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An Introduction to δ-open sets in a Pentapartitioned Neutrosophic Topological Spaces

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Abstract:- In this work , we present a stronger form of δ – open sets and δ – closed sets in pentapartitioned neutrosophic topological spaces. We examine some of their fundamantal characteristics and provide examples in pentapartitioned neutrosiphic topological spaces.

Keywords:.$\mathcal{PN}\delta - int$, $\mathcal{PN}\delta - cl$, $\mathcal{PN}\delta$ – *open sets*, $\mathcal{PN}\delta$ – *closed sets.*

1. Introduction

Zadeh (1965) was the first person in the field of logic and set theory to introduce the idea of a fuzzy set between real standard intervals in mathematics. Chang (1968) took the general topology as a fuzzy topological space and framed it with a fuzzy set. Lewine. K (1970), define Generalized closed sets in topology. Atanassov (1983) introduced intuitionistic fuzzy sets, which combine membership and non-membership values. Abdel- Monsef, El-Deep and Mohmoud (1983) introduced β-Open sets and β -Continuous mappings. Coker (1997) developed the intuitionistic fuzzy topological space. Chatterjee, Majumdar and Samanta (2016) contributed significantly to this development by introducing similarity measures and entropy for quadripartitioned single-valued neutrosophic sets. Arokiarani, Dhavaseelan, Jafari and Parimala (2017) introduced a variety of new notions and functions in neutrosophic topological spaces. Smarandache (2012) introduced the idea of neutrosophic set at the start of the 20th century. Salama and Alblowi (2012), neutrosophic set in a neutrosophic Topological spaces. δ-open sets in a fuzzy topological spaces defined by Saha. Vadivel, Seenivasan and John Sundar (2021) introduce δ -open sets in a Neutrosophic topological spaces.Navuluri Mohanarao and Sathish kumar(2025),proposed a new operator based on β -Open sets within quadri-partitioned neutrosophic topological spaces. Sai Anu and Arulselvam (2025) proposed on intuitionistic quadri-partitioned neutrosophic N-soft set model. In the context of Pentapartitioned neutrosophic topological spaces, we expand on the notion of δ -open sets and δ -closed sets in this study. we also provide examples of some of their fundamental characteristics.

2. Preliminaries

Definition 2.1 Assume $\mathfrak{Y}$ is a non – empty set. An object with the form $\mathfrak{L} = \{\mathfrak{y}, \dot{T}_{\mathfrak{L}}(\mathfrak{y}), \dot{C}_{\mathfrak{L}}(\mathfrak{y}), \dot{I}_{\mathfrak{L}}(\mathfrak{y}), \dot{U}_{\mathfrak{L}}(\mathfrak{y}), \dot{F}_{\mathfrak{L}}(\mathfrak{y}) : \mathfrak{y}\epsilon\mathfrak{Y}\}$where$\dot{T}_{\mathfrak{L}}(\mathfrak{y}) \to [0,1]\ Truth, \dot{C}_{\mathfrak{L}}(\mathfrak{y}) \to [0,1],\ contradiction, \dot{I}_{\mathfrak{L}}(\mathfrak{y}) \to [0,1]\ , ignorance, \dot{U}_{\mathfrak{L}}(\mathfrak{y}) \to [0,1], unknown, \dot{F}_{\mathfrak{L}}(\mathfrak{y}) \to [0,1]\ , falsity$
and $0 \leq \dot{T}_{\mathfrak{L}}(\mathfrak{y}) + \dot{C}_{\mathfrak{L}}(\mathfrak{y}) + \dot{I}_{\mathfrak{L}}(\mathfrak{y}) + \dot{U}_{\mathfrak{L}}(\mathfrak{y}) + \dot{F}_{\mathfrak{L}}(\mathfrak{y}) \leq 5$ for each $\mathfrak{y}\epsilon\ \mathfrak{Y}$.

Definition 2.2 Assume $\mathfrak{Y}$ is a non – empty set and the pentapartitioned neutrosophic sets $\mathfrak{L}$ *and* $\mathfrak{M}$ in the form$\mathfrak{L} = \{\mathfrak{y}, \dot{T}_{\mathfrak{L}}(\mathfrak{y}), \dot{C}_{\mathfrak{L}}(\mathfrak{y}), \dot{I}_{\mathfrak{L}}(\mathfrak{y}), \dot{U}_{\mathfrak{L}}(\mathfrak{y}), \dot{F}_{\mathfrak{L}}(\mathfrak{y}) : \mathfrak{y}\epsilon\mathfrak{Y}\}$, $\mathfrak{M} = \{\mathfrak{y}, \dot{T}_{\mathfrak{M}}(\mathfrak{y}), \dot{C}_{\mathfrak{M}}(\mathfrak{y}), \dot{I}_{\mathfrak{M}}(\mathfrak{y}), \dot{U}_{\mathfrak{M}}(\mathfrak{y}), \dot{F}_{\mathfrak{M}}(\mathfrak{y}) : \mathfrak{y}\epsilon\mathfrak{Y}\}$, then

(a) $\mathbf{0}_{\mathcal{PN}} = \langle \mathfrak{y}, 0, 0, 1, 1, 1 \rangle$ **and** $\mathbf{1}_{\mathcal{PN}} = \langle \mathfrak{y}, 1, 1, 0, 0, 0 \rangle$ **,**

(b) $\mathfrak{L} \subseteq \mathfrak{M}$ if $\dot{T}_{\mathfrak{L}}(\mathfrak{y}) \leq \dot{T}_{\mathfrak{M}}(\mathfrak{y}), \dot{C}_{\mathfrak{L}}(\mathfrak{y}) \leq \dot{C}_{\mathfrak{M}}(\mathfrak{y}), \dot{I}_{\mathfrak{L}}(\mathfrak{y}) \geq \dot{I}_{\mathfrak{M}}(\mathfrak{y}), \dot{U}_{\mathfrak{L}}(\mathfrak{y}) \geq \dot{U}_{\mathfrak{M}}(\mathfrak{y}), \dot{F}_{\mathfrak{L}}(\mathfrak{y}) \geq \dot{F}_{\mathfrak{M}}(\mathfrak{y}) : \mathfrak{y}\epsilon\mathfrak{Y}$,

(c) $\mathfrak{L} = \mathfrak{M}$ iff $\mathfrak{L} \subseteq \mathfrak{M}$ and $\mathfrak{M} \subseteq \mathfrak{L}$,

