Tuijin Jishu/Journal of Propulsion Technology
ISSN: 1001-4055
Vol. 47 No. 02 (2026)

Vertex-Magic Labeling And Maximum, Minimum Constant Of Cyclic And Path Graphs

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Abstract:- In this article, we discussed about the magic labeling of Vertex Set for cyclic and path graphs. Also, we find the maximum and minimum vertex magic constants for Magic labeling of vertex Set.

Keywords: *Magic Labeling of Vertex Set, Minimum-Vertex-Magic Constant Maximum Vertex-Magic Constant.*

1. Introduction

In this paper, $G = (p, q)$ graph with p the points set and q the line set of the graph, connected and undirected graph only. The number of points denoted by $|V(G)|$ and number of lines will be denoted by $|E(G)|$. Also, we discuss about the Magic labeling of Vertex Set of some graphs. Labeling concepts are available on J.A Gallian.

The Labeling of a Graph was first introduced by A. Rosa. The magic Labeling concept was introduced by Sedlacek. $G = (p, q)$ be undirected and simple graph then the magic labeling of Vertex Set is a bijection f from $V(G) \cup E(G)$ to $\{1,2, \dots, p + q\}$ with the property that for every vertex u in G, $f(u) + \sum_{v \in N(u)} f(uv) = k$ for some constant K.We studied about the vertex magic constant value, if the vertex and edge set of the graph $G = (p, q)$ are labeled as follows points set are labeled with 1 to p and edge set are labeled with $p + 1$ to $p + q$ then we get the magic constant Δ which is maximum. The value is $\Delta = \frac{p+4q}{2} + \frac{q(q+1)}{p} + \frac{1}{2}$, Also the vertex set are labeled with the numbers $\{q + 1, \ \dots, \ p + q - 1, \ p + q\}$ and edges are labeled with the number $\{1,2, \dots, q\}$ that the case we get a magic constant δ which is minimum the minimum constant $\delta = \frac{q}{p}(p + q + 1) + \frac{p+1}{2}$. When graph is cyclic then $\Delta = \frac{7P}{2} + \frac{3}{2}$ and $\delta = \frac{5p+3}{2}$, also we discuss the path graph with n vertices P_n. That the case $\delta = \frac{5p}{2} + 2$ and also for definitions about connected, connected graph and path one can see [5] which is useful to prove our ideas.

2. Main Results

This paper, discuss the vertex-magic constant S and it's maximum and minimum value. It is applied to some cyclic graph and path P_n of n vertices.They are called vertex-magic label graphs.

Definition 2.1 (Vertex-Magic Graph)

A graph $G = (p, q)$ is labeled by numbers 1 through $p + q$ with every vertex and its incident edges adds up to the same number S. then the graph G is called Vertex-Magic Graph. The same number S is called Vertex-Magic constant.

2.1 Find the Value of Vertex-Magic Constant

Let G be a (p,q) graph and f be a magic labeling of Vertex of G . Let $V(G) = \{v_1, v_2, v_3, \dots, v_p\}$.

A symmetric matrix $A = (a_{ij}), \forall\, i,j$ varies from 1 to p that is called a label adjacency matrix of G.

If $a_{ij} = f(v_i, v_j)$ by the definition of magic constant.

$$S = f(v_1) + a_{12} + a_{13} + \cdots + a_{1p}$$

$$S = f(v_2) + a_{21} + a_{23} + \cdots + a_{2p}$$

$$S = f(v_3) + a_{31} + a_{32} + \cdots + a_{3p}$$

$S = f(v_p) + a_{p1} + a_{p2} + \cdots + a_{(p-1)p}$.

The sum of all entries in row i together with $f(v_i)$ will be equal to the magic number k and $i = 1, \dots, p$ then, we have

$$pS = f(v_1) + f(v_2) + f(v_3) \dots + f(v_p) + 2[a_{12} + a_{13} + \cdots + a_{(p-1)p}]$$

where $f(v_1), f(v_2), \dots, f(v_p)$ are vertex labeled of the vertices $\{v_1, v_2, \dots, v_p\}$and $a_{12}, a_{13}, \dots a_{(p-1)p}$ are edge labeled.

