Homeomorphism via δβ-open Sets in Pythagorean Fuzzy Nano Topological Spaces with an Application to Decision Making

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Abstract

Classical set theory failed to cover non-probabilistic uncertain situations in to the set format, but fuzzy set theory can do this job perfectly with the fuzzy number of vagueness of non-probabilistic uncertainty. Topologist adopt this fuzzy set and fit in to the topological concepts to extent and apply their innovations for the needs and growth of the humans. Even though the fuzzy concept convert every situation, it hire the concept of intuitionistic fuzzy set to evident the importance of non-membership of the situation. Then Pythagorean fuzzy set is one of by membership and non-membership, more forceful to seize indeterminacy to cover the uncertain situations which are unable to covered by intuitionistic fuzzy sets. In this paper, we contribute our concept of δ (resp. δP, δS, δα, δβ)-homeomorphism, C-homeomorphism in Pythagorean fuzzy nano topological spaces and the properties for the field of fuzzy topological spaces. To register importance of Pythagorean fuzzy sets we applied a proposed similarity measure for multiple criteria decision making problem.

Keywords

Pythagorean fuzzy nano homeomorphism, Pythagorean fuzzy nano C-homeomorphism, similarity measure.

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1 Introduction

The fuzzy set concept, which has several applications in decision theory, artificial intelligence, operations research, expert systems, computer science, data analytics, pattern recognition, management science and robotics, was first introduced by Zadeh [40] in 1965. Fuzzy topological spaces were defined by Chang and Warren [14, 34] in 1968. They included the fundamental topological concepts of open, closed, neighborhood, interior, closure, continuity, and compactness in fuzzy topological spaces (FTS). Applications of fuzzy sets were investigated [1, 13, 26, 31]. Later, a large number of fuzzy topological spaces with special characteristics emerged. Dogan Coker [9, 16, 20] proposed intuitionistic fuzzy topological spaces in 1997 and investigated their compactness and continuity. Pythagorean fuzzy sets can be studied using intuitionistic fuzzy sets, which have numerous applications [31, 28]. Membership and non-membership are incorporated differently in each sets. Whereas in a Pythagorean fuzzy set, it is $\mu^2 + \lambda^2 \leq 1$, in an intuitionistic fuzzy set, the membership μ and non-membership λ are implemented so that $\mu + \lambda \leq 1$. Comparing Pythagorean fuzzy sets to intuitionistic fuzzy sets, Yager [37] introduced the non-standard fuzzy sets in 2013. The Pythagorean fuzzy set (PFS) and its use in decision-making were explained by him [3, 39, 38]. PFS is used in job placements based on academic performance [21], in the Pythagorean TOPSIS technique for mask selection during the COVID-19 pandemic [25], and more. Subsequently, Murat et al. [19] advanced the idea of Pythagorean fuzzy topological space (PFTS) by drawing on the belief in FTS [17, 18, 24]. The Pythagorean fuzzy continuous function between PFTS was defined by him. δ-open sets in fuzzy topological spaces were defined by Saha [27]. 2019 saw the definition of neutrosophic soft δ-topology by Acikgoz and Esenbel [2]. By introducing δ-open sets in neutrosophic, neutrosophic soft, neutrosophic hypersoft, and neutrosophic nano topological spaces, Aranganayagi et al., Surendra et al., and Vadivel et al. [7, 8, 29, 30, 32, 33] investigated its mappings and separation axioms. A useful tool for quantifying ambiguous information is the similarity measure. One metric that illustrates the proximity (difference) between fuzzy sets is the fuzzy similarity measure. Pythagorean fuzzy techniques to multi-attribute decision-making using similarity measurements were proposed by Zhang [15]. In order to address the problems of pattern recognition, medical diagnosis, and clustering analysis, Peng et al. [22] proposed numerous novel distance and similarity measures and talked about their transformation relations.Pythagorean fuzzy cosine functions were introduced by Wei and Wei [35] to address decision-making issues. Some of the current distance and similarity measures, however, are unable to resolve the division by zero issue, distinguish between positive and negative differences, or adhere to the third or fourth axiom. The mentioned counter-intuitive behaviours [35, 15, 22] of the current pfs similarity measures may make it difficult for DMs to select optimum or convincing alternatives. This paper's objective is to address the aforementioned problem by putting forth a novel similarity measure for Pythagorean fuzzy sets that is free of counter intuitive events.

Research Gap: No investigation on some stronger and weaker forms of Pythagorean fuzzy nano homeomorphism and nano C-homeomorphism maps such as $\mathcal{PFN}$ δ homeomorphism map, $\mathcal{PFN}$ δ-semi homeomorphism map, $\mathcal{PFN}$ δ-pre homeomorphism map, $\mathcal{PFN}$ δα homeomorphism map and $\mathcal{PFN}$ δβ homeomorphism map on $\mathcal{PFN}$ts has been reported in the $\mathcal{PFN}$ literature.

By introducing Pythagorean fuzzy nano δ (resp. $\delta\mathcal{P}$, $\delta\mathcal{S}$, δα & δβ or e^*)-homeomorphism mappings and C- homeomorphism mappings, as well as discussing their properties, this leads to embracing the idea of $\mathcal{PFN}$ts. By using this definition, we may determine the circumstances in which maps and inverse maps maintain their corresponding open sets.

The concept of fuzzy set [40], intuitionistic fuzzy set [9, 10, 11, 12], Pythagorean fuzzy set and their respective operations [36, 37, 39], Pythagorean fuzzy nano lower, upper and boundary approximations, Pythagorean fuzzy nano topology and their respective interior, closure, regular open and regular closed [4, 5], Pythagorean fuzzy nano continuous [6] and similarity measure [23] were introduced.

Later Murat et.al [19] introduced the conception of Pythagorean fuzzy topological space (PFTS) by provoking from the conviction of FTS [17, 18, 24]. He defined Pythagorean fuzzy continuous function between PFTS. The key objective of this paper is to encompass the notion of PFTS by introducing δ (resp. δα, $\delta\mathcal{S}$, $\delta\mathcal{P}$ & δβ or e^*)-homeomorphism and their respective C-homeomorphism in $\mathcal{PF}\mathfrak{N}$ts and discuss its properties. Finally, we apply one proposed similarity measure in the decision making of data analysis problem.

2 Preliminaries

We recall some basic notions of fuzzy sets, IFS's and pfs's .

Definition 2.1 [40] Let X be a nonempty set. A fuzzy set A in X is characterized by a membership function $\mu_A: X \to [0,1]$. That is:

$$\mu_A(x) = \begin{cases} 1, & \text{if} \quad x \in X \\ 0, & \text{if} \quad x \notin X \\ (0,1) & \text{if x ispartlyin X.} \end{cases}$$

Alternatively, a fuzzy set A in X is an object having the form $A = \{< x, \mu_A(x) > |x \in X\}$ or $A = \left\{\left\langle \frac{\mu_A(x)}{x} \right\rangle |x \in X\right\}$, where the function $\mu_A(x): X \to [0,1]$ defines the degree of membership of the element, $x \in X$.

The more x belongs to A, where the grades 1 and 0 denote full membership and full nonmembership, respectively, the closer the membership value $\mu_A(x)$ is to 1, a fuzzy set is a group of items with varying degrees of membership, or graded membership.

The traditional concept of a set is expanded upon by fuzzy sets. In classical set theory, an element's membership in a set is evaluated in binary terms based on a bivalent condition; it either belongs to the set or it doesn't.

Crisp sets are what fuzzy set theory refers to as classical bivalent sets. Since the indicator function of classical sets is a specific instance of the membership functions of fuzzy sets, fuzzy sets are generalized classical sets. If the latter only accept values 0 or1. With the use of a membership function valued in the real unit interval, fuzzy sets theory enables the incremental evaluation of an element's membership in a set [0,1]. Let's look at two instances:

(i) every employee of XYZ who is taller than 1.8m; (ii) every employee of XYZ who is tall.

In the first example, a classical set and a universe (all XYZ employees) are separated into members (those over 1.8m) and nonmembers using a membership rule. Because some employees are obviously in the set and some are definitely not, but some are borderline, the second example is a fuzzy set.

The membership function, μ, makes this distinction between the ins, the outs, and the borderline more precise.Using our second example once more, if x is a member of the universe and A is the fuzzy set of all tall employees, X (i.e., all employees), then $\mu_A(x)$ would be $\mu_A(x) = 1$ if x is unquestionably tall, or $\mu_A(x) = 0$ if x is non-tall, or $0 < \mu_A(x) < 1$ for borderline circumstances.

Definition 2.2 [9, 10, 11, 12] Let a nonempty set X be fixed. An IFS A in X is an object having the form: $A = \{< x, \mu_A(x), \lambda_A(x) > | x \in X\}$ or $A = \left\{\left\langle\frac{\mu_A(x),\lambda_A(x)}{x}\right\rangle | x \in X\right\}$, where the functions $\mu_A(x): X \to [0,1]$ and $\lambda_A(x): X \to [0,1]$ define the degree of membership and the degree of nonmembership, respectively, of the element $x \in X$ to A, which is a subset of X, and for every $x \in X$: $0 \leq \mu_A(x) + \lambda_A(x) \leq 1$. For each A in X: $\pi_A(x) = 1 - \mu_A(x) - \lambda_A(x)$ is the intuitionistic fuzzy set index or hesitation margin of x in X. The hesitation margin $\pi_A(x)$ is the degree of nondeterminacy of $x \in X$ to the set A and $\pi_A(x) \in [0,1]$. The hesitation margin is the function that expresses lack of knowledge of whether $x \in X$ or $x \notin X$. Thus: $\mu_A(x) + \lambda_A(x) + \pi_A(x) = 1$.

Example 2.1 Let $X = \{x, y, z\}$ be a fixed universe of discourse and $A = \left\{\left\langle\frac{0.6,0.1}{x}\right\rangle, \left\langle\frac{0.8,0.1}{y}\right\rangle, \left\langle\frac{0.5,0.3}{z}\right\rangle\right\}$, be the intuitionistic fuzzy set in X. The hesitation margins of the elements x, y, z to A are as follows: $\pi_A(x) = 0.3, \pi_A(y) = 0.1$ and $\pi_A(z) = 0.2$.

Definition 2.3 [36, 37, 39] Let a non empty set X be a universal set. Then, a Pythagorean fuzzy set A, which is a set of ordered pairs over X, is defined by the following: $A = \{< x, \mu_A(x), \lambda_A(x) | x \in X\}$ or $A = \left\{\left\langle\frac{\mu_A(x),\lambda_A(x)}{x}\right\rangle | x \in X\right\}$, where the functions $\mu_A(x): X \to [0,1]$ and $\lambda_A(x): X \to [0,1]$ define the degree of membership and the degree of nonmembership, respectively, of the element $x \in X$ to A, which is a subset of X, and for every $x \in X$, $0 \leq (\mu_A(x))^2 + (\lambda_A(x))^2 \leq 1$. Supposing $(\mu_A(x))^2 + (\lambda_A(x))^2 \leq 1$, then there is a degree of indeterminacy of $x \in X$ to A defined by $\pi_A(x) = \sqrt{1 - [(\mu_A(x))^2 + (\lambda_A(x))^2]}$ and $\pi_A(x) \in [0,1]$. In what follows, $(\mu_A(x))^2 + (\lambda_A(x))^2 + (\pi_A(x))^2 = 1$. Otherwise, $\pi_A(x) = 0$ whenever $(\mu_A(x))^2 + (\lambda_A(x))^2 = 1$. We denote the set of all PFS's over X by pfs(X).