(d) $\mathfrak{L}^C = \{\mathfrak{y}, \dot{F}_{\mathfrak{L}}(\mathfrak{y})\ , \dot{U}_{\mathfrak{L}}(\mathfrak{y}), 1 - \dot{I}_{\mathfrak{L}}(\mathfrak{y}), \dot{C}_{\mathfrak{L}}(\mathfrak{y}), \dot{T}_{\mathfrak{L}}(\mathfrak{y}) : \mathfrak{y}\epsilon\mathfrak{Y}\}$,

(e) $\mathfrak{L} \cup \mathfrak{M} =$
$\{\langle \mathfrak{y}, max(\dot{T}_{\mathfrak{L}}(\mathfrak{y}), \dot{T}_{\mathfrak{M}}(\mathfrak{y})), max(\dot{C}_{\mathfrak{L}}(\mathfrak{y}), \dot{C}_{\mathfrak{M}}(\mathfrak{y})), min(\dot{I}_{\mathfrak{L}}(\mathfrak{y}), \dot{I}_{\mathfrak{M}}(\mathfrak{y})), min(U_{\mathfrak{L}}(\mathfrak{y}), \dot{U}_{\mathfrak{M}}(\mathfrak{y})), min(\dot{F}_{\mathfrak{L}}(\mathfrak{y}), \dot{F}_{\mathfrak{M}}(\mathfrak{y}) : \mathfrak{y} \epsilon \mathfrak{Y}\rangle\}$
(f) $\mathfrak{L} \cap \mathfrak{M} =$
$\{\langle \mathfrak{y}, min(\dot{T}_{\mathfrak{L}}(\mathfrak{y}), \dot{T}_{\mathfrak{M}}(\mathfrak{y})), min(\dot{C}_{\mathfrak{L}}(\mathfrak{y}), \dot{C}_{\mathfrak{M}}(\mathfrak{y})), max(\dot{I}_{\mathfrak{L}}(\mathfrak{y}), \dot{I}_{\mathfrak{M}}(\mathfrak{y})), max(U_{\mathfrak{L}}(\mathfrak{y}), \dot{U}_{\mathfrak{M}}(\mathfrak{y})), max(\dot{F}_{\mathfrak{L}}(\mathfrak{y}), \dot{F}_{\mathfrak{M}}(\mathfrak{y}) : \mathfrak{y} \epsilon \mathfrak{Y}\rangle\}$

Definition 2.3 A non-empty set possessing a pentapartitioned neutrosophic structure, there are pentapartitioned subsets of $\mathfrak{Y}$ that satisfy the family $\varrho_{\mathcal{PN}}$.

(a) $\mathbf{0}_{\mathcal{PN}}, \mathbf{1}_{\mathcal{PN}} \in \varrho_{\mathcal{PN}}$.

(b) $\mathfrak{L}_1 \cap \mathfrak{L}_2 \in \varrho_{\mathcal{PN}}$, for any $\mathfrak{L}_1, \mathfrak{L}_2 \in \varrho_{\mathcal{PN}}$.

(c) $\cup \mathfrak{L}_a \in \varrho_{\mathcal{PN}}, \forall \mathfrak{L}_a : a \in \mathcal{A} \subseteq \varrho_{\mathcal{PN}}$.

Then $(\mathfrak{Y}, \varrho_{\mathcal{PN}})$ is called pentapartitioned neutrosophic topological space in $\mathfrak{Y}$. In $\mathfrak{Y}$, the φ elements are referred as pentapartitioned neutrosophicopen sets. If the complement $\mathcal{C}^c$ of a $\mathcal{PN}_{CS}$ is $\mathcal{PN}_{OS}$, it is reffered as a pentapartitioned neutrosophic closed set.

Definition 2.4 If $\mathfrak{L}$ is a pentapartitioned neutrosophic set on $\mathfrak{Y}$and $(\mathfrak{Y}, \varrho_{\mathcal{PN}})$ is a pentapartitioned neutrosophic topological space on $\mathfrak{Y}$, then the pentapartitioned neutrosophic closure of $\mathfrak{L}$ and its pentapartitioned neutrosophic interior are defined as

$$\mathcal{PN}\ int(\mathfrak{L}) = \cup \{\mathfrak{J} : \mathfrak{J} \subseteq \mathfrak{L}\ and\ \mathfrak{J}\ is\ a\ \mathcal{PN}_{OS}\ in\ \mathfrak{Y}\}$$

$$\mathcal{PN}\ cl(\mathfrak{L}) = \cap \{\mathfrak{J} : \mathfrak{L} \subseteq \mathfrak{J}\ and\ \mathfrak{J}\ is\ a\ \mathcal{PN}_{CS}\ in\ \mathfrak{Y}\}.$$

Definition 2.5 If $\mathfrak{L}$ is a pentapartitioned neutrosophic set on $\mathfrak{Y}$and $(\mathfrak{Y}, \varrho_{\mathcal{PN}})$ is a pentapartitioned neutrosophic topological space on $\mathfrak{Y}$. Consequently, $\mathfrak{L}$ is regarding as a pentapartitioned neutrosophic regular(resp. pre, semi,α and β) open set (briefly, $\mathcal{PN}_{ros}(resp. \mathcal{PN}\mathbb{P}_{OS}, \mathcal{PN}\mathbb{S}_{OS}, and\ \mathcal{PN}\beta_{OS})$) if
$\mathfrak{L} = \mathcal{PN}int(\mathcal{PN}cl(\mathfrak{L})(resp. \mathfrak{L} \subseteq \mathcal{PN}int(\mathcal{PN}cl(\mathfrak{L})), \mathfrak{L} \subseteq \mathcal{PN}cl(\mathcal{PN}int(\mathfrak{L})), \mathfrak{L} \subseteq \mathcal{PN}cl(\mathfrak{L}))$
$(resp. \mathfrak{L} \subseteq \mathcal{PN}int(\mathcal{PN}cl(\mathfrak{L})), \mathfrak{L} \subseteq \mathcal{PN}cl(\mathcal{PN}int(\mathfrak{L})), \mathfrak{L} \subseteq \mathcal{PN}int(\mathcal{PN}cl(\mathcal{PN}cl(\mathfrak{L})))$ and $\mathfrak{L} \subseteq \mathcal{PN}cl(\mathcal{PN}int(\mathcal{PN}cl(\mathfrak{L})))$.

