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Z-connectedness in Fermatean Fuzzy Topological Spaces

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Abstract

In this paper, we study the notion of Fermatean fuzzy (resp. δ, $\delta\mathcal{S}$, $\mathcal{P}$ and Z) connected respective disconnectedness in Fermatean fuzzy topological spaces. We also give some properties and theorems of such concepts with Fermatean fuzzy connected spaces. Also we provide a comprehensive study of Fermatean fuzzy separated sets specifically the δ, $\delta\mathcal{S}$, $\mathcal{P}$ and Z-separated sets within the framework of Fermatean fuzzy topological spaces. We establish several fundamental results concerning these separation concepts. Additionally, we introduce the notion of Fermatean fuzzy compact spaces, including their δ, $\delta\mathcal{S}$, $\mathcal{P}$ and Z-variants, and examine their core properties with respect to the corresponding open sets.

Keywords: Fermatean fuzzy open set, Fermatean fuzzy Z connected, Fermatean fuzzy Z disconnected.
AMS(2000) Subject classification: 03E72, 54A05, 54A40.

1 Introduction

Fuzzy sets were introduced by Zadeh in 1965 [25], laying the foundation for the mathematical treatment of imprecise and vague concepts that naturally occur in the real world. This development led to the emergence of several new branches in mathematics and found applications in diverse fields such as statistics, data processing, and linguistics. Since then, extensive research has been carried out in this area.

In 1968, Chang [6] introduced the concept of fuzzy topological spaces and generalized classical topological notions such as open sets, closed sets, continuity, and compactness to the fuzzy context. Later, Atanassov [1] introduced intuitionistic fuzzy sets, which further enhanced the capacity to model uncertainty. His work, along with contributions from other researchers [2, 5], advanced the study of intuitionistic fuzzy topological spaces, a topic first explored by Coker [7].

Subsequently, Yager [23] proposed a non-standard fuzzy set known as the Pythagorean fuzzy set, which was later extended to topological frameworks by Olgun et al. [12], who introduced the concept of Pythagorean fuzzy topological spaces.

In 2020, Senapati and Yager [15] introduced Fermatean fuzzy sets, which provide greater flexibility in handling uncertain information, especially in decision-making processes. They also defined basic operations on Fermatean fuzzy sets. Building on this, Hariwan Z. Ibrahim defined Fermatean fuzzy topological spaces and studied the continuity of functions between such spaces.

In our work, we extend these ideas by developing several stronger and weaker forms of Fermatean fuzzy open sets in Fermatean fuzzy topological spaces. We also investigate some of their fundamental properties, supported by illustrative examples.

Saha [14] introduced the notion of δ-open sets in fuzzy topological spaces. The concept of topological spaces was further studied by Pankajam et al. [13], and neutrosophic topological spaces were explored by Vadivel et al. [21]. Lellis Thivagar et al. [11] investigated new developments in neutrosophic topology, intuitionistic topology, and fuzzy topology. In 2011, El-Maghrabi and Al-Juhani [8] proposed the concept of MM-open sets in classical topological spaces and analyzed several of their properties. Padma et al. [16] also studied M-open sets in the context of topological spaces. Vadivel et al. [17, 18, 19] contributed to the study of various types of open sets in fuzzy and neutrosophic topological settings. Further developments on M-open sets in fuzzy and neutrosophic topological spaces were presented by Kalaiyarsan et al. [10] and Vadivel et al. [20]. Section 2 of this paper provides a brief review of fundamental definitions related to fuzzy sets fs's, intuitionistic fuzzy sets (IFS's), Pythagorean fuzzy sets (pfs's), and Fermatean fuzzy sets ($\mathfrak{F}\mathcal{F}s$'s).

Research Gap: No investigation on connectedness such as Fermatean fuzzy Z connected, Fermatean fuzzy Z disconnectedness in Fermatean fuzzy topological spaces has been reported in the Fermatean fuzzy literature.

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This paper introduces the concept of Fermatean fuzzy Z connected, Fermatean fuzzy Z disconnectedness in Fermatean fuzzy topological spaces. We also give some properties and theorems of such concepts in Fermatean fuzzy topological spaces.