Case – I

Suppose first we labeled the numbers $(1,2, \dots, q)$ are for edges and after we labeled the numbers $(q+1, q+2, q+3, \dots, p+q-1, p+q)$ are in vertices then we get a magic constant S. That the case we get the value of the magic-constant δ is minimum.

Case – II

Suppose first we labeled the numbers $(1,2, \dots, p)$ are for vertices and after we labeled the numbers $(p+1, p+2, p+3, \dots, p+q-1, p+q)$ are in edges then we get a magic constant S. That the case we get the value of the magic-constant Δ is maximum.

2.2 The Maximum Value of the Vertex Magic Constant

If $G = (p,q)$ be a graph. The vertices of G are labeled with number $(1,2, \dots, p)$ and also the edges of the graph are labeled with the numbers $(p+1, p+2, p+3, \dots, p+q-1, p+q)$ then we get Δ is maximum value.

The magic constant is

$$p\Delta = (1 + 2 + 3 + \cdots + p) + 2(p + 1 + p + 2 + p + 3 + \cdots p + q - 1 + p + q)$$

$$\Delta = \frac{(p+1)}{2} + 2q + \left(\frac{q(q+1)}{p}\right).$$

The magic constant Δ is maximum.

2.3 The Minimum Value of the Vertex Magic Constant

Suppose the graph $G = (p,q)$ be connected. The vertices of G are labeled with numbers $(q+1, q+2, q+3, \dots, p+q-1, p+q)$ and also the edges of the graph are labeled with the numbers $(1,2, \dots, q)$ then we get the magic constant δ is minimum value

The magic constant is

$$p\delta = (q + 1 + q + 2 + q + 3 + \cdots p + q - 1 + p + q) + 2(1 + 2 + 3 + \cdots + q)$$

$$\delta = q + \frac{(p+1)}{2} + \left(\frac{q(q+1)}{p}\right).$$

The magic constant δ is minimum.

Theorem 2.2 If $G = (p, q)$ be a graph. Also which is cycle graph then the Maximum Vertex-Magic constant $\Delta = \frac{7p}{2} + \frac{3}{2}$.

Proof. Given $G = (p, q)$ be cycle graph. We know that the definition of Maximum Vertex-Magic constant

$$\Delta = \frac{p+1}{2} + 2q + \frac{q(q+1)}{p}.$$

If G is cyclic then $p = q$, we get

$$\Delta = \frac{7p}{2} + \frac{3}{2}.$$

Note: The value of the Maximum Vertex Magic constant is integer only when p is odd

Corollary 2.3 If G is cyclic graph with p-points and also p-is even then the Maximum Vertex-Magic constant $\Delta = \frac{7p}{2} + 1$.

Proof. Given G is a cyclic with p-points then the maximum vertex Magic constant

$$\Delta = 3p + 1 + \frac{p+1}{2}.$$

In the case of p is even, $p + 1$ does not divide by 2. Add and subtract $p/2$ to the vertex and edge sum, we get

$$p\Delta = \frac{p(p+1)}{2} + 2\left[pq + \frac{q(q+1)}{2}\right]$$

when G is cyclic graph

$$\Delta = \frac{7p}{2} + 1.$$

Example 2.4 Consider the cyclic graph $G = (3,3)$. The vertices are labeled with the number $\{1,2,3\}$ and also the edges are labeled with numbers $\{4,5,6\}$. We get the Maximum Vertex-Magic constant