The complement of an $\mathcal{PN}\mathbb{P}_{OS}$ (resp,$\mathcal{PN}\mathbb{S}_{OS}, \mathcal{PN}\alpha_{OS}\ and\ \mathcal{PN}\beta_{OS}$) is called a pentapartitioned neutrosophic pre(resp. semi, $\alpha, regular\ and\ \beta$) closed set (brief, $\mathcal{PN}_{OS}(resp. \mathcal{PN}\mathbb{S}_{CS}, \mathcal{PN}\alpha_{CS}\ and\ \mathcal{PN}\beta_{CS})$)in $\mathfrak{Y}$.

The family of all $\mathcal{PN}\mathbb{P}_{OS}$(resp, $\mathcal{PN}\mathbb{P}_{CS}, \mathcal{PN}\mathbb{S}_{OS}, \mathcal{PN}\mathbb{S}_{CS}, \mathcal{PN}\alpha_{OS}, \mathcal{PN}\alpha_{CS}, \mathcal{PN}\beta_{OS}\ and\ \mathcal{PN}\beta_{CS}$)$of\ \mathfrak{Y}\ is\ denoted\ by\ \mathcal{PN}\mathbb{P}_{OS}(\mathfrak{Y})$(resp.
$\mathcal{PN}\mathbb{P}_{CS}(\mathfrak{Y}), \mathcal{PN}\mathbb{S}_{OS}(\mathfrak{Y}), \mathcal{PN}\mathbb{S}_{CS}(\mathfrak{Y}), \mathcal{PN}\alpha_{OS}(\mathfrak{Y}), \mathcal{PN}\alpha_{CS}(\mathfrak{Y}), \mathcal{PN}\beta_{OS}(\mathfrak{Y}). \mathcal{PN}\beta_{CS}(\mathfrak{Y})$).

3. $\delta - open\ sets\ in\ \mathcal{PN}ts$

In the above sections, let $(\mathfrak{Y}, \varrho_{\mathcal{PN}})$ be any $\mathcal{PN}ts$. Let $\mathfrak{J}, \mathfrak{K}, \mathfrak{L}, \mathfrak{M}\ and\ \mathfrak{N}$ be an $\mathcal{PN}s'S$ in $\mathcal{PN}ts$.

Definition 3.1 A set $\mathfrak{L}$ is said to be a pentapartitioned neutrosophic

(a) δ interior of $\mathfrak{J}(short, \mathcal{PN}\delta$ int($\mathfrak{J}$)) is defined by $\mathcal{PN}\delta$ int($\mathfrak{J}$)) = $\cup\{\mathcal{I} : \mathcal{I} \subseteq \mathfrak{J}\ and\ \mathfrak{J}\ is\ a\ \mathcal{PN}_{ros}\ in\ \mathfrak{Y}\}$.

(b) δ closure of $\mathfrak{J}(short, \mathcal{PN}\delta$ cl($\mathfrak{J}$)) is defined by $\mathcal{PN}\delta$ cl($\mathfrak{J}$)) = $\cap\{\mathcal{B} : \mathfrak{J} \subseteq \mathcal{B}\ and\ \mathcal{B}\ is\ a\ \mathcal{PN}_{ros}\ in\ \mathfrak{Y}\}$.

Definition 3.2 A set $\mathfrak{J}$ is said to be a pentapartitioned neutrosophic

(a) $\delta - open\ set(breifly, \mathcal{PN}\delta_{OS})\ if\ \ \mathfrak{J} = \mathcal{PN}\delta$ int($\mathfrak{J}$).

(b) $\delta - \ pre\ open\ set(breifly, \mathcal{PN}\delta\mathbb{P}_{OS})\ if\ \ \mathfrak{J} \subseteq \mathcal{PN}$int($\mathcal{PN}\delta$cl($\mathfrak{J}$)).

(c) $\delta - \ semi\ open\ set(breifly, \mathcal{PN}\delta\mathbb{S}_{OS})\ if\ \ \mathfrak{J} \subseteq \mathcal{PN}$cl($\mathcal{PN}\delta$int($\mathfrak{J}$)).

(d) $\delta\alpha - \ open\ (breifly, \mathcal{PN}\delta\alpha_{OS})\ if\ \ \mathfrak{J} \subseteq \mathcal{PN}$int($\mathcal{PN}$cl($\mathcal{PN}\delta int(\mathfrak{J})$)).

(e) $\delta\beta - \ open\ (breifly, \mathcal{PN}\beta\alpha_{OS})\ if\ \ \mathfrak{J} \subseteq \mathcal{PN}$cl($\mathcal{PN}int(\mathcal{PN}\delta cl(\mathfrak{J})$)).

The complement of an $\mathcal{PN}\delta_{OS}$, $\mathcal{PN}\delta\mathbb{P}_{OS}$, $\mathcal{PN}\delta\mathbb{S}_{OS}$, $\mathcal{PN}\delta\alpha_{OS}$ *and* $\mathcal{PN}\delta\beta_{OS}$ is called a $\mathcal{PN}\delta_{CS}$, $\mathcal{PN}\delta\mathbb{P}_{CS}$, $\mathcal{PN}\delta\mathbb{S}_{CS}$, $\mathcal{PN}\delta\alpha_{CS}$ *and* $\mathcal{PN}\delta\beta_{CS}$ in $\mathfrak{Y}$ are represents the family of all $\mathcal{PN}\delta_{OS}$, $\mathcal{PN}\delta_{CS}$, $\mathcal{PN}\delta\mathbb{P}_{OS}$, $\mathcal{PN}\delta\mathbb{P}_{CS}$, $\mathcal{PN}\delta\mathbb{S}_{OS}$, $\mathcal{PN}\delta\mathbb{S}_{CS}$, $\mathcal{PN}\delta\alpha_{OS}$, $\mathcal{PN}\delta\alpha_{CS}$, $\mathcal{PN}\delta\beta_{OS}$ *and* $\mathcal{PN}\delta\beta_{CS}$ *of* $\mathfrak{Y}$.

Definition 3.3 A set $\mathfrak{J}$ is said to be a pentapartitioned neutrosophic $\delta - pre(resp. \mathcal{PN}\delta - semi, \mathcal{PN}\delta - \alpha \text{ and } \mathcal{PN}\delta - \beta)$ interior of $\mathfrak{J}$(briefly, $\mathcal{PN}\delta\mathbb{P}int(\mathfrak{Y})$), $(resp.\ \mathcal{PN}\delta\mathbb{S}int(\mathfrak{Y}), \mathcal{PN}\delta int\alpha(\mathfrak{Y}), \mathcal{PN}\delta\beta int(\mathfrak{Y}))$ is the union of all $\mathcal{PN}\delta\mathbb{P}_{OS}$ $(resp. \mathcal{PN}\delta\mathbb{S}_{OS}, \mathcal{PN}\delta\alpha_{OS}$ *and* $\mathcal{PN}\delta\beta_{OS}$ contained in $\mathfrak{J}$.

