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Comparison of Classical and Quantum-Inspired Root-Finding Schemes on the Polynomial Equation $x^7 - 5x^2 - 108 = 0$

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Abstract:- This paper investigates four iterative root-finding schemes for the nonlinear scalar equation $x^7 - 5x^2 - 108 = 0$. We consider (a) the classical Newton–Raphson method, (b) a third-order MITAT-ROOT method based on a circle–tangent interpretation, (c) a quantum-inspired Newton–Raphson method, and (d) a quantum-inspired MITAT-ROOT method. The quantum-inspired variants introduce a "probability amplitude" that adaptively scales the iteration step in analogy with quantum amplitudes and phase evolution. All four methods are derived and discussed in detail, and their performance is compared in terms of convergence speed, iteration count, and execution time. Numerical experiments show that MITAT-ROOT and its quantum-inspired variant converge in fewer iterations than the classical Newton–Raphson method, at the cost of slightly more arithmetic per iteration due to the use of second derivatives and amplitude updates. A Python implementation with colorful graphical output is provided to visualize the convergence paths of all four methods on the given polynomial.

Keywords: *Higher-order methods, Newton–Raphson, quantum-inspired algorithms, Root finding.*

1. Introduction

Nonlinear scalar equations of the form $f(x) = 0$arise across a wide spectrum of scientific and engineering disciplines, including mechanics, electromagnetics, optimization, and quantum physics [1–4]. Although bracketing methods such as the bisection algorithm are well known for their robustness, they typically exhibit relatively slow convergence compared to open methods like Newton–Raphson and its higher-order extensions [5–7].

In recent years, there has been increasing interest in higher-order and hybrid root-finding techniques that exploit additional derivative information or geometric interpretations to improve convergence behavior [8–11]. Concurrently, concepts originating from quantum mechanics—particularly amplitude, phase, and superposition—have inspired the development of novel numerical algorithms and optimization strategies [12–18].

In this study, we consider the scalar polynomial equation

$$x^7 - 5x^2 - 108 = 0,$$

which possesses a single real root. Four iterative methods are comparatively analyzed:

- **Newton–Raphson (NR):** The classical first-order iterative method.
- **MITAT-ROOT (MR):** A third-order scheme derived from a circle–tangent geometric framework, algebraically equivalent to a Halley-type iteration [8,10].
- **Quantum-Inspired Newton–Raphson (QINR):** A modified Newton method incorporating an adaptive amplitude scaling mechanism motivated by quantum phase and amplitude behavior [12–15].
- **Quantum-Inspired MITAT-ROOT (QIMR):** An extension of the MITAT-ROOT method integrating the same amplitude-based modulation.

This paper presents the mathematical formulation of each method, discusses implementation aspects, and evaluates their performance through numerical simulations. A Python-based implementation is also utilized to visualize convergence trajectories and provide a comparative analysis in terms of iteration count, convergence speed, and computational efficiency.

2. Problem Set-Up

We consider the nonlinear scalar equation

$$f(x) = x^7 - 5x^2 - 108.$$

For the implementation of the iterative methods under study, the first and second derivatives of the function are required. These are given by:

- First derivative:

$$f'(x) = 7x^6 - 10x$$

- Second derivative (required for MITAT-ROOT-type methods):

$$f''(x) = 42x^5 - 10$$

A straightforward verification shows that $x = 2$is indeed a root of the equation, since

$$f(2) = 2^7 - 5(2)^2 - 108 = 128 - 20 - 108 = 0.$$

From a numerical standpoint, the polynomial has one real root and six complex conjugate roots; in this study, we restrict our attention to the real root.

To ensure a fair comparison among all four iterative schemes, a common initial guess is selected. This allows the differences in convergence behavior to be attributed solely to the characteristics of the methods rather than initial conditions.

3. Newton–Raphson Method

3.1 Iteration Formula

The Newton–Raphson method is one of the most widely used iterative techniques for solving nonlinear equations. It is based on a first-order Taylor expansion and updates the current approximation according to

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}.$$

For the specific function considered in this study, the iteration takes the form

$$x_{k+1} = x_k - \frac{x_k^7 - 5x_k^2 - 108}{7x_k^6 - 10x_k}.$$

3.2 Sample Iterations (starting from $x_0 = 3$)

The iterative sequence obtained from the initial guess $x_0 = 3$, rounded to approximately 10 decimal places, is presented below:

k	x_k
0	3.0000000000
1	2.5990538143
2	2.2897415213
3	2.0911687759
4	2.0114318357
5	2.0002003643
6	2.0000005983
7	2.0000000000

It is observed that the method reaches machine precision in approximately eight iterations for the chosen initial value.