Definition 2.4 [39] Let A and B be pfs's of the forms $A = \{< a, \mu_A(a), \lambda_A(a) > | a \in X\}$ and $B = \{< a, \mu_B(a), \lambda_B(a) > | a \in X\}$. Then [(i)]

1. $A \subseteq B$ if and only if $\mu_A(a) \leq \mu_B(a)$ and $\lambda_A(a) \geq \lambda_B(a)$ for all $a \in X$.
2. $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.
3. $\overline{A} = \{< a, \mu_A(a), \lambda_A(a) > | a \in X\}$.
4. $A \cap B = \{< a, \mu_A(a) \wedge \mu_B(a), \lambda_A(a) \vee \lambda_B(a) > | a \in X\}$.

5. $A \cup B = \{< a, \mu_A(a) \vee \mu_B(a), \lambda_A(a) \wedge \lambda_B(a) > | a \in X\}$.
6. $0_P = \{< a, 0,1 > | a \in X\}$ and $1_P = \{< a, 1,0 > | a \in X\}$.
7. $\overline{1}_P = 0_P$ and $\overline{0}_P = 1_P$.

Definition 2.5 [4] Let U be a non-empty set and R be an equivalence relation on U. Let A be a Pythagorean fuzzy set in U with the membership function $\mu_A(x)$ and non membership function $\lambda_A(x)$, $\forall$ x $\in$ U. The Pythagorean fuzzy nano lower approximation, Pythagorean fuzzy nano upper approximation and Pythagorean fuzzy nano boundary approximation of A in (U, R) denoted by $\underline{\mathcal{PF}\mathfrak{N}}(A), \overline{\mathcal{PF}\mathfrak{N}}(A)$ and $B_{\mathcal{PF}\mathfrak{N}}(A)$ and they are respectively defined as follows: [(i)]

1. $\underline{\mathcal{PF}\mathfrak{N}}(A) = \left\{\langle x, \mu_{\underline{R}(A)}(x), \lambda_{\overline{R}(A)}(x)\rangle / y \in [x]_R, x \in U\right\}$
2. $\overline{\mathcal{PF}\mathfrak{N}}(A) = \left\{\langle x, \mu_{\overline{R}(A)}(x), \lambda_{\underline{R}(A)}(x)\rangle / y \in [x]_R, x \in U\right\}$
3. $B_{\mathcal{PF}\mathfrak{N}}(A) = \overline{\mathcal{PF}\mathfrak{N}}(A) - \underline{\mathcal{PF}\mathfrak{N}}(A)$

where $\mu_{\underline{R}(A)}(x) = \wedge_{y \in [x]_R} \mu_A(y)$

$\lambda_{\underline{R}(A)}(x) = \wedge_{y \in [x]_R} \lambda_A(y)$,

$\mu_{\overline{R}(A)}(x) = \vee_{y \in [x]_R} \mu_A(y)$,

$\lambda_{\overline{R}(A)}(x) = \vee_{y \in [x]_R} \lambda_A(y)$.

Definition 2.6 [4] Let U be an universe of discourse, R be an equivalence relation on U and A be a Pythagorean fuzzy set in U and if the collection $\tau_{\mathcal{R}}(A) = \left\{0_{\mathcal{P}}, 1_{\mathcal{P}}, \underline{\mathcal{PF}\mathfrak{N}}(A), \overline{\mathcal{PF}\mathfrak{N}}(A), B_{\mathcal{PF}\mathfrak{N}}(A)\right\}$ forms a topology then it is said to be a Pythagorean fuzzy nano topology. We call $(U, \tau_{\mathcal{R}}(A))$ (or simply U) as the Pythagorean fuzzy nano topological space. The elements of $\tau_{\mathcal{R}}(A)$ are called Pythagorean fuzzy nano open (briefly, $\mathcal{PF}\mathfrak{N}o$) sets.

Remark 2.1 [4] $[\tau_{\mathcal{R}}(A)]^c$ is called the dual fuzzy nano topology of $\tau_{\mathcal{R}}(A)$. In short, $\mathcal{PF}\mathfrak{N}c$ sets are Pythagorean fuzzy nano closed elements of $[\tau_{\mathcal{R}}(A)]^c$. Therefore, we see that if and only if $1_P - G$ is Pythagorean fuzzy nano open in $\tau_{\mathcal{R}}(A)$, then a Pythagorean fuzzy set G of U is pythagorean fuzzy nano closed in $\tau_{\mathcal{R}}(A)$.

Definition 2.7 [4, 5] Let $(U, \tau_{\mathcal{P}}(A))$ be a $\mathcal{PF}\mathfrak{N}ts$ with respect to A where A is a pfs of U. Let S be a pfs of U. Then the Pythagorean fuzzy nano [(i)]

1. interior of S (briefly, $\mathcal{PF}\mathfrak{N}int(S)$) is defined by $\mathcal{PF}\mathfrak{N}int(S) = \cup \{I : I \subseteq S \,\&\, I isa\mathcal{PF}\mathfrak{N}oset\ inU\}$.
2. closure of S (briefly, $\mathcal{PF}\mathfrak{N}cl(S)$) is defined by $\mathcal{PF}\mathfrak{N}cl(S) = \cap \{A : S \subseteq A \,\&\, A isa\mathcal{PF}\mathfrak{N}cset\ inU\}$.

3. regular open (briefly, $\mathcal{PFN}ro$) set if $S = \mathcal{PFN}int(\mathcal{PFN}cl(S))$.

4. regular closed (briefly, $\mathcal{PFN}rc$) set if $S = \mathcal{PFN}cl(\mathcal{PFN}int(S))$.

Definition 2.8 [6] Let $(U_1, \tau_P(A_1))$ and $(U_2, \tau_P(A_2))$ be two $\mathcal{PFN}ts$'s. Then a function $h_P: U_1 \to U_2$ is said to be a Pythagorean fuzzy nano continuous (briefly, $\mathcal{PFN}Cts$) function if $h_P^{-1}(G)$ is $\mathcal{PFN}o$ set in U_1 for all $\mathcal{PFN}o$ set G in U_2.

Definition 2.9 [23] Let M, N and O be three pfs's on X. A similarity measure $S(M,N)$ is mapping $S: pfs(X) \times pfs(X) \to [0,1]$, possessing the following properties: [(S1)]

1. $0 \le S(M,N) \le 1$;
2. $S(M,N) = S(N,M)$;
3. $S(M,N) = 1$ iff $M = N$;
4. $S(M,M^c) = 0$ iff M is a crisp set;
5. If $M \subseteq N \subseteq O$, then $S(M,O) \le S(M,N)$ and $S(M,O) \le S(N,O)$.

Let $X = x_1, x_2, \dots, x_n$ be a finite universe of discourse, and A and B be two PFS's in X, in which $A = \{< x_i, \mu_A(x_i), \lambda_A(x_i) > | x_i \in X\}$ and $B = \{< x_i, \mu_B(x_i), \lambda_B(x_i) > | x_i \in X\}$.

Using the similarity measure in section 4, we have the Chen and Chang [15] similarity measure and is defined by

$S_{CC}(A,B)= 1 - \frac{1}{n}\sum_{i=1}^{n} ((|\mu_A(x_i) - \lambda_B(x_i)| \times (1 - \frac{\pi_A(x_i)+\pi_B(x_i)}{2})) + ((\int_0^1 |\mu_{A_{x_i}}(z) - \mu_{B_{x_i}}(z))|dz) \times (\frac{\pi_A(x_i)+\pi_B(x_i)}{2}))).$ where

$$\mu_{A_{x_i}}(z) = \begin{cases} 1 & if z = \mu_A(x_i) = 1 - \lambda_A(x_i) \\ \frac{1-\lambda_A(x_i)-z}{1-\mu_A(x_i)-\lambda_A(x_i)} & if z \in [\mu_A(x_i), 1 - \lambda_A(x_i)] \\ 0 & otherwise. \end{cases}$$

We recall some basic notions of fuzzy sets, IFS's and PFS's .

3 Pythagorean fuzzy nano $\delta\beta$-homeomorphism

In this section, we introduce Pythagorean fuzzy nano δ (resp. δ pre, δ semi, $\delta\alpha$ and $\delta\beta$)-homeomorphism and discuss some of their properties.

Definition 3.1 Let $(U, \tau_{\mathcal{P}}(A))$be an $\mathcal{PFN}ts$ and $A = \{< a, \mu_A(a), \lambda_A(a) > | a \in X\}$ be an pfs in X. Then the δ-interior and the δ-closure of A are denoted by $\mathcal{PFN}\delta int(A)$ and $\mathcal{PFN}\delta cl(A)$ and are defined as follows. $\mathcal{PFN}\delta int(A) = \cup \{G | G$ is an $\mathcal{PFN}ros$ and $G \subseteq A\}$ and $\mathcal{PFN}\delta cl(A) = \cap \{K | K$ is an $\mathcal{PFN}rcs$ and $A \subseteq K\}$.

Definition 3.2 Let $(U,\tau_{\mathcal{P}}(A))$be an $\mathcal{PF}\mathfrak{N}ts$ and $A=\{<a,\mu_A(a),\lambda_A(a)>|a\in X\}$ be an pfs in X. A set A is said to be $\mathcal{PF}\mathfrak{N}$ [(i)]

1. δ-open set (briefly, $\mathcal{PF}\mathfrak{N}\delta os$) if $A=\mathcal{PF}\mathfrak{N}\delta int(A)$,
2. δ-pre open set (briefly, $\mathcal{PF}\mathfrak{N}\delta \mathcal{P}os$) if $A\subseteq \mathcal{PF}\mathfrak{N}int(\mathcal{PF}\mathfrak{N}\delta cl(A))$.
3. δ-semi open set (briefly, $\mathcal{PF}\mathfrak{N}\delta \mathcal{S}os$) if $A\subseteq \mathcal{PF}\mathfrak{N}cl(\mathcal{PF}\mathfrak{N}\delta int(A))$.
4. δ-α open set or a-open set (briefly, $\mathcal{PF}\mathfrak{N}\delta\alpha os$ or $\mathcal{PF}\mathfrak{N}aos$) if $A\subseteq \mathcal{PF}\mathfrak{N}int(\mathcal{PF}\mathfrak{N}cl(\mathcal{PF}\mathfrak{N}\delta int(A)))$.
5. δ-β open set or e^*-open set (briefly, $\mathcal{PF}\mathfrak{N}\delta\beta os$ or $\mathcal{PF}\mathfrak{N}e^*os$) if $A\subseteq \mathcal{PF}\mathfrak{N}cl(\mathcal{PF}\mathfrak{N}int(\mathcal{PF}\mathfrak{N}\delta cl(A)))$.
6. δ (resp. δ-pre, δ-semi, δ-α and δ-β) dense if $\mathcal{PF}\mathfrak{N}\delta cl(A)$ (resp. $\mathcal{PF}\mathfrak{N}\delta pcl(A), \mathcal{PF}\mathfrak{N}\delta Scl(A), \mathcal{PF}\mathfrak{N}\delta\alpha cl(A)$ and $\mathcal{PF}\mathfrak{N}\delta\beta cl(A)) = 1_P$.

The complement of an $\mathcal{PF}\mathfrak{N}\delta os$ (resp. $\mathcal{PF}\mathfrak{N}\delta\mathcal{P}os, \mathcal{PF}\mathfrak{N}\delta\mathcal{S}os, \mathcal{PF}\mathfrak{N}\delta\alpha os$ and $\mathcal{PF}\mathfrak{N}\delta\beta os$) is called an $\mathcal{PF}\mathfrak{N}\delta$ (resp. $\mathcal{PF}\mathfrak{N}\delta\mathcal{P}, \mathcal{PF}\mathfrak{N}\delta\mathcal{S}, \mathcal{PF}\mathfrak{N}\delta\alpha$ and $\mathcal{PF}\mathfrak{N}\delta\beta$) closed set (briefly, $\mathcal{PF}\mathfrak{N}\delta cs$ (resp. $\mathcal{PF}\mathfrak{N}\delta\mathcal{P}cs, \mathcal{PF}\mathfrak{N}\delta\mathcal{S}cs, \mathcal{PF}\mathfrak{N}\delta\alpha cs$ and $\mathcal{PF}\mathfrak{N}\delta\beta cs$ in X.