Definition 3.4 A set $\mathfrak{J}$ is said to be a pentapartitioned neutrosophic $\delta - pre(resp. \mathcal{PN}\delta - semi, \mathcal{PN}\delta - \alpha \text{ and } \mathcal{PN}\delta - \beta)$ closure of $\mathfrak{J}$(briefly, $\mathcal{PN}\delta\mathbb{P}cl(\mathfrak{Y})$), $(resp.\ \mathcal{PN}\delta\mathbb{S}cl(\mathfrak{Y}), \mathcal{PN}\delta cl\alpha(\mathfrak{Y}), \mathcal{PN}\delta\beta cl(\mathfrak{Y}))$ is the intersection of all $\mathcal{PN}\delta\mathbb{P}_{CS}$ $(resp. \mathcal{PN}\delta\mathbb{S}_{CS}, \mathcal{PN}\delta\alpha_{CS}$ *and* $\mathcal{PN}\delta\beta_{CS}$ contained in $\mathfrak{J}$.

Proposition 3.5 The pentapartitioned neutrosphic δ −interior operator satisfies

(a) $\mathcal{PN}\delta int(\mathfrak{K}) \subseteq \mathfrak{K}$.
(b) $\mathfrak{K} \subseteq \mathfrak{M} \Longrightarrow \mathcal{PN}\delta int(\mathfrak{K}) \subseteq \mathcal{PN}\delta int(\mathfrak{M})$.
(c) $\mathcal{PN}\delta int(\mathfrak{K} \cap \mathfrak{M}) = \mathcal{PN}\delta int(\mathfrak{K}) \cap \mathcal{PN}\delta int(\mathfrak{M})$.
(d) $\mathcal{PN}\delta int(\mathfrak{K})$ is the greatest $\mathcal{PN}\delta_{OS}$ containing $\mathfrak{K}$.
(e) $\mathcal{PN}\delta int(\mathfrak{K}) = \mathfrak{K}$ if and only if $\mathfrak{K}$ is an $\mathcal{PN}\delta_{OS}$.
(f) $\mathcal{PN}\delta int(\mathcal{PN}\delta int(\mathfrak{K})) = \mathcal{PN}\delta int(\mathfrak{K})$.
(g) $(\mathfrak{Y} - \mathcal{PN}\delta int(\mathfrak{K})) = \mathcal{PN}\delta cl(\mathfrak{Y} - \mathfrak{K})$.

Proof:

(a) $\mathcal{PN}\delta int(\mathfrak{K}) = \cup\{\mathfrak{C}: \mathfrak{C} \subseteq \mathfrak{K} \text{ and } \mathfrak{C} \text{ is a } \mathcal{PN}_{ros}\}. Thus, \mathcal{PN}\delta int(\mathfrak{K}) \subseteq \mathfrak{K}$.
(b) $\mathcal{PN}\delta int(\mathfrak{M}) = \cup\{\mathfrak{C}: \mathfrak{C} \subseteq \mathfrak{M} \text{ and } \mathfrak{C} \text{ is a } \mathcal{PN}_{ros}\} \supseteq \cup\{\mathfrak{C}: \mathfrak{C} \subseteq \mathfrak{K} \text{ and } \mathfrak{C} \text{ is a } \mathcal{PN}_{ros}\} \supseteq \mathcal{PN}\delta int(\mathfrak{K}). Thus\ \mathcal{PN}\delta int(\mathfrak{K}) \subseteq \mathcal{PN}\delta int(\mathfrak{M})$.
(c) $\mathcal{PN}\delta int(\mathfrak{K} \cap \mathfrak{M}) = \cup\{\mathfrak{C}: \mathfrak{C} \subseteq \mathfrak{K} \cap \mathfrak{M} \text{ and } \mathfrak{C} \text{ is a } \mathcal{PN}_{ros}\} = (\cup\{\mathfrak{C}: \mathfrak{C} \subseteq \mathfrak{K}) \text{ and } \mathfrak{C} \text{ is a } \mathcal{PN}_{ros}\} \cap (\cup \mathfrak{C}: \mathfrak{C} \subseteq \mathfrak{M} \text{ and} \mathfrak{C} \text{ is a } \mathcal{PN}_{ros}\}) = \mathcal{PN}\delta int(\mathfrak{K}) \cap \mathcal{PN}\delta int(\mathfrak{K} \cap \mathfrak{M}) = \mathcal{PN}\delta int(\mathfrak{M})$.
(d) If $\mathfrak{A}$ *is an* $\mathcal{PN}\delta_{OS}$ in $\mathfrak{K}$, then $\mathfrak{A} \subseteq \mathcal{PN}\delta int(\mathfrak{K})$.Hance , $\mathcal{PN}\delta int(\mathfrak{K})$ *is the biggest* $\mathcal{PN}_{OS}$ *containing* $\mathfrak{K}$.
(e) Assume $\mathfrak{K}$ *represents any* $\mathcal{PN}\delta_{OS}$ *of* $\mathfrak{Y}$. The biggest $\mathcal{PN}\delta_{OS}$ found in $\mathfrak{K}$ *is itself*. *Thus* $\mathcal{PN}\delta int(\mathfrak{K}) = \mathfrak{K}$.
(f) According to (d), itself is the largest $\mathcal{PN}\delta_{OS}$ *that contains* $\mathcal{PN}\delta int(\mathfrak{K})$.Consequently, $\mathcal{PN}\delta int(\mathcal{PN}\delta int(\mathfrak{K})) = \mathcal{PN}\delta int(\mathfrak{K})$.
(g) The maximum number of $\mathcal{PN}\delta_{OS}$ that contain $\mathfrak{K}$ is a $\mathcal{PN}\delta int(\mathfrak{K})$. The smallest $\mathcal{PN}_{CS}$ in $\mathfrak{Y} - \mathfrak{K}$ *is the complement*. *Consequently*, $\mathcal{PN}\delta cl\ (\mathfrak{Y} - \mathfrak{K}) = (\mathfrak{Y} - \mathcal{PN}\delta int(\mathfrak{K}))$.