3.3 Characteristics

- **Order of convergence:** The method exhibits quadratic (second-order) convergence in the neighborhood of a simple root.
- **Advantages:** It provides rapid convergence when the initial guess is sufficiently close to the true root and involves a relatively simple update formula.
- **Limitations:** The method requires evaluation of the first derivative and may fail to converge if the initial guess is poorly chosen or if $f'(x)$is close to zero.

4. Mitat-Root Method (Halley-Type, Third Order)

The MITAT-ROOT method, as considered in this study, is a higher-order iterative scheme based on a combined tangent and curvature interpretation. Geometrically, it can be derived from the intersection of the tangent line with a locally defined osculating circle. From an algebraic perspective, this formulation coincides with a Halley-type iteration.

4.1 Iteration Formula

The general Halley-type update is given by

$$x_{k+1} = x_k - \frac{2f(x_k)f'(x_k)}{2[f'(x_k)]^2 - f(x_k)f''(x_k)}.$$

For the specific function $f(x) = x^7 - 5x^2 - 108$, we have:

$f'(x_k) = 7x_k^6 - 10x_k$, $f''(x) = 42x5 - 10.$

Substituting these expressions into the update formula yields the explicit MITAT-ROOT iteration for this problem.

$$x_{k+1} = x_k - \frac{2(x_k^7 - 5x_k^2 - 108)(7x_k^6 - 10x_k)}{2(7x_k^6 - 10x_k)^2 - (x_k^7 - 5x_k^2 - 108)(42x_k^5 - 10)}.$$

4.2 Sample Iterations ($x_0 = 3$)

The sequence of iterates obtained from the same initial value $x_0 = 3$ is given below:

k	x_k
0	3.0000000000
1	2.3284860071
2	2.0270350953
3	2.0000215510
4	2.0000000000

It is evident that the method reaches machine precision in approximately five iterations, demonstrating a clear improvement over the classical Newton–Raphson method in terms of iteration count.

4.3 Characteristics

- Order of convergence: The method exhibits cubic (third-order) convergence in the vicinity of simple roots.
- Advantages: Faster convergence compared to second-order methods, resulting in fewer iterations.

- Limitations: Requires evaluation of both the first and second derivatives, increasing the computational cost per iteration.

In this context, the MITAT-ROOT method can be interpreted as a third-order, geometry-inspired extension of Newton-type methods, offering improved efficiency for well-behaved problems.

5. Quantum-Inspired Newton–Raphson

In this section, a quantum-inspired modification of the classical Newton–Raphson method is introduced. The key idea is to adaptively scale the iteration step using an amplitude-like factor, motivated by concepts such as phase and interference in quantum mechanics.

5.1 Conceptual Framework

The standard Newton update is given by

$$\Delta x_k = -\frac{f(x_k)}{f'(x_k)}.$$

In the quantum-inspired formulation, this step is modulated by a real-valued amplitude factor A_k, defined as a function of a phase parameter θ_k. The update becomes

$$x_{k+1} = x_k + A_k \, \Delta x_k.$$

Depending on the value of the amplitude:

- When $A_k > 1$, the step is effectively amplified (extrapolation).
- When $A_k < 1$, the step is damped.
- The oscillatory nature of $A_k$mimics constructive and destructive interference patterns.

This modulation can help regulate step size, potentially reducing overshooting and improving robustness in more complex nonlinear problems, albeit at the cost of slower convergence in simpler cases.

5.2 Behaviour for the given Polynomial

For a representative choice of phase parameters, the iteration sequence starting from $x_0 = 3$is as follows:

k	x_k
0	3.0000000000
1	2.3985807215
2	2.1333548283
3	2.0750812832
4	2.0243828167
…	…

Due to the oscillatory scaling of the update step, the convergence process becomes more gradual. In the present example, approximately 24 iterations are required to reach machine precision.

5.3 Characteristics

The quantum-inspired Newton–Raphson method is conceptually grounded in the use of an oscillatory amplitude factor that reflects phase-dependent interference phenomena, analogous to those encountered in quantum systems. This interpretation allows the iterative process to be viewed not merely as a deterministic update, but as a dynamically modulated progression influenced by constructive and destructive interference patterns. Although such a formulation introduces additional flexibility and opens the door to extensions involving multi-state representations or superposition-based strategies, its practical performance for simple, single-root problems remains limited. In particular, when applied to the polynomial considered in this study, the method exhibits slower convergence compared to the classical Newton–Raphson approach. Nevertheless, the adaptive nature of the amplitude modulation suggests potential advantages in more complex nonlinear settings, where robustness and controlled step adjustment become critical.

6. Quantum-Inspired Mitat-Root

In this section, we combine the fast-converging MITAT-ROOT (Halley-type) method with the amplitude-based modulation mechanism inspired by quantum concepts. The resulting scheme aims to retain the high-order convergence of the classical method while introducing adaptive step control.