The family of all $\mathcal{PF}\mathfrak{N}\delta os$ (resp.$\mathcal{PF}\mathfrak{N}\delta cs, \mathcal{PF}\mathfrak{N}\delta\mathcal{P}os, \mathcal{PF}\mathfrak{N}\delta\mathcal{P}cs, \mathcal{PF}\mathfrak{N}\delta\mathcal{S}os$, $\mathcal{PF}\mathfrak{N}\delta\mathcal{S}cs, \mathcal{PF}\mathfrak{N}\delta\alpha os, \mathcal{PF}\mathfrak{N}\delta\alpha cs, \mathcal{PF}\mathfrak{N}\delta\beta os$ and $\mathcal{PF}\mathfrak{N}\delta\beta cs$) of X is denoted by $\mathcal{PF}\mathfrak{N}\delta OS(X)$,
(resp.$\mathcal{PF}\mathfrak{N}\delta CS(X), \mathcal{PF}\mathfrak{N}\delta POS(X), \mathcal{PF}\mathfrak{N}\delta PCS(X), \mathcal{PF}\mathfrak{N}\delta SOS(X), \mathcal{PF}\mathfrak{N}\delta SCS(X)$, $\mathcal{PF}\mathfrak{N}\delta\alpha OS(X), \mathcal{PF}\mathfrak{N}\delta\alpha CS(X)$, $\mathcal{PF}\mathfrak{N}\delta\beta OS(X)$ and $\mathcal{PF}\mathfrak{N}\delta\beta CS(X)$).

Definition 3.3 Let $(U,\tau_{\mathcal{P}}(A))$be an $\mathcal{PF}\mathfrak{N}ts$ and $A=\{<a,\mu_A(a),\lambda_A(a)>|a\in X\}$ be an pfs in X. Then the $\mathcal{PF}\mathfrak{N}\delta$-pre (resp. $\mathcal{PF}\mathfrak{N}\delta$-semi, $\mathcal{PF}\mathfrak{N}\delta\alpha$ and $\mathcal{PF}\mathfrak{N}\delta\beta$)-interior and the $\mathcal{PF}\mathfrak{N}\delta$-pre (resp. $\mathcal{PF}\mathfrak{N}\delta$-semi, $\mathcal{PF}\mathfrak{N}\delta\alpha$ and $\mathcal{PF}\mathfrak{N}\delta\beta$)-closure of A are denoted by $\mathcal{PF}\mathfrak{N}\delta\mathcal{P}int(A)$ (resp. $\mathcal{PF}\mathfrak{N}\delta\mathcal{S}int(A)$, $\mathcal{PF}\mathfrak{N}\delta\alpha int(A)$ and $\mathcal{PF}\mathfrak{N}\delta\beta int(A)$) and the $\mathcal{PF}\mathfrak{N}\delta\mathcal{P}cl(A)$ (resp. $\mathcal{PF}\mathfrak{N}\delta\mathcal{S}cl(A), \mathcal{PF}\mathfrak{N}\delta\alpha cl(A)$ and $\mathcal{PF}\mathfrak{N}\delta\beta cl(A)$ and are defined as follows:

$\mathcal{PF}\mathfrak{N}\delta\mathcal{P}int(A)$ (resp. $\mathcal{PF}\mathfrak{N}\delta\mathcal{S}int(A), \mathcal{PF}\mathfrak{N}\delta\alpha int(A)$ and $\mathcal{PF}\mathfrak{N}\delta\beta int(A)$) $=\cup\{G|G$ in a $\mathcal{PF}\mathfrak{N}\delta\mathcal{P}os$ (resp. $\mathcal{PF}\mathfrak{N}\delta\mathcal{S}os, \mathcal{PF}\mathfrak{N}\delta\ \alpha os$ and $\mathcal{PF}\mathfrak{N}\delta\beta os$)and $G\subseteq A\}$ and $\mathcal{PF}\mathfrak{N}\delta\mathcal{P}cl(A)$ (resp. $\mathcal{PF}\mathfrak{N}\delta\mathcal{S}cl(A), \mathcal{PF}\mathfrak{N}\delta\alpha cl(A)$ and $\mathcal{PF}\mathfrak{N}\delta\beta cl(A)$) $=\cap\{K|K$ is an $\mathcal{PF}\mathfrak{N}\delta\mathcal{P}cs$ (resp. $\mathcal{PF}\mathfrak{N}\delta\mathcal{S}cs, \mathcal{PF}\mathfrak{N}\delta\alpha cs$, $\mathcal{PF}\mathfrak{N}\delta\beta cs$) and $A\subseteq K\}$.

Definition 3.4 Let $(U_1,\tau_{\mathcal{P}}(A_1))$ and $(U_2,\tau_{\mathcal{P}}(A_2))$ be any two $\mathcal{PF}\mathfrak{N}ts$'s. A mapping $h_P:(U_1,\tau_{\mathcal{P}}(A_1))\to(U_2,\tau_{\mathcal{P}}(A_2))$ is said to be a Pythagorean fuzzy

1. continuous (briefly, $\mathcal{PF}\mathfrak{N}Cts$), if the inverse image of every $\mathcal{PF}\mathfrak{N}os$ in $(U_2,\tau_{\mathcal{P}}(A_2))$ is a $\mathcal{PF}\mathfrak{N}os$ in $(U_1,\tau_{\mathcal{P}}(A_1))$.

2. δ-continuous (briefly, $\mathcal{PF}\mathfrak{N}\delta Cts$), if the inverse image of every $\mathcal{PF}\mathfrak{N}os$ in $(U_2, \tau_{\mathcal{P}}(A_2))$ is a $\mathcal{PF}\mathfrak{N}\delta os$ in $(U_1, \tau_{\mathcal{P}}(A_1))$.

3. δP-continuous (briefly, $\mathcal{PF}\mathfrak{N}\delta PCts$), if the inverse image of every $\mathcal{PF}\mathfrak{N}os$ in $(U_2, \tau_{\mathcal{P}}(A_2))$ is a $\mathcal{PF}\mathfrak{N}\delta Pos$ in $(U_1, \tau_{\mathcal{P}}(A_1))$.

4. δS-continuous (briefly, $\mathcal{PF}\mathfrak{N}\delta SCts$), if the inverse image of every $\mathcal{PF}\mathfrak{N}os$ in $(U_2, \tau_{\mathcal{P}}(A_2))$ is a $\mathcal{PF}\mathfrak{N}\delta Sos$ in $(U_1, \tau_{\mathcal{P}}(A_1))$.

5. $\delta\alpha$-continuous (briefly, $\mathcal{PF}\mathfrak{N}\delta\alpha Cts$), if the inverse image of every $\mathcal{PF}\mathfrak{N}os$ in $(U_2, \tau_{\mathcal{P}}(A_2))$ is a $\mathcal{PF}\mathfrak{N}\delta\alpha os$ in $(U_1, \tau_{\mathcal{P}}(A_1))$.

6. $\delta\beta$-continuous (briefly, $\mathcal{PF}\mathfrak{N}\delta\beta Cts$), if the inverse image of every $\mathcal{PF}\mathfrak{N}os$ in $(U_2, \tau_{\mathcal{P}}(A_2))$ is a $\mathcal{PF}\mathfrak{N}\delta\beta os$ in $(U_1, \tau_{\mathcal{P}}(A_1))$.

Definition 3.5 Let $(U_1, \tau_{\mathcal{P}}(A_1))$ & $(U_2, \tau_{\mathcal{P}}(A_2))$ be a $\mathcal{PF}\mathfrak{N}ts$'s. A mapping $h_P: (U_1, \tau_{\mathcal{P}}(A_1)) \to (U_2, \tau_{\mathcal{P}}(A_2))$ is said to be a Pythagorean fuzzy nano (resp. δ, $\delta\alpha$, δS, $\delta\mathcal{P}$ & $\delta\beta$ or e^*)-open map (briefly, $\mathcal{PF}\mathfrak{N}O$ (resp. $\mathcal{PF}\mathfrak{N}\delta O$, $\mathcal{PF}\mathfrak{N}\delta\alpha O$, $\mathcal{PF}\mathfrak{N}\delta SO$, $\mathcal{PF}\mathfrak{N}\delta PO$ & $\mathcal{PF}\mathfrak{N}\delta\beta O$ or $\mathcal{PF}\mathfrak{N}e^*O$)) if the image of every $\mathcal{PF}\mathfrak{N}os$ in $(U_1, \tau_{\mathcal{P}}(A_1))$ is a $\mathcal{PF}\mathfrak{N}os$ (resp. $\mathcal{PF}\mathfrak{N}\delta os$, $\mathcal{PF}\mathfrak{N}\delta\alpha os$, $\mathcal{PF}\mathfrak{N}\delta Sos$, $\mathcal{PF}\mathfrak{N}\delta Pos$ & $\mathcal{PF}\mathfrak{N}\delta\beta os$ or $\mathcal{PF}\mathfrak{N}e^*os$) in $(U_2, \tau_{\mathcal{P}}(A_2))$.

Definition 3.6 Let $(U_1, \tau_{\mathcal{P}}(A_1))$ & $(U_2, \tau_{\mathcal{P}}(A_2))$ be any two $\mathcal{PF}\mathfrak{N}ts$'s. A mapping $h_P: (U_1, \tau_{\mathcal{P}}(A_1)) \to (U_2, \tau_{\mathcal{P}}(A_2))$ is said to be Pythagorean fuzzy nano (resp. δ, δS, $\delta\mathcal{P}$ and $\delta\beta$) closed map (briefly, $\mathcal{PF}\mathfrak{N}C$ (resp. $\mathcal{PF}\mathfrak{N}\delta C$, $\mathcal{PF}\mathfrak{N}\delta SC$, $\mathcal{PF}\mathfrak{N}\delta PC$ and $\mathcal{PF}\mathfrak{N}\delta\beta C$)) if the image of every $\mathcal{PF}\mathfrak{N}cs$ in $(U_1, \tau_{\mathcal{P}}(A_1))$ is a $\mathcal{PF}\mathfrak{N}cs$ (resp. $\mathcal{PF}\mathfrak{N}\delta cs$, $\mathcal{PF}\mathfrak{N}\delta Scs$, $\mathcal{PF}\mathfrak{N}\delta Pcs$ and $\mathcal{PF}\mathfrak{N}\delta\beta cs$) in $(U_2, \tau_{\mathcal{P}}(A_2))$.

Definition 3.7 A bijection $h_P: (U_1, \tau_{\mathcal{P}}(A_1)) \to (U_2, \tau_{\mathcal{P}}(A_2))$ is called a Pythagorean fuzzy (resp. δ, $\delta\alpha$, δS, $\delta\mathcal{P}$ & $\delta\beta$ or e^*)-homeomorphism (briefly, $\mathcal{PF}\mathfrak{N}Hom$ (resp. $\mathcal{PF}\mathfrak{N}\delta Hom$, $\mathcal{PF}\mathfrak{N}\delta\alpha Hom$, $\mathcal{PF}\mathfrak{N}\delta SHom$, $\mathcal{PF}\mathfrak{N}\delta PHom$ & $\mathcal{PF}\mathfrak{N}\delta\beta Hom$ or $\mathcal{PF}\mathfrak{N}e^*Hom$)) if h_P and h_P^{-1} are $\mathcal{PF}\mathfrak{N}Cts$ (resp. $\mathcal{PF}\mathfrak{N}\delta Cts$, $\mathcal{PF}\mathfrak{N}\delta\alpha Cts$, $\mathcal{PF}\mathfrak{N}\delta SCts$, $\mathcal{PF}\mathfrak{N}\delta PCts$ & $\mathcal{PF}\mathfrak{N}\delta\beta Cts$ or $\mathcal{PF}\mathfrak{N}e^*Cts$) mappings.