Proposition 3.6 The pentapartitioned neutrosophic δ − closure operator satisfies

(a) $\mathfrak{N} \subseteq \mathcal{PN}\delta cl(\mathfrak{N})$.
(b) $\mathfrak{K} \subseteq \mathfrak{M} \Longrightarrow \mathcal{PN}\delta cl(\mathfrak{K}) \subseteq \mathcal{PN}\delta int(\mathfrak{M})$.
(c) $\mathcal{PN}\delta cl(\mathfrak{K}) \cup \mathcal{PN}\delta int(\mathfrak{M})$.

(d) $\mathcal{PN}\delta cl(\mathfrak{K})$ is the smallest $\mathcal{PN}\delta_{CS} \subseteq \mathfrak{K}$.

(e) $\mathcal{PN}\delta cl(\mathfrak{K}) = \mathfrak{K}$ iff $\mathfrak{K}$ is an $\mathcal{PN}\delta_{CS}$.

(f) $\mathcal{PN}\delta cl(\mathcal{PN}\delta cl(\mathfrak{K})) = \mathcal{PN}\delta cl(\mathfrak{K})$.

(g) $(\mathfrak{J} - \mathcal{PN}\delta\ )\text{int}(\mathfrak{J} - \mathfrak{K}) = \mathcal{PN}\delta int(\mathfrak{J} - \mathfrak{K})$.

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(h) $\jmath \in \mathcal{PN}\delta cl(\mathfrak{K})$ iff $\mathfrak{K} \cap \mathfrak{C} \neq \emptyset$ for every $\mathcal{PN}\delta_{OS}$ $\mathfrak{C}$ containing j.

Proof. (h) Suppose $\jmath \in \mathcal{PN}\delta cl(\mathfrak{K})$. Let $\mathfrak{C}$ be a $\mathcal{PN}\delta_{OS}$ containing $\mathfrak{N}$. If $\mathfrak{K} \cap \mathfrak{C} = \emptyset$, then $\mathfrak{N} - \mathfrak{C}$ is a $\mathcal{PN}\delta_{CS}$ containing $\mathfrak{K}$ and so $\mathfrak{N} \notin \mathcal{PN}\delta cl(\mathfrak{K})$, a contradiction . Therefore, $\mathfrak{K} \cap \mathfrak{C} = \emptyset$. If $\jmath \notin \mathcal{PN}\delta cl(\mathfrak{K})$, then $\exists$ a $\mathcal{PN}\delta_{CS}$ $\mathfrak{C}^C$ containing $\mathfrak{K}\mathfrak{N} \notin \mathfrak{C}^C$. Then $\mathfrak{C} = \mathfrak{N} - \mathfrak{C}^C$ is a $\mathcal{PN}\delta_{OS}$ containing $\mathfrak{N}$ such that $\mathfrak{K} \cap \mathfrak{C} = \emptyset$, a contradiction. Therefore, $\mathfrak{N} \in \mathcal{PN}\delta cl(\mathfrak{K})$. The other cases are follows from proposition 3.5.

Proposition 3.7 The proposition 3.5 and 3.6 are also true for

Proposition 3.8 $\mathcal{PN}ts$ are covered by the assertions $\mathcal{PN}\delta\mathbb{P}_{OS}$, $\mathcal{PN}\delta\mathbb{S}_{OS}$,$\mathcal{PN}\delta\alpha_{OS}$,$\mathcal{PN}\delta\beta_{OS}$ of their respective interior and closure operators.

(a) All $\mathcal{PN}\delta_{OS}$ (resp. $\mathcal{PN}\delta_{CS}$) are $\mathcal{PN}_{OS}$ $(resp. \mathcal{PN}_{CS})$.
(b) All $\mathcal{PN}\delta_{OS}$ (resp. $\mathcal{PN}\delta_{CS}$) are $\mathcal{PN}\delta\mathbb{S}_{OS}$ $(resp. \mathcal{PN}\delta\mathbb{S}_{CS})$.
(c) All $\mathcal{PN}\delta_{OS}$ (resp. $\mathcal{PN}\delta_{CS}$) are $\mathcal{PN}\delta\mathbb{P}_{OS}$ $(resp. \mathcal{PN}\delta\mathbb{P}_{CS})$.
(d) All $\mathcal{PN}\delta\mathbb{S}_{OS}$ (resp. $\mathcal{PN}\delta\mathbb{S}_{CS}$) are $\mathcal{PN}\delta\beta_{OS}$ $(resp. \mathcal{PN}\delta\beta_{CS})$.
(e) All $\mathcal{PN}\delta\mathbb{P}_{OS}$ (resp. $\mathcal{PN}\delta\mathbb{P}_{CS}$) are $\mathcal{PN}\delta\beta_{OS}$ $(resp. \mathcal{PN}\delta\beta_{CS})$.
(f) All $\mathcal{PN}\delta\alpha_{OS}$ (resp. $\mathcal{PN}\delta\alpha_{CS}$) are $\mathcal{PN}\delta\mathbb{S}_{OS}$ $(resp. \mathcal{PN}\delta\mathbb{S}_{CS})$.
(g) All $\mathcal{PN}\delta\alpha_{OS}$ (resp. $\mathcal{PN}\delta\alpha_{CS}$) are $\mathcal{PN}\delta\mathbb{P}_{OS}$ $(resp. \mathcal{PN}\delta\mathbb{P}_{CS})$.

But not converse.

Proof.