2 Preliminaries

We recall some basic notions of fuzzy sets, IFS's, pfs's and $\mathfrak{F}\mathcal{F}s$'s.

Definition 2.1 [25] Let X be a nonempty set. A fuzzy set A in X is characterized by a membership function $\mu_A : X \to [0,1]$. That is:

$$\mu_A(x) = \begin{cases} 1, & \text{if} \quad x \in X \\ 0, & \text{if} \quad x \notin X \\ (0,1) & \text{if } x \text{ is partly in } X. \end{cases}$$

Alternatively, a fuzzy set A in X is an object having the form $A = \{< x, \mu_A(x) > | x \in X\}$ or $A = \left\{ \left\langle \frac{\mu_A(x)}{x} \right\rangle | x \in X \right\}$, where the function $\mu_A(x) : X \to [0,1]$ defines the degree of membership of the element, $x \in X$.

The closer the membership value $\mu_A(x)$ to 1, the more x belongs to A, where the grades 1 and 0 represent full membership and full nonmembership. Fuzzy set is a collection of objects with graded membership, that is, having degree of membership. Fuzzy set is an extension of the classical notion of set. In classical set theory, the membership of elements in a set is assessed in a binary terms according to a bivalent condition; an element either belongs or does not belong to the set. Classical bivalent sets are in fuzzy set theory called crisp sets. Fuzzy sets are generalized classical sets, since the indicator function of classical sets is special cases of the membership functions of fuzzy sets, if the latter only take values 0 or 1. Fuzzy sets theory permits the gradual assessment of the membership of element in a set; this is described with the aid of a membership function valued in the real unit interval $[0,1]$.

Let us consider two examples:

(i) all employees of XYZ who are over $1.8m$ in height; (ii) all employees of XYZ who are tall. The first example is a classical set with a universe (all XYZ employees) and a membership rule that divides the universe into members (those over $1.8m$) and nonmembers. The second example is a fuzzy set, because some employees are definitely in the set and some are definitely not in the set, but some are borderline.

This distinction between the ins, the outs, and the borderline is made more exact by the membership function, μ. If we return to our second example and let A represent the fuzzy set of all tall employees and x represent a member of the universe X (i.e. all employees), then $\mu_A(x)$ would be $\mu_A(x) = 1$ if x is definitely tall or $\mu_A(x) = 0$ if x is definitely not tall or $0 < \mu_A(x) < 1$ for borderline cases.

Definition 2.2 [1] The intuitionistic fuzzy sets are defined on a non-empty sets X as objects having the form $I = \{\langle x, \mu_I(x), \lambda_I(x)\rangle : x \in X\}$, where $\mu_I(x) : X \to [0,1]$ and $\lambda_I(x) : X \to [0,1]$ denote the degree of memebership and the degree of non-memebership of each element $x \in X$ to the set I, respectively, and $0 \le \mu_I(x) + \lambda_I(x) \le 1$, for all $x \in X$.

Definition 2.3 [1, 2, 3, 4] Let a nonempty set X be fixed. An IFS A in X is an object having the form: $A = \{< x, \mu_A(x), \lambda_A(x) > | x \in X\}$ or $A = \left\{ \left\langle \frac{\mu_A(x), \lambda_A(x)}{x} \right\rangle | x \in X \right\}$, where the functions $\mu_A(x) : X \to [0,1]$ and $\lambda_A(x) : X \to [0,1]$ define the degree of membership and the degree of nonmembership, respectively, of the element $x \in X$ to A, which is a subset of X, and for every $x \in X : 0 \le \mu_A(x) + \lambda_A(x) \le 1$. For each A in X: $\pi_A(x) = 1 - \mu_A(x) - \lambda_A(x)$ is the intuitionistic fuzzy set index or hesitation margin of x in X. The hesitation margin $\pi_A(x)$ is the degree of nondeterminacy of $x \in X$ to the set A and $\pi_A(x) \in [0,1]$. The hesitation margin is the function that expresses lack of knowledge of whether $x \in X$ or $x \notin X$. Thus: $\mu_A(x) + \lambda_A(x) + \pi_A(x) = 1$.