$$\Delta = \frac{7p}{2} + \frac{3}{2} = \frac{7(3)}{2} + \frac{3}{2} = 12$$

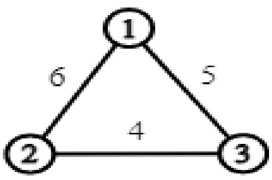

Figure 2.3.1

Example 2.5 Consider the cyclic graph $G = (5,5)$. The vertices are labeled with the number $\{1,2,3,4,5\}$ and also the edges are labeled with numbers $\{6,7,8,9,10\}$, We get the Maximum Vertex-Magic constant

$$\Delta = \frac{7p}{2} + \frac{3}{2} = \frac{7(5)}{2} + \frac{3}{2} = 19.$$

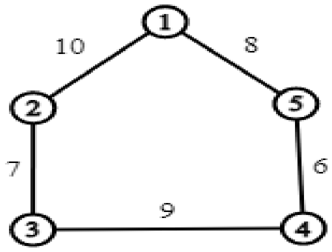

Figure 2.3.2

2.4 The General Method for Maximum Magic Labeling of Vertex Setof Cyclic Graph

Consider the cyclic graph with p-points. We follow the producer of labeling method for vertices and edges with numbers $\{1,2,\ldots,p+q\}$

Vertex Label	Edge Label
$2i-1$	$e_{2i-1}=2p-i+1\ (1\le i<p)$
	$e_{2i-2}=\frac{3p}{2}+\frac{3}{2}-i$
	$e_p=e_0$

i.e. If $i=1,2,\ldots$, we get the vertex label, $v_1,v_2,\ldots,v_p$.

Also the edge set $e_i(i=1,2,3\ldots q-1,q)$are labeled as follows

$$e_{2i-1}=2p+1-i,$$

$$e_{2i-2}=\frac{3p}{2}+\frac{3}{2}-i$$

Consider the graph $G=(7,7)$

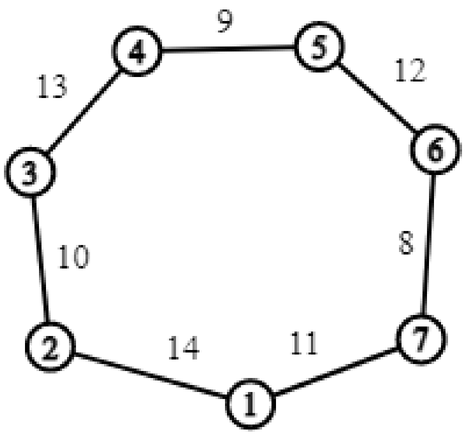

Figure 2.4.1

Even Vertex Label

Consider the walk of the sequence of the points and edges of the graph G is as follows $v_1e_1v_2e_2v_3e_3v_4e_4v_5e_5v_6e_6v_7e_7v_1$

i	**Vertex Label** $2i$	**Edge Label**

		$e_{2i-1} = 2p - i + 1, e_{2i} = \frac{3p}{2} + \frac{3}{2} - i$
1	2	$e_1 = 14,\ e_2 = 10$
2	4	$e_3 = 13, e_4 = 9$
3	6	$e_5 = 12, e_6 = 8$
4	8	$e_7 = 11,$

Odd Vertex Label

Consider the walk of the sequence of the points and edges of the graph G is as follows $v_1 e_1 v_2 e_2 v_3 e_3 v_4 e_4 v_5 e_5 v_6 e_6 v_7 e_7 v_1$

i	**Vertex Label** $(2i - 1)$	**Edge Label** $e_{2i-1} = 2p - i + 1, e_{2i-2} = \frac{3p}{2} + \frac{3}{2} - i$
1	1	$e_1 = 14,\ e_0 = 11$
2	3	$e_3 = 13, e_2 = 10$
3	5	$e_5 = 12, e_4 = 9$
4	7	$e_7 = 11, e_6 = 8$

Example 2.6 If G is a cyclic with p-point also p is even then the Maximum Vertex-Magic constant Δ is $\frac{7p}{2} + 1$.