(a) $\overline{\mathfrak{K}}$ is a $\mathcal{PN}\delta_{OS}$, then $\overline{\mathfrak{K}} = \mathcal{PN}\delta int(\overline{\mathfrak{K}}) \subseteq \mathcal{PN}int(\overline{\mathfrak{K}})$.∴ $\overline{\mathfrak{K}}$ is a $\mathcal{PN}_{OS}$.
(b) $\overline{\mathfrak{K}}$ is a $\mathcal{PN}\delta_{OS}$, then $\overline{\mathfrak{K}} \subseteq \mathcal{PN}\delta int(\overline{\mathfrak{K}})$. So $\overline{\mathfrak{K}} \subseteq \mathcal{PN}\delta int(\overline{\mathfrak{K}}) \subseteq \mathcal{PN}cl((\mathcal{PN}\delta int(\overline{\mathfrak{K}}))$. ∴ $\overline{\mathfrak{K}}$ is a $\mathcal{PN}\delta\mathbb{S}_{OS}$.
(c) $\overline{\mathfrak{K}}$ is a $\mathcal{PN}\delta_{OS}$, then $\overline{\mathfrak{K}} \subseteq \mathcal{PN}\delta int(\overline{\mathfrak{K}})$. So $\overline{\mathfrak{K}} \subseteq \mathcal{PN}\delta int(\overline{\mathfrak{K}}) \subseteq \mathcal{PN}int((\mathcal{PN}\delta cl(\overline{\mathfrak{K}}))$. ∴ $\overline{\mathfrak{K}}$ is a $\mathcal{PN}\delta\mathbb{P}_{OS}$.
(d) $\overline{\mathfrak{K}}$ is a $\mathcal{PN}\delta\mathbb{S}_{OS}$, then $\overline{\mathfrak{K}} \subseteq \mathcal{PN}cl(\mathcal{PN}\delta int(\overline{\mathfrak{K}})) \subseteq \mathcal{PN}cl(\mathcal{PN}int(\mathcal{PN}\delta cl(\overline{\mathfrak{K}})))$. ∴ $\overline{\mathfrak{K}}$ is a $\mathcal{PN}\delta\beta_{OS}$.
(e) $\overline{\mathfrak{K}}$ is a $\mathcal{PN}\delta\mathbb{P}_{OS}$, then $\overline{\mathfrak{K}} \subseteq \mathcal{PN}int(\mathcal{PN}\delta cl(\overline{\mathfrak{K}})) \subseteq \mathcal{PN}cl(\mathcal{PN}int(\mathcal{PN}\delta cl(\overline{\mathfrak{K}})))$. ∴ $\overline{\mathfrak{K}}$ is a $\mathcal{PN}\delta\beta_{OS}$.
(f) $\overline{\mathfrak{K}}$ is a $\mathcal{PN}\delta\alpha_{OS}$, then $\overline{\mathfrak{K}} \subseteq \mathcal{PN}int(\mathcal{PN}cl(\mathcal{PN}\delta int(\overline{\mathfrak{K}})))$. So $\overline{\mathfrak{K}} \subseteq \mathcal{PN}int(\mathcal{PN}cl(\mathcal{PN}\delta int(\overline{\mathfrak{K}}))) \subseteq \mathcal{PN}cl(\mathcal{PN}\delta int(\overline{\mathfrak{K}}))$. ∴ $\overline{\mathfrak{K}}$ is a $\mathcal{PN}\delta\mathbb{S}_{OS}$.
(g) $\overline{\mathfrak{K}}$ is a $\mathcal{PN}\delta\alpha_{OS}$, then $\overline{\mathfrak{K}} \subseteq \mathcal{PN}int(\mathcal{PN}cl(\mathcal{PN}\delta int(\overline{\mathfrak{K}})))$. So $\overline{\mathfrak{K}} \subseteq \mathcal{PN}int(\mathcal{PN}cl(\mathcal{PN}\delta int(\overline{\mathfrak{K}}))) \subseteq \mathcal{PN}int(\mathcal{PN}\delta cl(\overline{\mathfrak{K}}))$. ∴ $\overline{\mathfrak{K}}$ is a $\mathcal{PN}\delta\mathbb{P}_{OS}$.
The closed sets are also in a similar way.

Example 3.9. Let $\mathfrak{Y} = u$ and define $\mathcal{PN'S}$ $\mathfrak{Y}_1$ and $\mathfrak{Y}_2$ in $\mathfrak{Y}$ are $\mathfrak{Y}_1 = \langle \mathfrak{Y}, (0.2,0.4,0.5,0.7,0.9) \rangle \mathfrak{Y}_2 = \langle \mathfrak{Y}, (0.3,0.6,0.5,0.6,0.3) \rangle$. Then we have $\varsigma_{\mathcal{PN}} = \{0_{\mathcal{PN}}, \mathfrak{Y}_1, \mathfrak{Y}_2, 1_{\mathcal{PN}}\}$ then $\mathfrak{Y}_1$ is a $\mathcal{PN}_{OS}$ but not $\mathcal{PN}\delta_{OS}$.

Example 3.10. Let $\mathfrak{Y} = \{l, m, n\}$ and define $\mathcal{PN'S}$ $\mathfrak{Y}_1, \mathfrak{Y}_2$ and $\mathfrak{Y}_3$ in $\mathfrak{Y}$ are $\mathfrak{Y}_1 = \langle \mathfrak{Y}, (0.3,0.4,0.5,0.7,0.8), (0.3,0.5,0.5,0.6,0.7), (0.4,0.6,0.5,0.7,0.8) \rangle, \mathfrak{Y}_2 = \langle \mathfrak{Y}, (0.2,0.3,0.5,0.8,0.9), (0.3,0.4,0.5,0.7,0.8), (0.4,0.5,0.5,0.8,0.9) \rangle, \mathfrak{Y}_3 = \langle \mathfrak{Y}, (0.4,0.6,0.5,0.6,0.7), (0.5,0.6,0.5,0.6,0.7), (0.6,0.7,0.5,0.7,0.6) \rangle$. Then we have $\varsigma_{\mathcal{PN}} = \{0_{\mathcal{PN}}, \mathfrak{Y}_1, \mathfrak{Y}_2, 1_{\mathcal{PN}}\}$, then

(a) $\mathfrak{Y}_3$ is a $\mathcal{PN}\delta\mathbb{S}_{OS}$ but not $\mathcal{PN}\delta_{OS}$ $(resp. \mathcal{PN}\delta\alpha_{OS})$.
(b) $\mathfrak{Y}_3$ is a $\mathcal{PN}\delta\beta_{OS}$ but not $\mathcal{PN}\delta\mathbb{P}_{OS}$.

Example 3.11. Let$\mathfrak{Y} = l, m, n$ and define $\mathcal{PN'S}$ $\mathfrak{Y}_1, \mathfrak{Y}_2, \mathfrak{Y}_3$ and $\mathfrak{Y}_4$ in $\mathfrak{Y}$ are

$\mathfrak{Y}_1 = \langle \mathfrak{Y}, (0.3,0.4,0.5,0.6,0.8), (0.3,0.5,0.5,0.7,0.9), (0.4,0.5,0.6,0.6,0.8) \rangle$,

$\mathfrak{Y}_2 = \langle \mathfrak{Y}, (0.4,0.5,0.6,0.7,0.9), (0.4,0.6,0.7,0.7,0.8), (0.5,0.6,0.6,0.8,0.9) \rangle$,

$\mathfrak{Y}_3 = \langle \mathfrak{Y}, (0.4,0.5,0.5,0.6,0.8), (0.4,0.6,0.5,0.6,0.7), (0.5,0.6,0.6,0.5,0.6) \rangle$,

$\mathfrak{Y}_4 = \langle \mathfrak{Y}, (0.3,0.4,0.5,0.7,0.9), (0.3,0.5,0.6,0.8,0.9), (0.4,0.5,0.7,0.8,0.9) \rangle$..Then we have $\varsigma_{\mathcal{PN}} = \{0_{\mathcal{PN}}, \mathfrak{Y}_1, \mathfrak{Y}_2, 1_{\mathcal{PN}}\}$, then

(a) $\mathfrak{Y}_4$ is a $\mathcal{PN}\delta\mathbb{P}_{OS}$ but not $\mathcal{PN}_{OS}$ $(resp.\mathcal{PN}\delta\alpha_{OS})$.