Example 2.1 Let $X = \{x, y, z\}$ be a fixed universe of discourse and $A = \left\{ \left\langle \frac{0.6, 0.1}{x} \right\rangle, \left\langle \frac{0.8, 0.1}{y} \right\rangle, \left\langle \frac{0.5, 0.3}{z} \right\rangle \right\}$, be the intuitionistic fuzzy set in X. The hesitation margins of the elements x, y, z to A are as follows: $\pi_A(x) = 0.3$, $\pi_A(y) = 0.1$ and $\pi_A(z) = 0.2$.

Definition 2.4 [22, 23, 24] Let X be a universal set. Then, a Pythagorean fuzzy set A, which is a set of ordered pairs over X, is defined by the following: $A = \{< x, \mu_A(x), \lambda_A(x) | x \in X\}$ or $A = \left\{ \left\langle \frac{\mu_A(x), \lambda_A(x)}{x} \right\rangle | x \in X \right\}$, where the functions $\mu_A(x) : X \to [0,1]$ and $\lambda_A(x) : X \to [0,1]$ define the degree of membership and the degree of nonmembership, respectively, of the element $x \in X$ to A, which is a subset of X, and for every $x \in X$, $0 \le (\mu_A(x))^2 + (\lambda_A(x))^2 \le 1$. Supposing $(\mu_A(x))^2 + (\lambda_A(x))^2 \le 1$, then there is a degree of indeterminacy of $x \in X$ to A defined by $\pi_A(x) = \sqrt{1 - [(\mu_A(x))^2 + (\lambda_A(x))^2]}$ and $\pi_A(x) \in [0,1]$. In what follows, $(\mu_A(x))^2 + (\lambda_A(x))^2 + (\pi_A(x))^2 = 1$. Otherwise, $\pi_A(x) = 0$ whenever $(\mu_A(x))^2 + (\lambda_A(x))^2 = 1$. We denote the set of all PFS's over X by $pfs(X)$.

Definition 2.5 [15] Let X be a universe of discourse. A Fermatean fuzzy set ($\mathfrak{F}\mathcal{F}s$) F in X is an object having the form $F = \{< x, \mu_F(x), \lambda_F(x) >: x \in X\}$ where $\mu_F(x) : X \to [0,1]$ and $\lambda_F(x) : X \to [0,1]$, including the condition $0 \leq (\mu_F(x))^3 + (\lambda_F(x))^3 \leq 1$, for all $x \in X$. The numbers $\mu_F(x)$ and $\lambda_F(x)$ denote, respectively, the degree of memebership and the degree of non-memebership of the element x in the set F. For any $\mathfrak{F}\mathcal{F}s$ F and $x \in X$, $\pi_F(x) = \sqrt[3]{1 - [(\mu_F(x))^3 - (\lambda_F(x))^3]}$ is identified as the degree of interminancy of x to F. In the interest of simplicity, we shall mention the symbol $F = (\mu_F, \lambda_F)$ for the $\mathfrak{F}\mathcal{F}s$ $F = \{< x, \mu_F(x), \lambda_F(x) : x \in X\}$.

Definition 2.6 [15] Let $F = (\mu_F, \lambda_F)$, $F_1 = (\mu_{F_1}, \lambda_{F_1})$ and $F_2 = (\mu_{F_2}, \lambda_{F_2})$, be three Fermatean fuzzy sets ($\mathfrak{F}\mathcal{F}s$'s), then their operations are defined as follows:

(i) $F_1 \cap F_2 = (min\{\mu_{F_1}, \mu_{F_2}\}, max\{\lambda_{F_1}, \lambda_{F_2}\})$.

(ii) $F_1 \cup F_2 = (max\{\mu_{F_1}, \mu_{F_2}\}, min\{\lambda_{F_1}, \lambda_{F_2}\})$.

(iii) $F^c = (\lambda_F, \mu_F)$.