Theorem 3.1 Let $(U_1, \tau_{\mathcal{P}}(A_1))$ & $(U_2, \tau_{\mathcal{P}}(A_2))$ be a $\mathcal{PF}\mathfrak{N}ts$'s. Let $h_P: (U_1, \tau_{\mathcal{P}}(A_1)) \to (U_2, \tau_{\mathcal{P}}(A_2))$ be a mapping. Then the following statements are hold for $\mathcal{PF}\mathfrak{N}ts$, but not conversely. [(i)]

1. Every $\mathcal{PF}\mathfrak{N}\delta Hom$ is a $\mathcal{PF}\mathfrak{N}Hom$.
2. Every $\mathcal{PF}\mathfrak{N}\delta Hom$ is a $\mathcal{PF}\mathfrak{N}\delta SHom$.
3. Every $\mathcal{PF}\mathfrak{N}\delta Hom$ is a $\mathcal{PF}\mathfrak{N}\delta\mathcal{P}Hom$.
4. Every $\mathcal{PF}\mathfrak{N}\delta SHom$ is a $\mathcal{PF}\mathfrak{N}\delta\beta Hom$.
5. Every $\mathcal{PF}\mathfrak{N}\delta\mathcal{P}Hom$ is a $\mathcal{PF}\mathfrak{N}\delta\beta Hom$.
6. Every $\mathcal{PF}\mathfrak{N}\delta\alpha Hom$ is a $\mathcal{PF}\mathfrak{N}\delta SHom$.
7. Every $\mathcal{PF}\mathfrak{N}\delta\alpha Hom$ is a $\mathcal{PF}\mathfrak{N}\delta\mathcal{P}Hom$.

Proof. (i) Let h_P be $\mathcal{PF}\mathfrak{N}\delta Hom$, then h_P and h_P^{-1} are $\mathcal{PF}\mathfrak{N}\delta Cts$. But every $\mathcal{PF}\mathfrak{N}\delta Cts$ function is $\mathcal{PF}\mathfrak{N}Cts$. Hence, h_P and h_P^{-1} are $\mathcal{PF}\mathfrak{N}Cts$. Therefore, h_P is a $\mathcal{PF}\mathfrak{N}Hom$.

(ii) Let h_P be $\mathcal{PF}\mathfrak{N}\delta Hom$, then h_P and h_P^{-1} are $\mathcal{PF}\mathfrak{N}\delta Cts$. But every $\mathcal{PF}\mathfrak{N}\delta Cts$ function is $\mathcal{PF}\mathfrak{N}\delta \mathcal{S}Cts$. Hence, h_P and h_P^{-1} are $\mathcal{PF}\mathfrak{N}\delta \mathcal{S}Cts$. Therefore, h_P is a $\mathcal{PF}\mathfrak{N}\delta \mathcal{S}Hom$.

(iii) Let h_P be $\mathcal{PF}\mathfrak{N}\delta Hom$, then h_P and h_P^{-1} are $\mathcal{PF}\mathfrak{N}\delta Cts$. But every $\mathcal{PF}\mathfrak{N}\delta Cts$ function is $\mathcal{PF}\mathfrak{N}\delta \mathcal{P}Cts$. Hence, h_P and h_P^{-1} are $\mathcal{PF}\mathfrak{N}\delta \mathcal{P}Cts$. Therefore, h_P is a $\mathcal{PF}\mathfrak{N}\delta \mathcal{P}Hom$.

(iv) Let h_P be $\mathcal{PF}\mathfrak{N}\delta \mathcal{S}Hom$, then h_P and h_P^{-1} are $\mathcal{PF}\mathfrak{N}\delta \mathcal{S}Cts$. But every $\mathcal{PF}\mathfrak{N}\delta \mathcal{S}Cts$ function is $\mathcal{PF}\mathfrak{N}\delta\beta Cts$. Hence, h_P and h_P^{-1} are $\mathcal{PF}\mathfrak{N}\delta\beta Cts$. Therefore, h_P is a $\mathcal{PF}\mathfrak{N}\delta\beta Hom$.

(v) Let h_P be $\mathcal{PF}\mathfrak{N}\delta \mathcal{P}Hom$, then h_P and h_P^{-1} are $\mathcal{PF}\mathfrak{N}\delta \mathcal{P}Cts$. But every $\mathcal{PF}\mathfrak{N}\delta \mathcal{P}Cts$ function is $\mathcal{PF}\mathfrak{N}\delta\beta Cts$. Hence, h_P and h_P^{-1} are $\mathcal{PF}\mathfrak{N}\delta\beta Cts$. Therefore, h_P is a $\mathcal{PF}\mathfrak{N}\delta\beta Hom$.

(vi) Let h_P be $\mathcal{PF}\mathfrak{N}\delta\alpha Hom$, then h_P and h_P^{-1} are $\mathcal{PF}\mathfrak{N}\delta\alpha Cts$. But every $\mathcal{PF}\mathfrak{N}\delta\alpha Cts$ function is $\mathcal{PF}\mathfrak{N}\delta \mathcal{S}Cts$. Hence, h_P and h_P^{-1} are $\mathcal{PF}\mathfrak{N}\delta \mathcal{S}Cts$. Therefore, h_P is a $\mathcal{PF}\mathfrak{N}\delta \mathcal{S}Hom$.

(vii) Let h_P be $\mathcal{PF}\mathfrak{N}\delta\alpha Hom$, then h_P and h_P^{-1} are $\mathcal{PF}\mathfrak{N}\delta\alpha Cts$. But every $\mathcal{PF}\mathfrak{N}\delta\alpha Cts$ function is $\mathcal{PF}\mathfrak{N}\delta \mathcal{P}Cts$. Hence, h_P and h_P^{-1} are $\mathcal{PF}\mathfrak{N}\delta \mathcal{P}Cts$. Therefore, h_P is a $\mathcal{PF}\mathfrak{N}\delta \mathcal{P}Hom$.

Example 3.1 Assume $U_1 = U_2 = U = \{s_1, s_2, s_3, s_4\}$ be the universe set and the equivalence relation is $U/R = \{\{s_1, s_4\}, \{s_2\}, \{s_3\}\}$. Let $A = \left\{\left\langle\frac{s_1}{0.3,0.1}\right\rangle, \left\langle\frac{s_2}{0.1,0.5}\right\rangle, \left\langle\frac{s_3}{0.2,0.45}\right\rangle, \left\langle\frac{s_4}{0.4,0.25}\right\rangle\right\}$ be a Pythagorean fuzzy subset of U.

$$\underline{\mathcal{PF}\mathfrak{N}}(A) = \left\{\left\langle\frac{s_1, s_4}{0.3,0.25}\right\rangle, \left\langle\frac{s_2}{0.1,0.5}\right\rangle, \left\langle\frac{s_3}{0.2,0.45}\right\rangle\right\},$$

$$\overline{\mathcal{PF}\mathfrak{N}}(A) = \left\{\left\langle\frac{s_1, s_4}{0.4,0.1}\right\rangle, \left\langle\frac{s_2}{0.1,0.5}\right\rangle, \left\langle\frac{s_3}{0.2,0.45}\right\rangle\right\},$$

$$B_{\mathcal{PF}\mathfrak{N}}(A) = \left\{\left\langle\frac{s_1, s_4}{0.25,0.3}\right\rangle, \left\langle\frac{s_2}{0.1,0.5}\right\rangle, \left\langle\frac{s_3}{0.2,0.45}\right\rangle\right\}.$$

Now $\tau_P(A_1) = \tau_P(A_2) = \tau_P(A) = \{0_{\mathcal{P}}, 1_{\mathcal{P}}, \underline{\mathcal{PF}\mathfrak{N}}(A), \overline{\mathcal{PF}\mathfrak{N}}(A), B_{\mathcal{PF}\mathfrak{N}}(A)\}$. Let $h_P: (U, \tau_P(A_1)) \to (U, \tau_P(A_2))$ be an identity function, Then h_P is $\mathcal{PFN}Hom$ (resp. $\mathcal{PFN}\delta\mathcal{P}Hom$, $\mathcal{PFN}\delta\beta Hom$ and $\mathcal{PFN}\delta\mathcal{P}Hom$) but not $\mathcal{PFN}\delta Hom$ (resp. $\mathcal{PFN}\delta Hom$, $\mathcal{PFN}\delta\mathcal{S}Hom$ and $\mathcal{PFN}\delta\alpha Hom$). Since, $\overline{\mathcal{PF}\mathfrak{N}}(A)$ is a $\mathcal{PFN}o$ set in U_2 but $h_P^{-1}(\overline{\mathcal{PF}\mathfrak{N}}(A)) = \overline{\mathcal{PF}\mathfrak{N}}(A)$ is not $\mathcal{PFN}\delta o$ (resp. $\mathcal{PFN}\delta o$, $\mathcal{PFN}\delta\mathcal{S}o$ and $\mathcal{PFN}\delta\alpha o$) set in U_1.

Example 3.2 Let $U_1 = \{s_1, s_2, s_3, s_4\}$, $U_2 = \{t_1, t_2, t_3, t_4\}$ are the universe sets and the equivalence relations are $U_1/R = \{\{s_1, s_4\}, \{s_2\}, \{s_3\}\}$ and $U_2/R = \{\{t_1, t_4\}, \{t_2\}, \{t_3\}\}$. Let $A_1 = \left\{\left\langle\frac{s_1}{0.3,0.1}\right\rangle, \left\langle\frac{s_2}{0.1,0.5}\right\rangle, \left\langle\frac{s_3}{0.2,0.45}\right\rangle, \left\langle\frac{s_4}{0.4,0.25}\right\rangle\right\}$ and $A_2 = \left\{\left\langle\frac{t_1}{0.4,0.3}\right\rangle, \left\langle\frac{t_2}{0.4,0.2}\right\rangle, \left\langle\frac{t_3}{0.5,0.3}\right\rangle, \left\langle\frac{t_4}{0.5,0.2}\right\rangle\right\}$ be a Pythagorean fuzzy subsets of U.

$$\underline{\mathcal{PF}\mathfrak{N}}(A_1) = \left\{\left\langle\frac{s_1, s_4}{0.3,0.25}\right\rangle, \left\langle\frac{s_2}{0.1,0.5}\right\rangle, \left\langle\frac{s_3}{0.2,0.45}\right\rangle\right\},$$

$$\overline{\mathcal{PFN}}(A_1) = \left\{\left\langle\frac{s_1,s_4}{0.4,0.1}\right\rangle, \left\langle\frac{s_2}{0.1,0.5}\right\rangle, \left\langle\frac{s_3}{0.2,0.45}\right\rangle\right\},$$

$$B_{\mathcal{PFN}}(A_1) = \left\{\left\langle\frac{s_1,s_4}{0.25,0.3}\right\rangle, \left\langle\frac{s_2}{0.1,0.5}\right\rangle, \left\langle\frac{s_3}{0.2,0.45}\right\rangle\right\},$$

$$\underline{\mathcal{PFN}}(A_2) = \left\{\left\langle\frac{t_1,t_4}{0.4,0.3}\right\rangle, \left\langle\frac{t_2}{0.4,0.2}\right\rangle, \left\langle\frac{t_3}{0.5,0.3}\right\rangle\right\},$$

$$\overline{\mathcal{PFN}}(A_2) = \left\{\left\langle\frac{t_1,t_4}{0.5,0.2}\right\rangle, \left\langle\frac{t_2}{0.4,0.2}\right\rangle, \left\langle\frac{t_3}{0.5,0.3}\right\rangle\right\},$$

$$B_{\mathcal{PFN}}(A_2) = \left\{\left\langle\frac{t_1,t_4}{0.3,0.4}\right\rangle, \left\langle\frac{t_2}{0.2,0.4}\right\rangle, \left\langle\frac{t_3}{0.3,0.5}\right\rangle\right\}.$$

Now $\tau_P(A_1) = \{0_{\mathcal{P}}, 1_{\mathcal{P}}, \underline{\mathcal{PFN}}(A_1), \overline{\mathcal{PFN}}(A_1), B_{\mathcal{PFN}}(A_1)\}$, $\tau_P(A_2) = \{0_{\mathcal{P}}, 1_{\mathcal{P}}, \underline{\mathcal{PFN}}(A_2), \overline{\mathcal{PFN}}(A_2), B_{\mathcal{PFN}}(A_2)\}$. Let $h_P: (U_1, \tau_P(A_1)) \to (U_2, \tau_P(A_2))$ be an identity function, Then h_P is $\mathcal{PFN}\delta\beta Hom$ but not $\mathcal{PFN}\delta\mathcal{P}Hom$. Since, $B_{\mathcal{PFN}}(A_2)$ is a $\mathcal{PFN}o$ set in U_2 but $h_P^{-1}(B_{\mathcal{PFN}}(A_2)) = B_{\mathcal{PFN}}(A_2)$ is not $\mathcal{PFN}\delta\mathcal{P}o$ set in U_1.