(b) $\mathfrak{Y}_4$ is a $\mathcal{PN}\delta\beta_{OS}$ but not $\mathcal{PN}\delta\mathbb{S}_{OS}$.

Properties 3.12. The union (resp. intersection) of any family of $\mathcal{PN}\delta\beta_{OS}$ $(\mathfrak{Y})$ (resp. $\mathcal{PN}\delta\beta_{CS}$ $(\mathfrak{Y})$) is a $\mathcal{PN}\delta\beta_{OS}$ $(\mathfrak{Y})$ (resp. $\mathcal{PN}\delta\beta_{CS}$ $(\mathfrak{Y})$).

Proof.

Let $\{\mathfrak{K}_a: a\in \varsigma_{\mathcal{PN}}\}$ be a family of $\mathcal{PN}\delta\beta_{OS}$ 'S. For each a$\in \varsigma_{\mathcal{PN}}$, $\mathfrak{K}_a \subseteq \mathcal{PN}cl(\mathcal{PN}int(\mathcal{PN}\delta cl(\mathfrak{K}_a)))$.

$\cup a\subseteq \varsigma_{\mathcal{PN}}\ \mathfrak{K}_a \subseteq \cup\, a\subseteq \varsigma_{\mathcal{PN}} \mathcal{PN}cl(\mathcal{PN}int(\mathcal{PN}\delta cl(\mathfrak{K}_a)))$

$\subseteq \mathcal{PN}cl(\mathcal{PN}\delta cl(\cup\ \mathfrak{K}_a))$.

The other case is similar.

The proposition 3.12 is also true for $\mathcal{PN}\delta\mathbb{S}_{OS}$ $(\mathfrak{Y})$, $\mathcal{PN}\delta\mathbb{S}_{CS}$ $(\mathfrak{Y})$(resp. $\mathcal{PN}\delta\mathbb{P}_{OS}$ $(\mathfrak{Y})$, $\mathcal{PN}\delta\mathbb{P}_{CS}$ $(\mathfrak{Y})$).

4. Properties of $\mathcal{PN}\delta_{OS}$

Proposition 4.1. If $\overline{\mathfrak{L}}$ is a $\mathcal{PN}\delta_{OS}$, then $\overline{\mathfrak{L}} \cap \overline{\mathfrak{K}}$ is a $\mathcal{PN}\delta\beta_{OS}$.

Proof. $\overline{\mathfrak{L}} \cap \overline{\mathfrak{K}} \subseteq \overline{\mathfrak{L}} \cap \mathcal{PN}cl(\mathcal{PN}int(\mathcal{PN}\delta cl(\overline{\mathfrak{K}}))) \subseteq \mathcal{PN}cl(\overline{\mathfrak{L}} \cap \mathcal{PN}int(\mathcal{PN}\delta cl(\overline{\mathfrak{K}}))) \subseteq \mathcal{PN}cl(\mathcal{PN}int(\mathcal{PN}\delta cl(\overline{\mathfrak{L}} \cap \overline{\mathfrak{K}})$. Therefore, $\overline{\mathfrak{L}} \cap \overline{\mathfrak{K}}$ is a $\mathcal{PN}\delta\beta$ is a $\mathcal{PN}\delta\beta_{OS}$.

Remark 4.2. The proposition 4.1 is also true if $\overline{\mathfrak{K}}$ is a $\mathcal{PN}\delta\mathbb{S}_{OS}$, $\mathcal{PN}\delta\mathbb{P}_{OS}$ *and* $\mathcal{PN}\delta\alpha_{OS}$.

Proposition 4.3. If $\overline{\mathfrak{L}}$ is a $\mathcal{PN}\delta\mathbb{P}_{OS}$ *and* $\overline{\mathfrak{K}}$ is a $\mathcal{PN}\delta\alpha_{OS}$ then $\overline{\mathfrak{L}} \cap \overline{\mathfrak{K}}$ is a $\mathcal{PN}\delta\beta_{OS}$.

Proof. $\overline{\mathfrak{L}} \cap \overline{\mathfrak{K}} \subseteq \mathcal{PN}int(\mathcal{PN}\delta cl(\overline{\mathfrak{L}})) \cap \mathcal{PN}int(\mathcal{PN}cl(\mathcal{PN}\delta int(\overline{\mathfrak{K}}))) \subseteq \mathcal{PN}int(\mathcal{PN}int(\mathcal{PN}cl(\overline{\mathfrak{L}}))) \cap$

$\mathcal{PN}cl(\mathcal{PN}\delta int(\overline{\mathfrak{L}})) \subseteq \mathcal{PN}int(\mathcal{PN}cl(\mathcal{PN}\delta cl(\overline{\mathfrak{L}}))) \cap \mathcal{PN}cl(\mathcal{PN}\delta int(\overline{\mathfrak{K}})) \subseteq \mathcal{PN}int(\mathcal{PN}cl(\mathcal{PN}\delta cl(\overline{\mathfrak{L}}))) \cap$

$\mathcal{PN}\delta int(\overline{\mathfrak{K}}) \subseteq \mathcal{PN}int(\mathcal{PN}cl(\mathcal{PN}\delta cl(\overline{\mathfrak{L}} \cap \overline{\mathfrak{K}}\ ))) = \mathcal{PN}int(\mathcal{PN}\delta cl(\overline{\mathfrak{L}} \cap \overline{\mathfrak{K}}))$). Therefore, $\overline{\mathfrak{L}} \cap \overline{\mathfrak{K}}$ is a $\mathcal{PN}\delta\mathbb{P}_{OS}$.