Remark 2.1 If $\mu_{F_1} = \mu_{F_2}$ and $\lambda_{F_1} = \lambda_{F_2}$, then $F_1 = F_2$

Definition 2.7 [9] Let X be a non empty set and τ be a family of Fermatean fuzzy subsets of X. If

(i) $1_F, 0_F \in \tau$

(ii) for any $F_1, F_2 \in \tau$, we have $F_1 \cap F_2 \in \tau$,

(iii) for any $\{F_i\}_{i \in I} \subset \tau$, we have $\bigcup_{i \in I} F_i \in \tau$ where I is an arbitrary index set then τ is called a Fermatean fuzzy topology on X.
The pair (X, τ) is said to be a Fermatean fuzzy topological space. Each member of τ is called an Fermatean fuzzy open set. The complement of an Fermatean fuzzy open set is called a Fermatean fuzzy closed set.

Remark 2.2 [9] As any Intuitionistic fuzzy subset or Pythagorean fuzzy subset of a set can be considered as Fermatean fuzzy subset, we observe that any Intuitionstic fuzzy topological space or Pythagorean fuzzy topological space is a Fermatean fuzzy topological space as well. On the other hand, it is obvious that a Fermatean fuzzy topological space need not be Intuitionistic fuzzy topological space and Pythagorean fuzzy topological space. Even an Fermatean fuzzy open set maybe neither an Intuitionistic fuzzy set nor Pythagorean fuzzy set.

Example 2.2 [9] Let $X = \{c_1, c_2\}$. Consider the following family Fermatean fuzzy subsets $\tau = \{1_F, 0_F, F_1, F2\}$ where $F_1 = \{\langle c_1, \mu_{F_1}(c_1) = 0.4, \lambda_{F_1}(c_1) = 0.6\rangle, \langle c_2, \mu_{F_1}(c_2) = 0.1, \lambda_{F_1}(c_2) = 0.3\rangle\}$ and $F_2 = \{\langle c_1, \mu_{F_2}(c_1) = 0.9, \lambda_{F_2}(c_1) = 0.6\rangle, \langle c_2, \mu_{F_2}(c_2) = 0.2, \lambda_{F_2}(c_2) = 0.3\rangle\}$. Observe that (X, τ) is a Fermatean fuzzy topological space but (X, τ) is neither Intuitionistic fuzzy topological space nor Pythagorean fuzzy topological space.

Definition 2.8 [9] Let (X, τ) be an $\mathfrak{F}\mathcal{F}ts$ and $A = \{< a, \mu_A(a), \lambda_A(a) > | a \in X\}$ be an $\mathfrak{F}\mathcal{F}s$ in X. Then the Fermatean fuzzy interior and the Fermatean fuzzy closure of A are denoted by $\mathfrak{F}\mathcal{F}int(A)$ and $\mathfrak{F}\mathcal{F}cl(A)$ and are defined as follows: $\mathfrak{F}\mathcal{F}int(A) = \cup\{G | G$ is a $\mathfrak{F}\mathcal{F}os$ and $G \subseteq A\}$ and $\mathfrak{F}\mathcal{F}cl(A) = \cap\{K | K$ is a $\mathfrak{F}\mathcal{F}cs$ and $A \subseteq K\}$. Also, it can be established that $\mathfrak{F}\mathcal{F}cl(A)$ is an $\mathfrak{F}\mathcal{F}cs$ and $\mathfrak{F}\mathcal{F}int(A)$ is an $\mathfrak{F}\mathcal{F}os$, A is an $\mathfrak{F}\mathcal{F}cs$ if and only if $\mathfrak{F}\mathcal{F}cl(A) = A$ and A is an $\mathfrak{F}\mathcal{F}os$ if and only if $\mathfrak{F}\mathcal{F}int(A) = A$. We say that A is $\mathfrak{F}\mathcal{F}$-dense if $\mathfrak{F}\mathcal{F}cl(A) = 1_{\mathfrak{F}}$.

Lemma 2.1 [9] For any Fermatean fuzzy set A in (X, τ), we have $1_{\mathfrak{F}} - \mathfrak{F}\mathcal{F}int(A) = \mathfrak{F}\mathcal{F}cl(1_{\mathfrak{F}} - A)$ and $1_{\mathfrak{F}} - \mathfrak{F}\mathcal{F}cl(A) = \mathfrak{F}\mathcal{F}int(1_{\mathfrak{F}} - A)$.