Theorem 3.2 Let $h_P: (U_1, \tau_{\mathcal{P}}(A_1)) \to (U_2, \tau_{\mathcal{P}}(A_2))$ be a bijective mapping. If h_P is $\mathcal{PFN}Cts$ (resp. $\mathcal{PFN}\delta Cts$, $\mathcal{PFN}\delta\alpha Cts$, $\mathcal{PFN}\delta\mathcal{S}Cts$, $\mathcal{PFN}\delta\mathcal{P}Cts$ & $\mathcal{PFN}\delta\beta Cts$), then the followings statements are equivalent:

1. h_P is a $\mathcal{PFN}C$ (resp. $\mathcal{PFN}\delta C$, $\mathcal{PFN}\delta\alpha C$, $\mathcal{PFN}\delta\mathcal{S}C$, $\mathcal{PFN}\delta\mathcal{P}C$ & $\mathcal{PFN}\delta\beta C$ or $\mathcal{PFN}e^*C$) mapping.
2. h_P is a $\mathcal{PFN}O$ (resp. $\mathcal{PFN}\delta O$, $\mathcal{PFN}\delta\alpha O$, $\mathcal{PFN}\delta\mathcal{S}O$, $\mathcal{PFN}\delta\mathcal{P}O$ & $\mathcal{PFN}\delta\beta O$ or $\mathcal{PFN}e^*C$) mapping.
3. h_P^{-1} is a $\mathcal{PFN}Hom$ (resp. $\mathcal{PFN}\delta Hom$, $\mathcal{PFN}\delta\alpha Hom$, $\mathcal{PFN}\delta\mathcal{S}Hom$, $\mathcal{PFN}\delta\mathcal{P}Hom$ & $\mathcal{PFN}\delta\beta Hom$ or $\mathcal{PFN}e^*Hom$) .

Proof. (i) ⇒ (ii) : Assume that h_P is a bijective mapping and a $\mathcal{PFN}\delta\beta C$ mapping. Hence, h_P^{-1} is a $\mathcal{PFN}\delta\beta Cts$ mapping. We know that each $\mathcal{PFN}os$ in $(U_1, \tau_{\mathcal{P}}(A_1))$ is a $\mathcal{PFN}\delta\beta os$ in $(U_2, \tau_{\mathcal{P}}(A_2))$. Hence, h_P is a $\mathcal{PFN}\delta\beta O$ mapping.

(ii) ⇒ (iii) : Let h_P be a bijective and $\mathcal{PFN}\delta\beta O$ mapping. Further, h_P^{-1} is a $\mathcal{PFN}\delta\beta Cts$ mapping. Hence, h_P and h_P^{-1} are $\mathcal{PFN}\delta\beta Cts$. Therefore, h_P is a $\mathcal{PFN}\delta\beta Hom$.

(iii) ⇒ (i): Let h_P be a $\mathcal{PFN}\delta\beta Hom$. Then h_P and h_P^{-1} are $\mathcal{PFN}\delta\beta Cts$. Since each $\mathcal{PFN}\delta cs$ in $(U_1, \tau_{\mathcal{P}}(A_1))$ is $\mathcal{PFN}cs$ in $(U_1, \tau_{\mathcal{P}}(A_1))$ is a $\mathcal{PFN}\delta\beta cs$ in $(U_2, \tau_{\mathcal{P}}(A_2))$, h_P is a $\mathcal{PFN}\delta\beta C$ mapping. The proof of other cases are similar. width 0.22 true cm height 0.22 true cm depth 0pt

Definition 3.8 A $\mathcal{PFN}ts$ $(U, \tau_{\mathcal{P}}(A))$ is said to be a Pythagorean fuzzy nano $\alpha T_{\frac{1}{2}}$ (resp. $\delta\mathcal{S}T_{\frac{1}{2}}$, $\delta\mathcal{P}T_{\frac{1}{2}}$ and $\delta\beta T_{\frac{1}{2}}$) (briefly, $\mathcal{PFN}\delta\alpha T_{\frac{1}{2}}$ (resp. $\mathcal{PFN}\delta\mathcal{S}T_{\frac{1}{2}}$, $\mathcal{PFN}\delta\mathcal{P}T_{\frac{1}{2}}$ and $\mathcal{PFN}\delta\beta T_{\frac{1}{2}}$))-space if every $\mathcal{PFN}\delta\alpha cs$ (resp. $\mathcal{PFN}\delta\mathcal{S}cs, \mathcal{PFN}\delta\mathcal{P}cs$, and $\mathcal{PFN}\delta\beta cs$) is $\mathcal{PFN}cs$ in $(U, \tau_{\mathcal{P}}(A))$.

Theorem 3.3 Let $h_P: (U_1, \tau_{\mathcal{P}}(A_1)) \to (U_2, \tau_{\mathcal{P}}(A_2))$ be a $\mathcal{PFN}\delta\alpha Hom$ (resp. $\mathcal{PFN}\delta SHom$, $\mathcal{PFN}\delta PHom$ & $\mathcal{PFN}\delta\beta Hom$ or $\mathcal{PFN}e^* Hom$). Then h_P is a $\mathcal{PFN}Hom$ if $(U_1, \tau_{\mathcal{P}}(A_1))$ and $(U_2, \tau_{\mathcal{P}}(A_2))$ are $\mathcal{PFN}\delta\alpha T_{\frac{1}{2}}$ (resp. $\mathcal{PFN}\delta ST_{\frac{1}{2}}$, $\mathcal{PFN}\delta PT_{\frac{1}{2}}$ and $\mathcal{PFN}\delta\beta T_{\frac{1}{2}}$)-space.

Proof. Assume that K is a $\mathcal{PFN}cs$ in $(U_2, \tau_{\mathcal{P}}(A_2))$. Then $h_P^{-1}(K)$ is a $\mathcal{PFN}\delta\beta cs$ in $(U_1, \tau_{\mathcal{P}}(A_1))$. Since $(U_1, \tau_{\mathcal{P}}(A_1))$ is a $\mathcal{PFN}\delta\beta T_{\frac{1}{2}}$-space, $h_P^{-1}(K)$ is a $\mathcal{PFN}cs$ in $(U_1, \tau_{\mathcal{P}}(A_1))$. Therefore, h_P is $\mathcal{PFN}Cts$. By hypothesis, h_P^{-1} is $\mathcal{PFN}\delta\beta Cts$. Let L be a $\mathcal{PFN}cs$ in $(U_1, \tau_{\mathcal{P}}(A_1))$. Then, $(h_P^{-1})^{-1}(L) = h_P(L)$ is a $\mathcal{PFN}cs$ in $(U_2, \tau_{\mathcal{P}}(A_2))$, by presumption. Since $(U_2, \tau_{\mathcal{P}}(A_2))$ is a $\mathcal{PFN}\delta\beta T_{\frac{1}{2}}$-space, $h_P(L)$ is a $\mathcal{PFN}cs$ in $(U_2, \tau_{\mathcal{P}}(A_2))$. Hence, h_P^{-1} is $\mathcal{PFN}Cts$. Hence, h_P is a $\mathcal{PFN}Hom$. Proof of other cases are similar.

Theorem 3.4 Let $h_P: (U_1, \tau_{\mathcal{P}}(A_1)) \to (U_2, \tau_{\mathcal{P}}(A_2))$ be a mapping. Then the following are equivalent if $(U_2, \tau_{\mathcal{P}}(A_2))$ is a $\mathcal{PFN}\delta\alpha T_{\frac{1}{2}}$ (resp. $\mathcal{PFN}\delta ST_{\frac{1}{2}}$, $\mathcal{PFN}\delta PT_{\frac{1}{2}}$ and $\mathcal{PFN}\delta\beta T_{\frac{1}{2}}$)-space:

1. h_P is $\mathcal{PFN}\delta\alpha C$ (resp. $\mathcal{PFN}\delta SC$, $\mathcal{PFN}\delta PC$ and $\mathcal{PFN}\delta\beta C$) mapping.
2. If K is a $\mathcal{PFN}os$ in $(U_1, \tau_{\mathcal{P}}(A_1))$, then $h_P(U_1, \tau_{\mathcal{P}}(A_1))$ is $\mathcal{PFN}\delta\alpha os$ (resp. $\mathcal{PFN}\delta Sos$, $\mathcal{PFN}\delta Pos$ and $\mathcal{PFN}\delta\beta os$) in $(U_2, \tau_{\mathcal{P}}(A_2))$.
3. $h_P(\mathcal{PFN}int(K)) \subseteq \mathcal{PFN}cl(\mathcal{PFN}int(h_P(K)))$ for every $\mathcal{PFN}s$ K in $(U_1, \tau_{\mathcal{P}}(A_1))$.

Proof. (i) ⇒ (ii): Obvious.

(ii) ⇒ (iii): Let K be a pfs in $(U_1, \tau_{\mathcal{P}}(A_1))$. Then, $\mathcal{PFN}int(K)$ is a $\mathcal{PFN}os$ in $(U_1, \tau_{\mathcal{P}}(A_1))$. Then, $h_P(\mathcal{PFN}int(K)$ is a $\mathcal{PFN}\delta\beta os$ in $(U_2, \tau_{\mathcal{P}}(A_2))$. Since $(U_2, \tau_{\mathcal{P}}(A_2))$ is a $\mathcal{PFN}\delta\beta T_{\frac{1}{2}}$-space, so $h_P(\mathcal{PFN}int(K))$ is a $\mathcal{PFN}os$ in $(U_2, \tau_{\mathcal{P}}(A_2))$. Therefore, $h_P(\mathcal{PFN}int(K)) = \mathcal{PFN}int(h_P(\mathcal{PFN}int(K))) \subseteq \mathcal{PFN}cl(\mathcal{PFN}int(h_P(K)))$.