Corollary 4.4. If $\overline{\mathfrak{L}}$ is a $\mathcal{PN}\delta\mathbb{P}_{CS}$ and $\overline{\mathfrak{K}}$ is a $\mathcal{PN}\delta\alpha_{OS}$ *, then* $\overline{\mathfrak{L}} \cup \overline{\mathfrak{K}}$ is a $\mathcal{PN}\delta\mathbb{P}_{CS}$.

Proposition 4.5. $\overline{\mathfrak{K}}$ is a pentapartitioned neutrosophic subset of $\mathfrak{Y}$ and $\overline{\mathfrak{L}}$ is a $\mathcal{PN}\delta\mathbb{P}_{OS}$ on $\mathfrak{Y}$ such that

$\overline{\mathfrak{L}} \subseteq \overline{\mathfrak{K}} \subseteq \mathcal{PN}cl(\mathcal{PN}cl(\mathcal{PN}\delta int(\overline{\mathfrak{L}})))$. Then $\overline{\mathfrak{K}}$ is a $\mathcal{PN}\delta\beta_{OS}$.

Proof. Let If $\overline{\mathfrak{L}}$ is a $\mathcal{PN}\delta\mathbb{P}_{OS}$, $\overline{\mathfrak{L}} \subseteq \mathcal{PN}int(\mathcal{PN}\delta cl(\overline{\mathfrak{L}}))$. Now $\overline{\mathfrak{K}} \subseteq (\mathcal{PN}cl(\mathcal{PN}\delta int(\overline{\mathfrak{L}})) \subseteq \mathcal{PN}cl(\mathcal{PN}int(\mathcal{PN}\delta int(\mathcal{PN}\delta cl(\overline{\mathfrak{L}})))) = \mathcal{PN}cl(\mathcal{PN}int(\mathcal{PN}\delta cl(\overline{\mathfrak{L}})))$. Hence $\overline{\mathfrak{K}} \subseteq \mathcal{PN}cl(\mathcal{PN}int(\mathcal{PN}\delta cl(\overline{\mathfrak{L}})))$.

Therefore, $\overline{\mathfrak{K}}$ is a $\mathcal{PN}\delta\beta_{OS}$.

Proposition 4.6. *If each* $\overline{\mathfrak{L}}$ is a $\mathcal{PN}\delta\beta_{OS}$ which is a $\mathcal{PN}\delta\mathbb{S}_{CS}$ is also a $\mathcal{PN}\delta\mathbb{S}_{OS}$.

Proof. Let $\overline{\mathfrak{L}}$ be a $\mathcal{PN}\delta\beta_{OS}$ and $\mathcal{PN}\delta\mathbb{S}_{CS}$. Then $\overline{\mathfrak{L}} \subseteq \mathcal{PN}cl(\mathcal{PN}int(\mathcal{PN}\delta cl(\overline{\mathfrak{L}})))$ and $\mathcal{PN}int(\mathcal{PN}\delta cl(\overline{\mathfrak{L}})) \subseteq \overline{\mathfrak{L}}$. Therefore, $\mathcal{PN}int(\mathcal{PN}\delta cl(\overline{\mathfrak{L}})) \subseteq \overline{\mathfrak{L}}$ and so, $\mathcal{PN}cl(\mathcal{PN}int(\mathcal{PN}\delta cl(\overline{\mathfrak{L}}))) \subseteq \mathcal{PN}cl(\mathcal{PN}\delta int(\overline{\mathfrak{L}}))$. Hence, $\overline{\mathfrak{L}} \subseteq \mathcal{PN}cl(\mathcal{PN}int(\mathcal{PN}\delta cl(\overline{\mathfrak{L}}))) \subseteq \mathcal{PN}cl(\mathcal{PN}\delta int(\overline{\mathfrak{L}}))$. Therefore, $\overline{\mathfrak{L}}$ is a $\mathcal{PN}\delta\mathbb{S}_{OS}$.

Proposition 4.7. *If each* $\overline{\mathfrak{L}}$ is a $\mathcal{PN}\delta\beta_{CS}$ and $\mathcal{PN}\delta\mathbb{S}_{OS}$ *, then it is* a $\mathcal{PN}\delta\mathbb{S}_{CS}$.

Proof. Since $\overline{\mathfrak{L}}$ is a $\mathcal{PN}\delta\beta_{CS}$ and $\mathcal{PN}\delta\mathbb{S}_{OS}$ *. Then* $\mathfrak{Y} - \overline{\mathfrak{L}}$ is $\mathcal{PN}\delta\beta_{OS}$ and $\mathcal{PN}\delta\mathbb{S}_{CS}$ and so by proposition 4.6, $\mathfrak{Y} - \overline{\mathfrak{L}}$ is a $\mathcal{PN}\delta\mathbb{S}_{OS}$. Therefore, $\overline{\mathfrak{L}}$ is a $\mathcal{PN}\delta\mathbb{S}_{CS}$.

Proposition 4.8. If each $\overline{\mathfrak{L}}$ is a $\mathcal{PN}\delta\beta_{OS}$ which is a $\mathcal{PN}\delta\alpha_{CS}$ is also a $\mathcal{PN}\delta_{CS}$.

Proof. Let $\overline{\mathfrak{L}}$ be a $\mathcal{PN}\delta\beta_{OS}$ and $\mathcal{PN}\delta\alpha_{CS}$ .Then, $\overline{\mathfrak{L}} \subseteq \mathcal{PN}cl(\mathcal{PN}int(\mathcal{PN}\delta cl(\overline{\mathfrak{L}})))$ and $\mathcal{PN}cl(\mathcal{PN}int(\mathcal{PN}\delta cl(\overline{\mathfrak{L}}))) \subseteq \overline{\mathfrak{L}}$.Therefore, $\mathcal{PN}cl(\mathcal{PN}int(\mathcal{PN}\delta cl(\overline{\mathfrak{L}}))) \subseteq \overline{\mathfrak{L}} \subseteq \mathcal{PN}cl(\mathcal{PN}int(\mathcal{PN}\delta cl(\overline{\mathfrak{L}})))$.So, $\overline{\mathfrak{L}} = \mathcal{PN}cl(\mathcal{PN}int(\mathcal{PN}\delta cl(\overline{\mathfrak{L}})))$. Therefore, $\overline{\mathfrak{L}}$ is a $\mathcal{PN}\delta_{CS}$.

Corollary 4.9. If each $\overline{\mathfrak{K}}$ is a $\mathcal{PN}\delta\beta_{CS}$ which is $\mathcal{PN}\delta\alpha_{OS}$ is also a $\mathcal{PN}\delta_{OS}$.

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