3 Fermatean fuzzy (resp. δ, $\delta\mathcal{S}$, pre and Z)-connected spaces

Definition 3.1 A function $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ is said to be Fermatean fuzzy

(i) continuous (briefly, $\mathfrak{F}\mathcal{F}Cts$), if for each $\mathfrak{F}\mathcal{F}o$ set M of X_2, the set $h_{\mathfrak{F}}^{-1}(M)$ is $\mathfrak{F}\mathcal{F}o$ set of X_1.

(ii) δ continuous (briefly, $\mathfrak{F}\mathcal{F}\delta Cts$), if for each $\mathfrak{F}\mathcal{F}o$ set M of X_2, the set $h_{\mathfrak{F}}^{-1}(M)$ is $\mathfrak{F}\mathcal{F}\delta o$ set of X_1.

(iii) δ semi continuous (briefly, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Cts$), if for each $\mathfrak{F}\mathcal{F}o$ set M of X_2, the set $h_{\mathfrak{F}}^{-1}(M)$ is $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$ set of X_1.

(iv) pre continuous (briefly, $\mathfrak{F}\mathcal{F}\mathcal{P}Cts$), if for each $\mathfrak{F}\mathcal{F}o$ set M of X_2, the set $h_{\mathfrak{F}}^{-1}(M)$ is $\mathfrak{F}\mathcal{F}\mathcal{P}o$ set of X_1.

(v) Z continuous (briefly, $\mathfrak{F}\mathcal{F}ZCts$), if for each $\mathfrak{F}\mathcal{F}o$ set M of X_2, the set $h_{\mathfrak{F}}^{-1}(M)$ is $\mathfrak{F}\mathcal{F}Zo$ set of X_1.

Definition 3.2 A function $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ is called Fermatean fuzzy

(i) irresolute (briefly, $\mathfrak{F}\mathcal{F}ZIrr$) function, if for each $\mathfrak{F}\mathcal{F}o$ subset M of X_2, the set $h_{\mathfrak{F}}^{-1}(M)$ is $\mathfrak{F}\mathcal{F}\mathcal{S}o$ subset of X_1.

(ii) pre irresolute (briefly, $\mathfrak{F}\mathcal{F}\mathcal{P}Irr$) function, if for each $\mathfrak{F}\mathcal{F}\mathcal{P}o$ subset M of X_2, the set $h_{\mathfrak{F}}^{-1}(M)$ is $\mathfrak{F}\mathcal{F}\mathcal{P}o$ subset of X_1.

(iii) δ semi irresolute (briefly, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Irr$) function, if for each $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$ subset M of X_2, the set $h_{\mathfrak{F}}^{-1}(M)$ is $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$ subset of X_1.

(iv) Z irresolute (briefly, $\mathfrak{F}\mathcal{F}ZIrr$) function, if for each $\mathfrak{F}\mathcal{F}Zo$ subset M of X_2, the set $h_{\mathfrak{F}}^{-1}(M)$ is $\mathfrak{F}\mathcal{F}Zo$ subset of X_1.

Definition 3.3 Let (X, τ) be a $\mathfrak{F}\mathcal{F}ts$. Then (X, τ) is Fermatean fuzzy (resp. δ, $\delta\mathcal{S}$, pre and Z) disconnected (briefly, $\mathfrak{F}\mathcal{F}DCon$ (resp. $\mathfrak{F}\mathcal{F}\delta DCon$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}DCon$, $\mathfrak{F}\mathcal{F}preDCon$ and $\mathfrak{F}\mathcal{F}ZDCon$)) if there exists $\mathfrak{F}\mathcal{F}o$ (resp. $\mathfrak{F}\mathcal{F}\delta o$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$, $\mathfrak{F}\mathcal{F}\mathcal{P}o$ and $\mathfrak{F}\mathcal{F}Zo$) sets A, B in X, $A \neq 0_{\mathfrak{F}}$, $B \neq 0_{\mathfrak{F}} \ni A \cup B = 1_{\mathfrak{F}}$ and $A \cap B = 0_{\mathfrak{F}}$. That is, $\mu_A \vee \mu_B = 1$ and $\lambda_A \wedge \lambda_B = 0$; $\mu_A \wedge \mu_B = 0$ and $\lambda_A \vee \lambda_B = 1$.