(iii) ⇒ (i): Let K be a $\mathcal{PFN}cs$ in $(U_1, \tau_{\mathcal{P}}(A_1))$. Then, $(K)^c$ is a $\mathcal{PFN}os$ in $(U_1, \tau_{\mathcal{P}}(A_1))$. From, $h_P(\mathcal{PFN}int(K)^c) \subseteq \mathcal{PFN}cl(\mathcal{PFN}int(h_P(K)^c))$, $h_P((K)^c) \subseteq \mathcal{PFN}cl(\mathcal{PFN}int(h_P(K)^c))$. Therefore, $h_P((K)^c)$ is $\mathcal{PFN}\delta\beta os$ in $(U_2, \tau_{\mathcal{P}}(A_2))$. Therefore, $h_P(K)$ is a $\mathcal{PFN}\delta\beta cs$ in $(U_1, \tau_{\mathcal{P}}(A_1))$. Hence, h_P is a $\mathcal{PFN}C$ mapping. The proof of other cases are similar.

Theorem 3.5 Let $h_P: (U_1, \tau_{\mathcal{P}}(A_1)) \to (U_2, \tau_{\mathcal{P}}(A_2))$ and $g_P: (U_2, \tau_{\mathcal{P}}(A_2)) \to (U_3, \tau_{\mathcal{P}}(A_3))$ be $\mathcal{PFN}\delta\alpha C$ (resp. $\mathcal{PFN}\delta SC$, $\mathcal{PFN}\delta PC$ and $\mathcal{PFN}\delta\beta C$), where $(U_1, \tau_{\mathcal{P}}(A_1))$ and $(U_3, \tau_{\mathcal{P}}(A_3))$ are two $\mathcal{PFN}ts$'s and $(U_2, \tau_{\mathcal{P}}(A_2))$ a $\mathcal{PFN}\delta T_{\frac{1}{2}}$ (resp. $\mathcal{PFN}\delta\alpha T_{\frac{1}{2}}$,

$\mathcal{PFN}\delta ST_{\frac{1}{2}}$, $\mathcal{PFN}\delta\mathcal{P}T_{\frac{1}{2}}$ and $\mathcal{PFN}\delta\beta T_{\frac{1}{2}}$)-space, then the composition $g_P \circ h_P$ is $\mathcal{PFN}\delta\alpha C$ (resp. $\mathcal{PFN}\delta SC$, $\mathcal{PFN}\delta\mathcal{P}C$ and $\mathcal{PFN}\delta\beta C$).

Proof. Let K be a $\mathcal{PFN}cs$ in $(U_1, \tau_{\mathcal{P}}(A_1))$. Since h_P is $\mathcal{PFN}\delta\beta C$ and $h_P(K)$ is a $\mathcal{PFN}\delta\beta cs$ in $(U_2, \tau_{\mathcal{P}}(A_2))$, by assumption, $h_P(K)$ is a $\mathcal{PFN}cs$ in $(U_2, \tau_{\mathcal{P}}(A_2))$. Since g_P is $\mathcal{PFN}\delta\beta C$, then $g_P(h_P(K))$ is $\mathcal{PFN}\delta\beta cs$ in $(U_3, \tau_{\mathcal{P}}(A_3))$ and $g_P(h_P(K)) = (g_P \circ h_P)(K)$. Therefore, $g_P \circ h_P$ is $\mathcal{PFN}\delta\beta C$. width 0.22 true cm height 0.22 true cm depth 0pt

Theorem 3.6 Let $h_P: (U_1, \tau_{\mathcal{P}}(A_1)) \to (U_2, \tau_{\mathcal{P}}(A_2))$ and $g_P: (U_2, \tau_{\mathcal{P}}(A_2)) \to (U_3, \tau_{\mathcal{P}}(A_3))$ be two $\mathcal{PFN}ts$'s, then the following hold: [(i)]

1. If $g_P \circ h_P$ is $\mathcal{PFN}\delta O$ (resp. $\mathcal{PFN}\delta\alpha O$, $\mathcal{PFN}\delta SO$, $\mathcal{PFN}\delta\mathcal{P}O$ and $\mathcal{PFN}\delta\beta O$) and h_P is $\mathcal{PFN}Cts$, then g_P is $\mathcal{PFN}\delta O$ (resp. $\mathcal{PFN}\delta\alpha O$, $\mathcal{PFN}\delta SO$, $\mathcal{PFN}\delta\mathcal{P}O$ and $\mathcal{PFN}\delta\beta O$).

2. If $g_P \circ h_P$ is $\mathcal{PFN}O$ and g_P is $\mathcal{PFN}\delta Cts$ (resp. $\mathcal{PFN}\delta\alpha Cts$, $\mathcal{PFN}\delta SCts$, $\mathcal{PFN}\delta\mathcal{P}Cts$ and $\mathcal{PFN}\delta\beta Cts$), then h_P is $\mathcal{PFN}\delta O$ (resp. $\mathcal{PFN}\delta\alpha O$, $\mathcal{PFN}\delta SO$, $\mathcal{PFN}\delta\mathcal{P}O$ and $\mathcal{PFN}\delta\beta O$).

Proof. (i) Let K be a $\mathcal{PFN}os$ in $(U_2, \tau_{\mathcal{P}}(A_2))$. As h_P is $\mathcal{PFN}Cts$ mapping, $h_P^{-1}(K)$ is $\mathcal{PFN}os$ in $(U_1, \tau_{\mathcal{P}}(A_1))$. As $g_P \circ h_P$ is $\mathcal{PFN}\delta\beta O$ mapping, $(g_P \circ h_P)(h_P^{-1}(K)) = g_P(h_P(h_P^{-1}(K))) = g_P(K)$ is $\mathcal{PFN}\delta\beta os$ in $(U_3, \tau_{\mathcal{P}}(A_3))$. Thus h_P is $\mathcal{PFN}\delta\beta O$ mapping.

The other case is similar.

4 Pythagorean fuzzy δβ-C homeomorphism

Definition 4.1 A bijection $h_P: (U_1, \tau_{\mathcal{P}}(A_1)) \to (U_2, \tau_{\mathcal{P}}(A_2))$ is called a Pythagorean fuzzy (resp. δ, $\delta\alpha$, δS, $\delta\mathcal{P}$ & $\delta\beta$ or e^*)-C homeomorphism (briefly, $\mathcal{PFN}CHom$ (resp. $\mathcal{PFN}\delta CHom$, $\mathcal{PFN}\delta\alpha CHom$, $\mathcal{PFN}\delta SCHom$, $\mathcal{PFN}\delta\mathcal{P}CHom$ & $\mathcal{PFN}\delta\beta CHom$ or $\mathcal{PFN}e^*CHom$)) if h_P and h_P^{-1} are $\mathcal{PFN}Irr$ (resp. $\mathcal{PFN}\delta Irr$, $\mathcal{PFN}\delta\alpha Irr$, $\mathcal{PFN}\delta SIrr$, $\mathcal{PFN}\delta\mathcal{P}Irr$ & $\mathcal{PFN}\delta\beta Irr$ or $\mathcal{PFN}e^*Irr$) mappings.

Theorem 4.1 Each $\mathcal{PFN}Hom$ (resp. $\mathcal{PFN}\delta Hom$, $\mathcal{PFN}\delta\alpha CHom$, $\mathcal{PFN}\delta SCHom$, $\mathcal{PFN}\delta\mathcal{P}CHom$ & $\mathcal{PFN}\delta\beta CHom$ or $\mathcal{PFN}e^*CHom$) is a $\mathcal{PFN}CHom$ (resp. $\mathcal{PFN}\delta CHom$, $\mathcal{PFN}\delta\alpha Hom$, $\mathcal{PFN}\delta SHom$, $\mathcal{PFN}\delta\mathcal{P}Hom$ & $\mathcal{PFN}\delta\beta Hom$ or $\mathcal{PFN}e^*Hom$). But not conversely.

Proof. Let us assume that K be a $\mathcal{PFN}\delta cs$ in $(U_2, \tau_{\mathcal{P}}(A_2))$ is $\mathcal{PFN}cs$. This shows that K is a $\mathcal{PFN}\delta\beta cs$ in $(U_2, \tau_{\mathcal{P}}(A_2))$. By assumption, $h_P^{-1}(K)$ is a $\mathcal{PFN}\delta\beta cs$ in $(U_1, \tau_{\mathcal{P}}(A_1))$. Hence, h_P is a $\mathcal{PFN}\delta\beta Cts$ mapping. Hence, h_P and h_P^{-1} are $\mathcal{PFN}\delta\beta Cts$ mappings. Hence h_P is a $\mathcal{PFN}\delta\beta Hom$. The proof of other cases are similar. width 0.22 true cm height 0.22 true cm depth 0pt

Example 4.1 Assume $U_1 = U_2 = U = \{s_1, s_2, s_3, s_4\}$ be the universe set and the equivalence relation is $U/R = \{\{s_1, s_4\}, \{s_2\}, \{s_3\}\}$. Let $A = \left\{\left\langle\frac{s_1}{0.6,0.7}\right\rangle, \left\langle\frac{s_2}{0.5,0.7}\right\rangle, \left\langle\frac{s_3}{0.3,0.5}\right\rangle, \left\langle\frac{s_4}{0.7,0.3}\right\rangle\right\}$ be a Pythagorean fuzzy subset of U.

$$\underline{\mathcal{PF}\mathfrak{N}}(A) = \left\{\left\langle\frac{s_1,s_4}{0.6,0.7}\right\rangle, \left\langle\frac{s_2}{0.5,0.7}\right\rangle, \left\langle\frac{s_3}{0.3,0.5}\right\rangle\right\},$$

$$\overline{\mathcal{PF}\mathfrak{N}}(A) = \left\{\left\langle\frac{s_1,s_4}{0.7,0.3}\right\rangle, \left\langle\frac{s_2}{0.5,0.7}\right\rangle, \left\langle\frac{s_3}{0.3,0.5}\right\rangle\right\},$$

$$B_{\mathcal{PF}\mathfrak{N}}(A) = \left\{\left\langle\frac{s_1,s_4}{0.7,0.6}\right\rangle, \left\langle\frac{s_2}{0.5,0.7}\right\rangle, \left\langle\frac{s_3}{0.3,0.5}\right\rangle\right\}.$$

Now $\tau_P(A_1) = \tau_P(A_2) = \tau_P(A) = \{0_{\mathcal{P}}, 1_{\mathcal{P}}, \underline{\mathcal{PF}\mathfrak{N}}(A), \overline{\mathcal{PF}\mathfrak{N}}(A), B_{\mathcal{PF}\mathfrak{N}}(A)\}$. Let $h_P: (U, \tau_P(A_1)) \to (U, \tau_P(A_2))$ be an identity function, Then h_P is $\mathcal{PFN}\delta CHom$ but not $\mathcal{PFN}\delta Hom$. Since, $\overline{\mathcal{PF}\mathfrak{N}}(A)$ is a $\mathcal{PFN}o$ set in U_2 but $h_P^{-1}(\overline{\mathcal{PF}\mathfrak{N}}(A)) = \overline{\mathcal{PF}\mathfrak{N}}(A)$ is not $\mathcal{PFN}\delta o$ set in U_1.