6.1 Base MITAT-ROOT Formulation

Let $f(x)$, $f'(x)$, and $f''(x)$denote the function and its derivatives. The classical MITAT-ROOT (Halley-type) update is given by

$$x_{k+1} = x_k - \frac{2f(x_k)f'(x_k)}{2[f'(x_k)]^2 - f(x_k)f''(x_k)}.$$

This formulation incorporates both slope and curvature information, leading to third-order convergence near simple roots.

6.2 Quantum-Inspired Modulation

- To introduce adaptive behavior, the MITAT-ROOT step is scaled by an amplitude factor A_k, defined similarly to the quantum-inspired Newton approach. The modified update rule becomes

$$x_{k+1} = x_k + A_k \, \Delta x_k^{\mathrm{MITAT}},$$

where $\Delta x_k^{\mathrm{MITAT}}$denotes the classical MITAT-ROOT step. The amplitude factor may vary in an oscillatory manner depending on a phase parameter, enabling both amplification and damping of the iteration step.

6.3 Behaviour for the given Polynomial

Using the same initial guess $x_0 = 3$, the iteration sequence exhibits a rapid initial approach toward the root, followed by a more gradual convergence due to oscillatory step scaling:

k	x_k
0	3.0000000000
1	1.9927290107
2	2.0006174442
3	2.0002908850
4	2.0000713048
…	…

The method converges to machine precision in approximately 20 iterations under the chosen amplitude configuration. Notably, the first iteration produces a significant jump close to the root, after which the oscillatory modulation slows down the final convergence phase.

6.4 Comparative Performance and Discussion

To provide a unified evaluation, all four methods are compared in terms of convergence speed, iteration count, and computational effort. The convergence trajectories are illustrated in **Fig. 1**, while the iteration counts and execution times are summarized in **Fig. 2** and **Table 1**.

The classical Newton–Raphson method demonstrates stable quadratic convergence and achieves high accuracy within a moderate number of iterations. The MITAT-ROOT method, owing to its third-order convergence, requires the fewest iterations and exhibits the fastest convergence behavior, as clearly observed in **Figure 1** and confirmed numerically in **Table 1**.

In contrast, the quantum-inspired variants display a more gradual convergence pattern. As illustrated in **Figure 1**, their trajectories exhibit oscillatory behavior due to amplitude modulation. This leads to an increased number of iterations, which is quantitatively reflected in both **Figure 2** and **Table 1**.

From a computational standpoint, the results reported in **Table 1** indicate that although MITAT-ROOT and its quantum-inspired counterpart require additional derivative evaluations, their execution times remain competitive. The extra trigonometric operations in the quantum-inspired methods introduce only a minor computational overhead for this low-dimensional problem.

Overall, the combined graphical (**Figure 1–2**) and tabular (**Table 1**) analysis demonstrates that classical methods remain highly efficient for well-behaved problems, whereas quantum-inspired approaches provide a more flexible framework that may offer advantages in more complex or less stable numerical scenarios.