If X is not $\mathfrak{F}\mathcal{F}DCon$ (resp. $\mathfrak{F}\mathcal{F}\delta DCon$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}DCon$, $\mathfrak{F}\mathcal{F}\mathcal{P}DCon$ and $\mathfrak{F}\mathcal{F}ZDCon$) then it is said to be Fermatean fuzzy (resp. δ, $\delta\mathcal{S}$, $\mathcal{P}$ and Z) connected (briefly, $\mathfrak{F}\mathcal{F}Con$ (resp. $\mathfrak{F}\mathcal{F}\delta Con$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Con$, $\mathfrak{F}\mathcal{F}\mathcal{P}Con$ and $\mathfrak{F}\mathcal{F}ZCon$)).

Example 3.1 Let $X = \{a, b\}$ and the $\mathfrak{F}\mathcal{F}s$'s A_1, A_2, A_3 and A_4 are defined as
$\mu_{A_1}(a) = 0.2,\ \lambda_{A_1}(a) = 0.8,$
$\mu_{A_1}(b) = 0.4,\ \lambda_{A_1}(b) = 0.6;$
$\mu_{A_2}(a) = 0.1,\ \lambda_{A_2}(a) = 0.9,$
$\mu_{A_2}(b) = 0.3,\ \lambda_{A_2}(b) = 0.7;$
$\mu_{A_3}(a) = 0.9,\ \lambda_{A_3}(a) = 0.1,$
$\mu_{A_3}(b) = 0.7,\ \lambda_{A_2}(b) = 0.3;$
$\mu_{A_4}(a) = 0.2,\ \lambda_{A_4}(a) = 0.8,$
$\mu_{A_4}(b) = 0.3,\ \lambda_{A_4}(b) = 0.7;$

Let $\tau = \{0_{\mathfrak{F}},\ 1_{\mathfrak{F}},\ A_1,\ A_2,\ A_3,\ A_4\}$ be a $\mathfrak{F}\mathcal{F}ts$ on X.
Let A_1 and A_2 are $\mathfrak{F}\mathcal{F}o$ (resp. $\mathfrak{F}\mathcal{F}\delta o$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$, $\mathfrak{F}\mathcal{F}\mathcal{P}o$ and $\mathfrak{F}\mathcal{F}Zo$) sets. Then X is $\mathfrak{F}\mathcal{F}Con$ (resp. $\mathfrak{F}\mathcal{F}\delta Con$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Con$, $\mathfrak{F}\mathcal{F}\mathcal{P}Con$ and $\mathfrak{F}\mathcal{F}ZCon$).

Example 3.2 In Example 3.1, Let A_3 and A_4 are $\mathfrak{F}\mathcal{F}Zo$ sets. Then X is $\mathfrak{F}\mathcal{F}ZDCon$.

Definition 3.4 Let (X, τ) be a $\mathfrak{F}\mathcal{F}ts$. Let S be a Fermatean fuzzy subset of X.

(a) If there exists $\mathfrak{F}\mathcal{F}o$ (resp. $\mathfrak{F}\mathcal{F}\delta o$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$, $\mathfrak{F}\mathcal{F}\mathcal{P}o$ and $\mathfrak{F}\mathcal{F}Zo$) sets L and M in X satisfying the following properties, then S is called Fermatean fuzzy (resp. δ, $\delta\mathcal{S}$, $\mathcal{P}$ and Z) C_i-disconnected (briefly, $\mathfrak{F}\mathcal{F}C_iDCon$ (resp. $\mathfrak{F}\mathcal{F}\delta C_iDCon$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}C_iDCon$, $\mathfrak{F}\mathcal{F}\mathcal{P}C_iDCon$ and $\mathfrak{F}\mathcal{F}ZC_iDCon$)) $(i = 1, 2, 3, 4)$:

$C_1 :\ S \subseteq L \cup M, L \cap M \subseteq S^c, S \cap L \neq 0_{\mathfrak{F}}, S \cap M \neq 0_{\mathfrak{F}}.$