Example 4.2 Let $U_1 = \{s_1, s_2, s_3, s_4\}$, $U_2 = \{t_1, t_2, t_3, t_4\}$ are the universe sets and the equivalence relations are $U_1/R = \{\{s_1, s_4\}, \{s_2\}, \{s_3\}\}$ and $U_2/R = \{\{t_1, t_4\}, \{t_2\}, \{t_3\}\}$. Let $A_1 = \left\{\left\langle\frac{s_1}{0.3,0.1}\right\rangle, \left\langle\frac{s_2}{0.1,0.5}\right\rangle, \left\langle\frac{s_3}{0.2,0.45}\right\rangle, \left\langle\frac{s_4}{0.4,0.25}\right\rangle\right\}$ and $A_2 = \left\{\left\langle\frac{t_1}{0.9,0.3}\right\rangle, \left\langle\frac{t_2}{0.5,0.7}\right\rangle, \left\langle\frac{t_3}{0.3,0.5}\right\rangle, \left\langle\frac{t_4}{0.6,0.7}\right\rangle\right\}$ be a Pythagorean fuzzy subsets of U_1 and U_2 respectively.

$$\underline{\mathcal{PF}\mathfrak{N}}(A_1) = \left\{\left\langle\frac{s_1,s_4}{0.3,0.25}\right\rangle, \left\langle\frac{s_2}{0.1,0.5}\right\rangle, \left\langle\frac{s_3}{0.2,0.45}\right\rangle\right\},$$

$$\overline{\mathcal{PF}\mathfrak{N}}(A_1) = \left\{\left\langle\frac{s_1,s_4}{0.4,0.1}\right\rangle, \left\langle\frac{s_2}{0.1,0.5}\right\rangle, \left\langle\frac{s_3}{0.2,0.45}\right\rangle\right\},$$

$$B_{\mathcal{PF}\mathfrak{N}}(A_1) = \left\{\left\langle\frac{s_1,s_4}{0.25,0.3}\right\rangle, \left\langle\frac{s_2}{0.1,0.5}\right\rangle, \left\langle\frac{s_3}{0.2,0.45}\right\rangle\right\},$$

$$\underline{\mathcal{PF}\mathfrak{N}}(A_2) = \left\{\left\langle\frac{t_1,t_4}{0.6,0.7}\right\rangle, \left\langle\frac{t_2}{0.5,0.7}\right\rangle, \left\langle\frac{t_3}{0.3,0.5}\right\rangle\right\},$$

$$\overline{\mathcal{PF}\mathfrak{N}}(A_2) = \left\{\left\langle\frac{t_1,t_4}{0.9,0.3}\right\rangle, \left\langle\frac{t_2}{0.5,0.7}\right\rangle, \left\langle\frac{t_3}{0.3,0.5}\right\rangle\right\},$$

$$B_{\mathcal{PF}\mathfrak{N}}(A_2) = \left\{\left\langle\frac{t_1,t_4}{0.7,0.6}\right\rangle, \left\langle\frac{t_2}{0.5,0.7}\right\rangle, \left\langle\frac{t_3}{0.3,0.5}\right\rangle\right\}.$$

Here $\tau_p(A_1) = \{0_P, 1_P, \underline{\mathcal{PF}\mathfrak{N}}(A_1), \overline{\mathcal{PF}\mathfrak{N}}(A_1), B_{\mathcal{PF}\mathfrak{N}}(A_1)\}$ and $\tau_p(A_2) = \{0_P, 1_P, \underline{\mathcal{PF}\mathfrak{N}}(A_2), \overline{\mathcal{PF}\mathfrak{N}}(A_2), B_{\mathcal{PF}\mathfrak{N}}(A_2)\}$ are the $\mathcal{PFN}ts's$ on U_1 and U_2 respectively. Let $h_P: (U_1, \tau_P(A_1)) \to (U_2, \tau_P(A_2))$ be an identity function, then h_P is $\mathcal{PFN}\delta\mathcal{P}Hom$ but not $\mathcal{PFN}\delta\mathcal{P}CHom$, because the set

$$B_1 = \left\{\left\langle\frac{s_1,s_4}{0.25,0.35}\right\rangle, \left\langle\frac{s_2}{0.25,0.45}\right\rangle, \left\langle\frac{s_3}{0.3,0.2}\right\rangle\right\}$$

is a $\mathcal{PFN}\delta\mathcal{P}os$ in U_2 but $h_P^{-1}(B_1) = B_1$ is not $\mathcal{PFN}\delta\mathcal{P}os$ in U_1.

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Example 4.3 Let $U_1 = \{s_1, s_2, s_3, s_4\}$, $U_2 = \{t_1, t_2, t_3, t_4\}$ are the universe sets and the equivalence relations are $U_1/R = \{\{s_1, s_4\}, \{s_2\}, \{s_3\}\}$ and $U_2/R = \{\{t_1, t_4\}, \{t_2\}, \{t_3\}\}$. Let $A_1 = \left\{\left\langle\frac{s_1}{0.3,0.1}\right\rangle, \left\langle\frac{s_2}{0.1,0.5}\right\rangle, \left\langle\frac{s_3}{0.2,0.45}\right\rangle, \left\langle\frac{s_4}{0.4,0.25}\right\rangle\right\}$ and $A_2 = \left\{\left\langle\frac{t_1}{0.9,0.3}\right\rangle, \left\langle\frac{t_2}{0.5,0.7}\right\rangle, \left\langle\frac{t_3}{0.3,0.5}\right\rangle, \left\langle\frac{t_4}{0.6,0.7}\right\rangle\right\}$ be a Pythagorean fuzzy subsets of U_1 and U_2 respectively.

$$\underline{\mathcal{PF}\mathfrak{N}}(A_1) = \left\{\left\langle\frac{s_1,s_4}{0.3,0.25}\right\rangle, \left\langle\frac{s_2}{0.1,0.5}\right\rangle, \left\langle\frac{s_3}{0.2,0.45}\right\rangle\right\},$$

$$\overline{\mathcal{PF}\mathfrak{N}}(A_1) = \left\{\left\langle\frac{s_1,s_4}{0.4,0.1}\right\rangle, \left\langle\frac{s_2}{0.1,0.5}\right\rangle, \left\langle\frac{s_3}{0.2,0.45}\right\rangle\right\},$$

$$B_{\mathcal{PF}\mathfrak{N}}(A_1) = \left\{\left\langle\frac{s_1,s_4}{0.25,0.3}\right\rangle, \left\langle\frac{s_2}{0.1,0.5}\right\rangle, \left\langle\frac{s_3}{0.2,0.45}\right\rangle\right\}$$

$$\underline{\mathcal{PF}\mathfrak{N}}(A_2) = \left\{\left\langle\frac{t_1,t_4}{0.6,0.7}\right\rangle, \left\langle\frac{t_2}{0.5,0.7}\right\rangle, \left\langle\frac{t_3}{0.3,0.5}\right\rangle\right\},$$

$$\overline{\mathcal{PF}\mathfrak{N}}(A_2) = \left\{\left\langle\frac{t_1,t_4}{0.9,0.3}\right\rangle, \left\langle\frac{t_2}{0.5,0.7}\right\rangle, \left\langle\frac{t_3}{0.3,0.5}\right\rangle\right\},$$

$$B_{\mathcal{PF}\mathfrak{N}}(A_2) = \left\{\left\langle\frac{t_1,t_4}{0.7,0.6}\right\rangle, \left\langle\frac{t_2}{0.5,0.7}\right\rangle, \left\langle\frac{t_3}{0.3,0.5}\right\rangle\right\}.$$

Here $\tau_p(A_1) = \{0_P, 1_P, \underline{\mathcal{PF}\mathfrak{N}}(A_1), \overline{\mathcal{PF}\mathfrak{N}}(A_1), B_{\mathcal{PF}\mathfrak{N}}(A_1)\}$ and $\tau_p(A_2) = \{0_P, 1_P, \underline{\mathcal{PF}\mathfrak{N}}(A_2), \overline{\mathcal{PF}\mathfrak{N}}(A_2), B_{\mathcal{PF}\mathfrak{N}}(A_2)\}$ are the $\mathcal{PFN}ts's$ on U_1 and U_2 respectively. Let $h_P: (U_1, \tau_P(A_1)) \to (U_2, \tau_P(A_2))$ be an identity function, then h_P is $\mathcal{PFN}\delta\beta Hom$ but not $\mathcal{PFN}\delta\beta CHom$, because the set

$$C_1 = \left\{\left\langle\frac{s_1,s_4}{0.1,0.4}\right\rangle, \left\langle\frac{s_2}{0.2,0.4}\right\rangle, \left\langle\frac{s_3}{0.3,0.2}\right\rangle\right\}$$

is a $\mathcal{PFN}\delta\beta os$ in U_2 but $h_P^{-1}(C_1) = C_1$ is not $\mathcal{PFN}\delta\beta os$ in U_1.

Theorem 4.2 If $h_P: (U_1, \tau_{\mathcal{P}}(A_1)) \to (U_2, \tau_{\mathcal{P}}(A_2))$ is a $\mathcal{PF}\mathfrak{N}\delta\beta CHom$, then $\mathcal{PF}\mathfrak{N}\delta\beta cl(h_P^{-1}(K)) \subseteq h_P^{-1}(\mathcal{PF}\mathfrak{N}cl(K))$ for each pfs K in $(U_2, \tau_{\mathcal{P}}(A_2))$.

Proof. Let K be a pfs in $(U_2, \tau_{\mathcal{P}}(A_2))$. Then, $\mathcal{PF}\mathfrak{N}cl(K)$ is a $\mathcal{PF}\mathfrak{N}cs$ in $(U_2, \tau_{\mathcal{P}}(A_2))$, and every $\mathcal{PF}\mathfrak{N}cs$ is a $\mathcal{PF}\mathfrak{N}\delta\beta cs$ in $(U_2, \tau_{\mathcal{P}}(A_2))$. Assume h_P is $\mathcal{PF}\mathfrak{N}\delta\beta Irr$ and $h_P^{-1}(\mathcal{PF}\mathfrak{N}\mathfrak{N}cl(K))$ is a $\mathcal{PF}\mathfrak{N}\delta\beta cs$ in $(U_1, \tau_{\mathcal{P}}(A_1))$. Then, $\mathcal{PF}\mathfrak{N}cl(h_P^{-1}(\mathcal{PF}\mathfrak{N}cl(K))) = h_P^{-1}(\mathcal{PF}\mathfrak{N}\mathfrak{N}cl(K))$. Here, $\mathcal{PF}\mathfrak{N}\delta\beta cl(h_P^{-1}(K)) \subseteq \mathcal{PF}\mathfrak{N}\delta\beta cl(h_P^{-1}(\mathcal{PF}\mathfrak{N}cl(K))) = h_P^{-1}(\mathcal{PF}\mathfrak{N}\, cl(K))$. Therefore, $\mathcal{PF}\mathfrak{N}\delta\beta cl(h_P^{-1}(K)) \subseteq h_P^{-1}(\mathcal{PF}\mathfrak{N}\mathfrak{N}cl(K))$ for every pfs K in $(U_2, \tau_{\mathcal{P}}(A_2))$.

Theorem 4.3 Let $h_P: (U_1, \tau_{\mathcal{P}}(A_1)) \to (U_2, \tau_{\mathcal{P}}(A_2))$ be a $\mathcal{PF}\mathfrak{N}\delta\beta CHom$. Then $\mathcal{PF}\mathfrak{N}\delta\beta cl(h_P^{-1}(K)) = h_P^{-1}(\mathcal{PF}\mathfrak{N}\delta\beta cl(K))$ for each $\mathcal{PF}\mathfrak{N}s$ K in $(U_2, \tau_{\mathcal{P}}(A_2))$.