Proof. Consider the graph$G = (4,4)$, then $S = 15$. Where p is even

$$V_{sum} = 1 + 2 + 3 + 4 = 10$$

$$E_{sum} = 5 + 6 + 7 + 8 = 26$$

Therefore, add $\frac{p}{2}$ to V_{sum} and subtract $\frac{p}{2}$ to E_{sum}

$V_{sum} + \frac{p}{2} = 12$ and $E_{sum} - \frac{p}{2} = 24$.

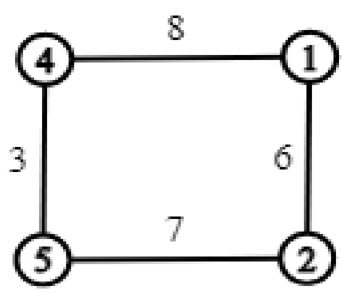

Figure 2.4.2

So, the vertices are labeled with the numbers {1,2,4,5} and the edges are labeled with the number {3,6,7,8}.

Example 2.7 If G is a cyclic with p-point also p is even then the Maximum Vertex-Magic constant Δ is $\frac{7p}{2}+1$.

Proof. Consider the graph $G=(6,6)$, then $\Delta = 22$. Where p is even

$$V_{sum} = 1+2+3+4+5+6 = 21$$

$$E_{sum} = 7+8+9+10+11 = 45$$

Therefore, add $\frac{p}{2}$ to V_{sum} and subtract $\frac{p}{2}$ to E_{sum}

$V_{sum}+\frac{p}{2} = 24$ and $E_{sum}-\frac{p}{2} = 42$.

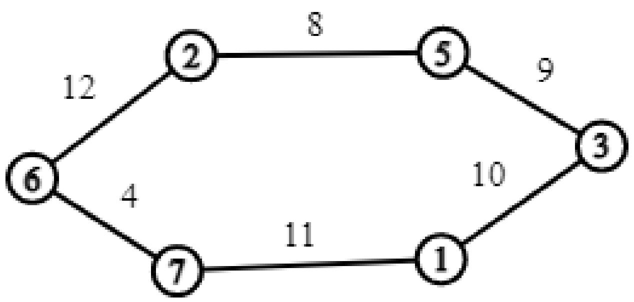

Figure 2.4.3

So, the vertices are labeled with the numbers {1,2,3,5,6,7}and the edges are labeled with the numbers{4,8,9,10,11,12}.

Theorem 2.8 Let $G=(p,q)$ is a cyclic graph with p-points. Then the Minimum Vertex Magic constant $\delta = \frac{5p+3}{2}$.

Proof. We know that the minimum vertex magic constant

$p\delta = pq + \frac{q(q+1)}{2} + p(p+1)$. Then we get the minimum Vertex Magic-constant $\delta = \frac{5p+3}{2}$.

Note: The minimum constant δ exist only when p is odd

Example 2.9 Let $G=(3,3)$. So, $\delta = \frac{5(3)+3}{2}$ which gives $\delta = 9$.

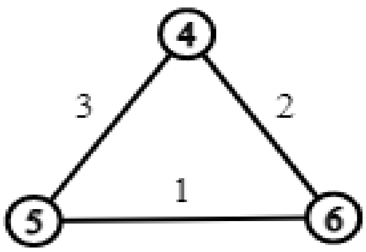

Figure 2.4.4

Example 2.10 Consider the cyclic graph $G=(5,5)$. So, $\delta = \frac{5(5)+3}{2}$ which gives $\delta = 14$.

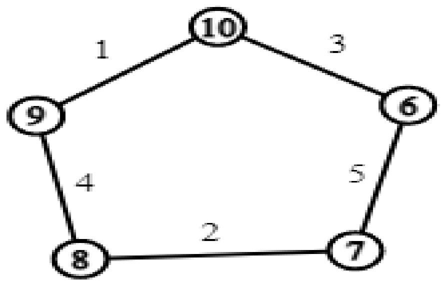

Figure 2.4.5

Theorem 2.11 The minimum magic constant δ of the cyclic graph with p-points also p is even is $\delta = \frac{5p}{2} + 2$.

