3Star\ \{0.9,0.2,0.1\}, Single\ door\{0.5,0.3,0.4\}, 15years\{(0.8,0.4,0.2)\}$

Definition 3.8. PNSHSS for AND

Let $f(T_{PNSHSS})_1$, $f(T_{PNSHSS})_2$ be two PNSHSS in the universe Z. For k≥1, let $(b_1, b_2, b_3 \ldots\ldots b_k\ )_{PNSHS}$ be k-distinct attributes, each of whose associated attributive values is the set $(B_1, B_2, B_3 \ldots\ldots B_k\ )_{PNSHS}$ with $(B_m \cap B_n)_{PNSHSS} = \emptyset$ as well as m≠n and m, n $\in \{1,2, \ldots, k\}$. $G(B_1)_{PNSHS}, G(B_2)_{PNSHS}, G(B_3)_{PNSHS}, \ldots\ldots, G(B_k)_{PNSHS} = T$ be the power sets of the set $(B_1, B_2, B_3 \ldots\ldots B_k\ )_{PNSHS}$ respectively.

Then $f(T_{PNSHSS})_1 \wedge f(T_{PNSHSS})_2 = f(T_1 \times T_2)_{PNSHSS}$ is

$\mathrm{T}(T_1 \times T_2)_{PNSHSS} = min(T(f(T_{PNSHSS})_1), T(f(T_{PNSHSS})_2))$. (15)

$\mathrm{I}(T_1 \times T_2)_{PNSHSS} = max(I(f(T_{PNSHSS})_1), I(f(T_{PNSHSS})_2))$ (16)

$\mathrm{F}(T_1 \times T_2)_{PNSHSS} = max(F(f(T_{PNSHSS})_1), F(f(T_{PNSHSS})_2))$
(17)

Example

Let the two PNSHSS $f(T_{PNSHSS})_1$, $f(T_{PNSHSS})_2$ over the same universe.

$Z = \{m^1, m^2, m^3, m^4, m^5\}$.

$f(T_{PNSHSS})_1 = f(Haier, (Single\ door, Double\ door), 10Years) = \{m^2, m^3\}$ and

$f(T_{PNSHSS})_2 = f(Haier, 10Years) = \{m^3\}$ given below

TABLE 14. PNSHSS $f(T_{PNSHSS})_1$

$f(T_{PNSHSS})_1$	m^2	m^3
Haier	(0.3,0.2,0.1)	(0.3,0.6,0.2)
(Single door, Double door)	(0.7,0.5,0.2)	(0.5,0.5,0.2)
10 Years	(0.6,0.5,0.3)	(0.5,0.4,0.3)

TABLE 15. PNSHSS $f(T_{PNSHSS})_2$

$f(T_{PNSHSS})_2$	m^3
Haier	(0.6,0.2,0.4)
10 Years	(0.7,0.6,0.2)

Then the AND operation of above PNSHSS

TABLE 16. AND of PNSHSS $f(T_{PNSHSS})_1$ *and* $f(T_{PNSHSS})_2$

$f(T_{PNSHSS})_1 \wedge f(T_{PNSHSS})_2$	m^2	m^3
(Haier)×(Haier)	(0.0,0.2,0.1)	(0.3,0.6,0.4)
(Haier)×(10years)	(0.0,0.2,0.1)	(0.3,0.6,0.2)
(Single door, Double door)×(Haier)	(0.0,0.5,0.2)	(0.5,0.5,0.4)
(Single door, Double door)×(10Years)	(0.0,0.5,0.2)	(0.5,0.6,0.2)
(10years)×(Haier)	(0.0,0.5,0.3)	(0.5,0.4,0.4)
(10years)×(10Years)	(0.0,0.5,0.3)	(0.5,0.6,0.3)

Definition 3.9. PNSHSS for OR

Let $f(T_{PNSHSS})_1$, $f(T_{PNSHSS})_2$ be two PNSHSS in the universe Z. For k≥1, let $(b_1, b_2, b_3 \ldots\ldots b_k)_{PNSHS}$ be k-distinct attributes, each of whose associated attributive values is the set $(B_1, B_2, B_3 \ldots\ldots B_k)_{PNSHS}$ with $(B_m \cap B_n)_{PNSHSS} = \emptyset$ as well as m≠n and m, n ∈ $\{1,2,\ldots,k\}$
$G(B_1)_{PNSHS}, G(B_2)_{PNSHS}, G(B_3)_{PNSHS}, \ldots\ldots, G(B_k)_{PNSHS} = T$ be the power sets of the set $(B_1, B_2, B_3 \ldots\ldots B_k)_{PNSHS}$ respectively.
Then $f(T_{PNSHSS})_1 \vee f(T_{PNSHSS})_2 = f(T_1 \times T_2)_{PNSHSS}$ is given as

$$\mathrm{T}(T_1 \times T_2)_{PNSHSS} = max(T(f(T_{PNSHSS})_1), T(f(T_{PNSHSS})_2)), \quad (18)$$

$$\mathrm{I}(T_1 \times T_2)_{PNSHSS} = min(I(f(T_{PNSHSS})_1), I(f(T_{PNSHSS})_2)) \quad (19)$$

$$\mathrm{F}(T_1 \times T_2)_{PNSHSS} = min(F(f(T_{PNSHSS})_1), F(f(T_{PNSHSS})_2)) \quad (20)$$

Example

Let two PNSHSS $f(T_{PNSHSS})_1$, $f(T_{PNSHSS})_2$ over the universe $Z = \{m^1, m^2, m^3, m^4, m^5\}$. Tabular representation of PNSHSS
$f(T_{PNSHSS})_1 = f(Haier, (Single\ door, Double\ door), 10Years) = \{m^2, m^3\}$ and
$f(T_{PNSHSS})_2 = f(Haier, 10Years) = \{m^3\}$

TABLE 17. Table representation of PNSHSS $f(T_{PNSHSS})_1$

$f(T_{PNSHSS})_1$	m^2	m^3
Haier	(0.3,0.2,0.1)	(0.3,0.6,0.2)
(Single door, Double door)	(0.7,0.5,0.2)	(0.5,0.5,0.2)

10 Years	(0.6,0.5,0.3)	(0.5,0.4,0.3)

TABLE 18. Table representation of PNSHSS $f(T_{PNSHSS})_2$

$f(T_{PNSHSS})_2$	m^3
Haier	(0.6,0.2,0.4)
10 Years	(0.7,0.6,0.2)

TABLE 19. OR of PNSHSS $f(T_{PNSHSS})_1$ and $f(T_{PNSHSS})_2$

$f(T_{PNSHSS})_1 \wedge f(T_{PNSHSS})_2$	m^2	m^3
(Haier)×(Haier)	(0.3,0.0,0.0)	(0.6,0.2,0.2)
(Haier)×(10years)	(0.3,0.0,0.0)	(0.7,0.6,0.2)
(Single door, Double door)×(Haier)	(0.7,0.0,0.0)	(0.6,0.2,0.2)
(Single door, Double door)×(10Years)	(0.7,0.0,0.0)	(0.7,0.5,0.2)
(10years)×(Haier)	(0.6,0.0,0.0)	(0.6,0.2,0.3)
(10years)×(10Years)	(0.6,0.0,0.0)	(0.7,0.4,0.2)

4. CONCLUSION

This paper presents the union, intersection, AND, and OR operations of the PNSHSS. By providing a relevant example, the suggested operations and definitions' applicability and execution are confirmed. The Pythagorean Neutrosophic Super Hyper Soft Set, or PNSHSS, is a novel tool for appropriate selection in decision-making situations.

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