Proof. Since h_P is a $\mathcal{PF}\mathfrak{N}\delta\beta CHom$, h_P is a $\mathcal{PF}\mathfrak{N}\delta\beta Irr$ mapping. Let K be a pfs in $(U_2, \tau_{\mathcal{P}}(A_2))$. Clearly, $\mathcal{PF}\mathfrak{N}\delta\beta cl(K)$ is a $\mathcal{PF}\mathfrak{N}\delta\beta cs$ in $(U_2, \tau_{\mathcal{P}}(A_2))$. Then $\mathcal{PF}\mathfrak{N}\delta\beta cl(K)$ is a $\mathcal{PF}\mathfrak{N}\delta\beta cs$ in $(U_2, \tau_{\mathcal{P}}(A_2))$. Since $h_P^{-1}(K) \subseteq h_P^{-1}(\mathcal{PF}\mathfrak{N}\delta\beta cl(K))$, then $\mathcal{PF}\mathfrak{N}\delta\beta cl(h_P^{-1}(K)) \subseteq \mathcal{PF}\mathfrak{N}\delta\beta cl(h_P^{-1}(\mathcal{PF}\mathfrak{N}\delta\beta cl(K))) = h_P^{-1}(\mathcal{PF}\mathfrak{N}\delta\beta cl(K))$. Therefore, $\mathcal{PF}\mathfrak{N}\ \delta\beta cl(h_P^{-1}(K)) \subseteq$

$h_P^{-1}(\mathcal{PF}\mathfrak{N}\delta\beta cl(K))$. Let h_P be a $\mathcal{PF}\mathfrak{N}\delta\beta CHom$. h_P^{-1} is a $\mathcal{PF}\mathfrak{N}\delta\beta Irr$ mapping. Let us consider pfs $h_P^{-1}(K)$ in $(U_1, \tau_{\mathcal{P}}(A_1))$, which implies $\mathcal{PF}\mathfrak{N}\delta\beta cl(h_P^{-1}(K))$ is a $\mathcal{PF}\mathfrak{N}\delta\beta cs$ in $(U_1, \tau_{\mathcal{P}}(A_1))$. Hence, $\mathcal{PF}\mathfrak{N}\delta\beta cl(h_P^{-1}(K))$ is a $\mathcal{PF}\mathfrak{N}\delta\beta cs$ in $(U_1, \tau_{\mathcal{P}}(A_1))$. This implies that $(h_P^{-1})^{-1}(\mathcal{PF}\mathfrak{N}\delta\beta cl(h_P^{-1}(K))) = h_P(\mathcal{PF}\mathfrak{N}\delta\beta cl(h_P^{-1}(K)))$ is a $\mathcal{PF}\mathfrak{N}\delta\beta cs$ in $(U_2, \tau_{\mathcal{P}}(A_2))$. This proves $K = (h_P^{-1})^{-1}(h_P^{-1}(K)) \subseteq (h_P^{-1})^{-1}(\mathcal{PF}\mathfrak{N}\delta\beta cl(h_P^{-1}(K))) = h_P(\mathcal{PF}\mathfrak{N}\delta\beta cl(h_P^{-1}(K)))$. Therefore, $\mathcal{PF}\mathfrak{N}\delta\beta cl(K) \subseteq \mathcal{PF}\mathfrak{N}\delta\beta cl(h_p(\mathcal{PF}\mathfrak{N}\delta\beta cl(h_P^{-1}(K)))) = h_P(\mathcal{PF}\mathfrak{N}\delta\beta cl(h_P^{-1}(K)))$, since h_P^{-1} is a $\mathcal{PF}\mathfrak{N}\delta\beta Irr$ mapping. Hence, $h_P^{-1}(\mathcal{PF}\mathfrak{N}\delta\beta cl(K)) \subseteq h_P^{-1}(h_P(\mathcal{PF}\mathfrak{N}\delta\beta cl(h_P^{-1}(K)))) = \mathcal{PF}\mathfrak{N}\delta\beta cl(h_P^{-1}(K))$. That is, $h_P^{-1}(\mathcal{PF}\mathfrak{N}\delta\beta cl(K)) \subseteq \mathcal{PF}\mathfrak{N}\delta\beta cl(h_P^{-1}(K))$. Hence, $\mathcal{PF}\mathfrak{N}\delta\beta cl(h_P^{-1}(K)) = h_P^{-1}(\mathcal{PF}\mathfrak{N}\delta\beta cl(K))$.

Remark 4.1 Theorems 4.2 and 4.3 are also true if h_P is a $\mathcal{PF}\mathfrak{N}\mathfrak{N}CHom$ (resp. $\mathcal{PF}\mathfrak{N}\delta CHom$, $\mathcal{PF}\mathfrak{N}\delta\alpha CHom$, $\mathcal{PF}\mathfrak{N}\delta\mathcal{S}CHom$ & $\mathcal{PF}\mathfrak{N}\delta\mathcal{P}CHom$).

Theorem 4.4 If $h_P: (U_1, \tau_{\mathcal{P}}(A_1)) \to (U_2, \tau_{\mathcal{P}}(A_2))$ and $g_P: (U_2, \tau_{\mathcal{P}}(A_2)) \to (U_3, \tau_{\mathcal{P}}(A_3))$ are $\mathcal{PF}\mathfrak{N}CHom$ (resp. $\mathcal{PF}\mathfrak{N}\delta\ CHom$, $\mathcal{PF}\mathfrak{N}\delta\alpha CHom$, $\mathcal{PF}\mathfrak{N}\delta\mathcal{S}CHom$, $\mathcal{PF}\mathfrak{N}\delta\mathcal{P}CHom$ & $\mathcal{PF}\mathfrak{N}\delta\beta CHom$ or $\mathcal{PF}\mathfrak{N}e^*CHom$)'s, then $g_P \circ h_P$ is a $\mathcal{PF}\mathfrak{N}CHom$ (resp. $\mathcal{PF}\mathfrak{N}\delta CHom$, $\mathcal{PF}\mathfrak{N}\delta\alpha CHom$, $\mathcal{PF}\mathfrak{N}\delta\mathcal{S}CHom$, $\mathcal{PF}\mathfrak{N}\delta\mathcal{P}CHom$ & $\mathcal{PF}\mathfrak{N}\delta\beta CHom$ or $\mathcal{PF}\mathfrak{N}e^*CHom$).

Proof. Let h_P and g_P be two $\mathcal{PF}\mathfrak{N}\delta\beta CHom$'s. Assume K is a $\mathcal{PF}\mathfrak{N}\delta\beta cs$ in $(U_3, \tau_{\mathcal{P}}(A_3))$. Then, $g_P^{-1}(K)$ is a $\mathcal{PF}\mathfrak{N}\delta\beta cs$ in $(U_2, \tau_{\mathcal{P}}(A_2))$. Then, by hypothesis, $h_P^{-1}(g_P^{-1}(K))$ is a $\mathcal{PF}\mathfrak{N}\delta\beta cs$ in $(U_1, \tau_{\mathcal{P}}(A_1))$. Hence, $g_P \circ h_P$ is a $\mathcal{PF}\mathfrak{N}\delta\beta Irr$ mapping. Now, let K be a $\mathcal{PF}\mathfrak{N}\delta\beta cs$ in $(U_1, \tau_{\mathcal{P}}(A_1))$. Then, by presumption, $h_P(K)$ is a $\mathcal{PF}\mathfrak{N}\delta\beta cs$ in $(U_2, \tau_{\mathcal{P}}(A_2))$. Then, by hypothesis, $g_P(h_P(K))$ is a $\mathcal{PF}\mathfrak{N}\delta\beta cs$ in $(U_3, \tau_{\mathcal{P}}(A_3))$. This implies that $g_P \circ h_P$ is a $\mathcal{PF}\mathfrak{N}\delta\beta Irr$ mapping. Hence, $g_P \circ h_P$ is a $\mathcal{PF}\mathfrak{N}\delta\beta CHom$. The proof of other cases are similar.

5 Application

Now we are riding the surf of Artificial Intelligence, we are surfing more deeper and deeper into it in the search of replacing the human by the machine. The machines are programmed to perform as the human beings which mean the have to analysis the available data and take the decisions by itself. Pattern recognization is the one of the way of analysis and there are n number of methods are available for pattern recognization. Similarity measure is the mathematical method of measuring the distance between the data so that the datas can be classified easily. Here we have five online platforms for buying the electronic good namely platform 1, platform 2, platform 3, platform 4 and platform 5. Each of the platform have the different ratings for the attributes: performance, Durability, cost and user friendly feature of the electronic good. Now the reviews are non probabilistic vagueness based on the consumers expectation and satisfaction / disappointment. The satisfactions are reviewed as likes and the disappointments are reviewed as dislikes. This circumstances can be converted into Pythagorean fuzzy sets by likes are taken into account of membership grades and dislikes are taken into account of non membership grades. Now the decision is to classifying the similar platforms which considers all the attributes of the electronic good.

Here the decision is to made as whether any variations in the inference of portray the electronic good between the platforms. This can be easily solved with the evidenced numerical values which are called as similarity measures.

5.1 Flow chart

Identify the pfs's of each platforms with all attributes.

↓

Identify all possible combinations of the platforms.

↓

Compute the proposed similarities between the available combinations

↓

Conclude, the pairs with maximum measure among the pfs as the similar platforms

Here we use the Chen & Chang similarity measure for classifying the platforms. the following table gives the pfs of the platforms according to the attributes of the electronic good.

Table 1 pfs of the platforms according to the attributes

	Performance	Durability	Cost	User friendly
	$< \mu(x), \lambda(x) >$	$< \mu(x), \lambda(x) >$	$< \mu(x), \lambda(x) >$	$< \mu(x), \lambda(x) >$
Platform1	$< 0.67, 0.54 >$	$< 0.66, 0.65 >$	$< 0.63, 0.62 >$	$< 0.62, 0.60 >$
Platform2	$< 0.55, 0.55 >$	$< 0.52, 0.63 >$	$< 0.56, 0.45 >$	$< 0.69, 0.62 >$
Platform3	$< 0.53, 0.51 >$	$< 0.91, 0.17 >$	$< 0.70, 0.60 >$	$< 0.70, 0.60 >$
Platform4	$< 0.88, 0.28 >$	$< 0.30, 0.90 >$	$< 0.71, 0.58 >$	$< 0.50, 0.50 >$
Platform5	$< 0.45, 0.73 >$	$< 0.38, 0.81 >$	$< 0.52, 0.67 >$	$< 0.48, 0.71 >$

clearly they are pfs's.

Table.2 shows the Chen and Chang Similarity measures of the combinations of the above plot forms.

Combinations of platforms	$S_{CC}(Platform_i, am_j)$
$(Platform_1, Platform_2)$	0.94
$(Platform_1, Platform_3)$	0.92
$(Platform_1, Platform_4)$	0.74
$(Platform_1, Platform_5)$	0.89
$(Platform_2, Platform_3)$	0.93
$(Platform_2, Platform_4)$	0.75
$(Platform_2, Platform_5)$	0.94
$(Platform_3, Platform_4)$	0.70
$(Platform_3, Platform_5)$	0.86
$(Platform_4, Platform_5)$	0.90

From the above table we observe that the platform1 1, platform 2 and platform 5 are the similar platforms. Hence out of five platforms there are three platforms portray that electronic goods as uniformly and the remaining platforms has variations in portray the same good.

6 Conclusion

Fuzzy topological spaces are the classical topological spaces which characterize the membership values alone. Intuitionistic fuzzy topological spaces portray the membership as well as the non-membership values. Pythagorean fuzzy topological spaces extend their arm to cover the missed ones of the intuitionistic fuzzy topological spaces. Pythagorean fuzzy nano topological spaces shorten the Pythagorean fuzzy sets of any cardinality into a tiny set which represents the same in nano approximation with boundary space. Our contribution to this area is the concepts of PFNδβ-Hom, PFNδβ-CHom, and PFNδβT1 2-space and we have derived some of their related characteristics. Finally, we applied a similarity measure to multiple criteria decision making with the help of Pythagorean fuzzy sets. In future, we will employ some similarity measures for comparing or decision making in the field of medical diagnosis and teaching learning process. We also take up this idea into the diverse fuzzy environment for real-world application purposes. Acknowledgement The corresponding author K. Shantha lakshmi would like to thank the editor and anonymous reviewers for their valuable suggestions that helped improve the quality of this work.

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