Proof. We know that the Minimum Vertex-Magic Constant

$$p\delta = pq + \frac{q(q+1)}{2} + 2\left[\frac{p(p+1)}{2}\right].$$

Add and subtract $p/2$ to edge and vertex sum, we get

$$p\delta = -\frac{p}{2} + pq + \frac{q(q+1)}{2} + 2\left[\frac{p(p+1)}{2} + \frac{p}{2}\right].$$

When G is cyclic, we have $p = q$ we get

$$\delta = \frac{5p}{2} + 2\,.$$

Therefore, the Minimum Vertex-Magic constant$\delta = \frac{5p}{2} + 2$, when p is even.

Example 2.12 Consider the graph$G = (4,4)$. So, $\delta = \frac{5(4)}{2} + 2$ which gives $\delta = 12$.

Theorem 2.13 Consider the path p_n with n-vertices then the Minimum Vertex-Magic constant $\delta = \frac{5p-3}{2}$.

Proof. If $G$is a cyclic graph with p- point then the minimum vertex magic constant and G is a path p_n, then $q = p - 1$

$$\delta = p - 1 + \frac{p+1}{2} + \frac{(p-1)(p)}{p} = \frac{5p-3}{2}.$$

Note: The Minimum Vertex-Magic constant exists only p is odd.

Example 2.14 Consider the path p_3, we have $\delta = \frac{5(3)-3}{2}$ which gives $\delta = 6$.

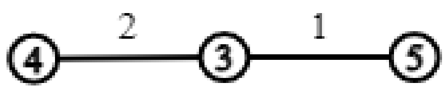

Figure 2.4.6

Example 2.15 Consider the path p_5, we have $\delta = \frac{5(5)-3}{2} = 11$.

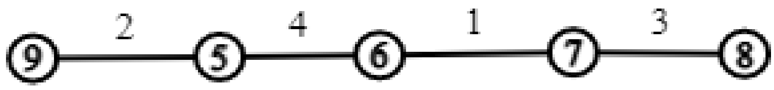

Figure 2.4.7

2.5 General Method of Vertex Minimum Labeling For The Path p_N

VERTEX LABEL	EDGE LABEL
$f(v_1) = p + q$	$e_{2i-1} = \frac{q}{2} - i + 1$
$f(v_{p+1-i}) = p + q - i$	$e_{2i} = q + 1 - i$

Here v_1 and $f(v_1)$ are the vertex and vertex label of the path.

Example 2.13 Consider the path p_7, we have $\delta = \frac{5(7)-3}{2}$ which gives $\delta = 16$.

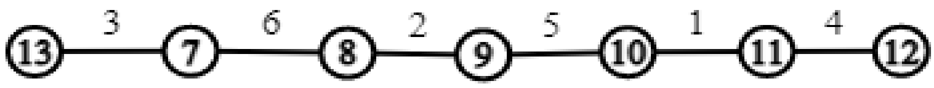

Figure 2.4.8

Here,

Vertex Label

$f(v_1) = 13;$

$$f(v_{7-1+1}) = f(v_7) = 12.$$

Similarly,

$$f(v_6) = 11, f(v_5) = 10, f(v_4) = 9, f(v_3) = 8, f(v_2) = 7.$$

Edge Label

i	$e_{2i-1} = \frac{p+1}{2} - i$	$e_{2i} = p - i$
1	3	6
2	2	5
3	1	4

3. Conclusion

This article discusses the magic number for the graph G. There are also several types of magic number, some of which are V-Magic and E-Magic. We discuss about Magic Labeling of the Vertex set on some graphs namely cyclic and path. The Vertex - Magic constant value is many kind when it is maximum, minimum are discussed this paper. We briefly deliberated about the maximum Vertex-Magic constant of same graph differ when the number of vertices of the cyclic graph p is odd and p is even.

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