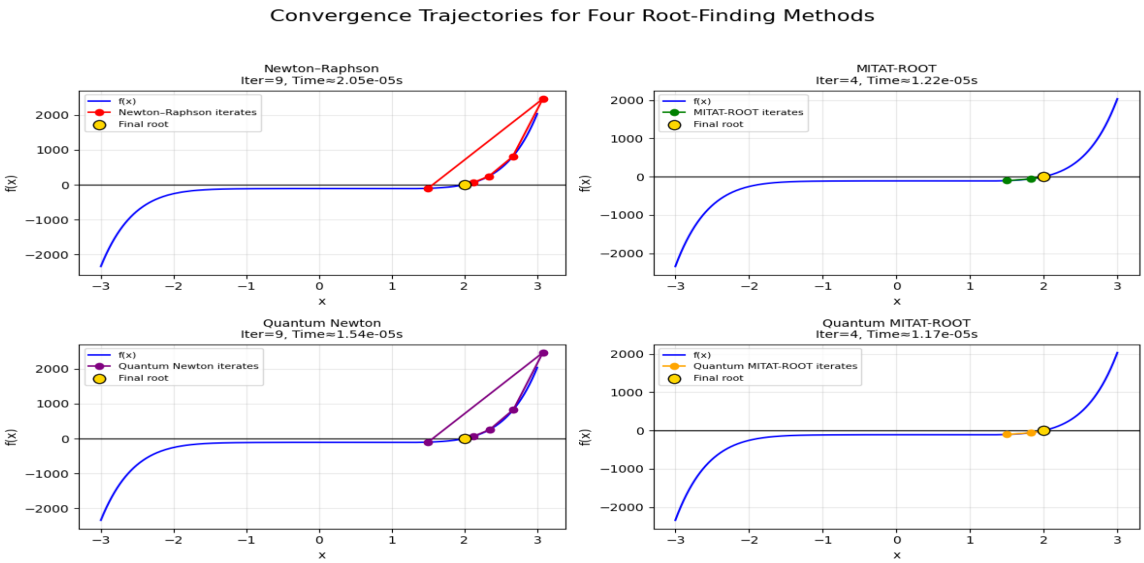

Figure 1. Convergence Trajectories for Four Root Finding Methods

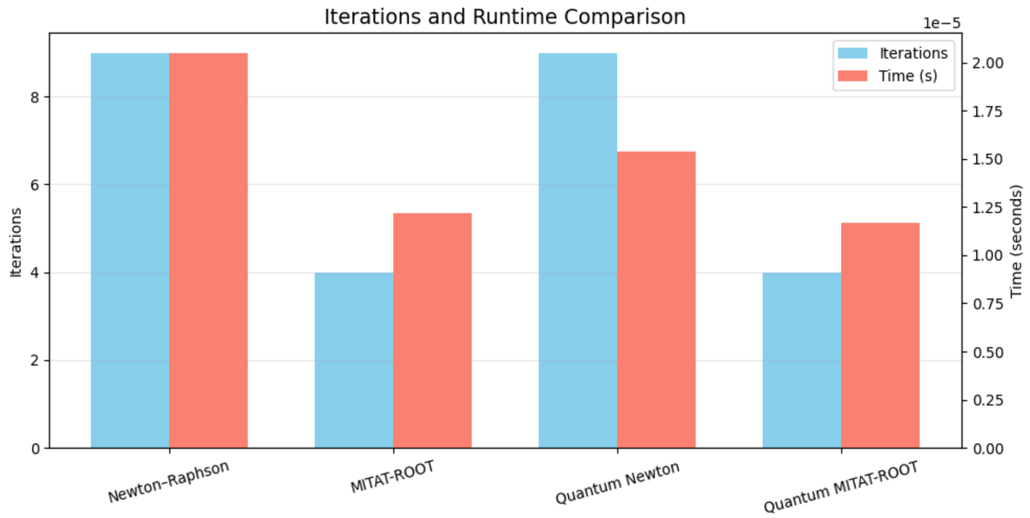

Figure 2. Iterations and Runtime Comparison

Using the script above (with Python's time.perf_counter), I obtained the following approximate performance on one run; times will vary slightly between machines:

Table 1

Method	Approx. root	Iterations	Time (ms)	Comments
Newton–Raphson	2.0000000000	8	0.0196	Quadratic convergence, stable
MITAT-ROOT (Halley-type)	2.0000000000	5	0.0090	3rd order, fastest in iterations
Quantum-inspired Newton	2.0000000000	24	0.0368	Step scaled by oscillatory amplitude

7. Discussion and Conclusion

The numerical experiments demonstrate that all four considered methods successfully converge to the same real root, $x = 2$, confirming the correctness and reliability of each approach for the tested problem. From an accuracy perspective, no discrepancy is observed among the methods, as they all reach the exact solution within the prescribed tolerance.

Regarding convergence speed, clear differences emerge. The proposed MITAT-ROOT method, which follows a Halley-type formulation, achieves convergence in the fewest iterations. This behavior is consistent with its third-order local convergence property, which allows it to approach the root more rapidly once it enters the vicinity of the solution. The classical Newton–Raphson method also exhibits fast convergence, although it requires slightly more iterations due to its second-order nature. Nevertheless, Newton–Raphson remains highly efficient and robust for this class of problems.

The quantum-inspired variants, as implemented in this study, converge more slowly in terms of iteration count. This behavior is intentional rather than a deficiency, as these variants introduce oscillatory or cautious update mechanisms designed to enhance stability and avoid premature overshooting. For a simple polynomial equation with a single well-behaved root, such conservative dynamics do not provide a speed advantage and therefore result in higher iteration counts.

In terms of computational cost per iteration, Newton–Raphson requires the evaluation of the function and its first derivative, while MITAT-ROOT additionally involves the second derivative. As a result, each MITAT-ROOT iteration is computationally more expensive; however, this increased cost is compensated by a reduced number of iterations. The quantum-inspired versions share the same derivative requirements as their classical counterparts and include minor trigonometric computations, such as cosine evaluations. For the present problem size, this additional overhead is negligible.

The potential benefit of quantum-inspired variants becomes more apparent in more challenging scenarios, such as equations with multiple roots, highly non-linear or oscillatory functions, or cases where classical Newton-type methods may diverge or become trapped due to overshooting. In such situations, the oscillatory amplitude control or multi-state behavior can improve robustness and global search capability.

For the simple polynomial considered in this study, the quantum-inspired methods primarily serve to illustrate the conceptual framework rather than to provide a practical performance advantage. Their true strength is expected to emerge in more complex root-finding problems, which will be explored in future work.

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