$C_2 :\ S \subseteq L \cup M, S \cap L \cap M = 0_{\mathfrak{F}}, S \cap L \neq 0_{\mathfrak{F}}, S \cap M \neq 0_{\mathfrak{F}}.$

$C_3 :\ S \subseteq L \cup M, L \cap M \subseteq S^c, L \nsubseteq S^c, M \nsubseteq S^c.$

$C_4 :\ S \subseteq L \cup M, S \cap L \cap M = 0_{\mathfrak{F}}, L \nsubseteq S^c, M \nsubseteq S^c.$

(b) S is said to be Fermatean fuzzy (resp. δ, $\delta\mathcal{S}$, $\mathcal{P}$ and Z) C_i-connected (briefly, $\mathfrak{F}\mathcal{F}C_iCon$ (resp. $\mathfrak{F}\mathcal{F}\delta C_iCon$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}C_iCon$, $\mathfrak{F}\mathcal{F}\mathcal{P}C_iCon$ and $\mathfrak{F}\mathcal{F}ZC_iCon$)), $(i = 1, 2, 3, 4)$ if N is not $\mathfrak{F}\mathcal{F}C_iDCon$ (resp. $\mathfrak{F}\mathcal{F}\delta C_iDCon$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}C_iDCon$, $\mathfrak{F}\mathcal{F}\mathcal{P}C_iDCon$ and $\mathfrak{F}\mathcal{F}ZC_iDCon$), $(i = 1, 2, 3, 4)$. Obviously, we can obtain the following implications between several types of $\mathfrak{F}\mathcal{F}C_iCon$ (resp. $\mathfrak{F}\mathcal{F}\delta C_iCon$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}C_iCon$, $\mathfrak{F}\mathcal{F}\mathcal{P}C_iCon$ and $\mathfrak{F}\mathcal{F}ZC_iCon$), $(i = 1, 2, 3, 4)$.

(1) $\mathfrak{F}\mathcal{F}C_1Con$ (resp. $\mathfrak{F}\mathcal{F}\delta C_1Con$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}C_1Con$, $\mathfrak{F}\mathcal{F}\mathcal{P}C_1Con$ and $\mathfrak{F}\mathcal{F}ZC_1Con$) $\Rightarrow$ $\mathfrak{F}\mathcal{F}C_2Con$ (resp. $\mathfrak{F}\mathcal{F}\delta C_2Con$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}C_2Con$, $\mathfrak{F}\mathcal{F}\mathcal{P}C_2Con$ and $\mathfrak{F}\mathcal{F}ZC_2Con$).

(2) $\mathfrak{F}\mathcal{F}C_1Con$ (resp. $\mathfrak{F}\mathcal{F}\delta C_1Con$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}C_1Con$, $\mathfrak{F}\mathcal{F}\mathcal{P}C_1Con$ and $\mathfrak{F}\mathcal{F}ZC_1Con$) $\Rightarrow$ $\mathfrak{F}\mathcal{F}C_3Con$ (resp. $\mathfrak{F}\mathcal{F}\delta C_3Con$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}C_3Con$, $\mathfrak{F}\mathcal{F}\mathcal{P}C_3Con$ and $\mathfrak{F}\mathcal{F}ZC_3Con$).

(3) $\mathfrak{F}\mathcal{F}C_3Con$ (resp. $\mathfrak{F}\mathcal{F}\delta C_3Con$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}C_3Con$, $\mathfrak{F}\mathcal{F}\mathcal{P}C_3Con$ and $\mathfrak{F}\mathcal{F}ZC_3Con$) $\Rightarrow$ $\mathfrak{F}\mathcal{F}C_4Con$ (resp. $\mathfrak{F}\mathcal{F}\delta C_4Con$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}C_4Con$, $\mathfrak{F}\mathcal{F}\mathcal{P}C_4Con$ and $\mathfrak{F}\mathcal{F}ZC_4Con$).

(4) $\mathfrak{F}\mathcal{F}C_2Con$ (resp. $\mathfrak{F}\mathcal{F}\delta C_2Con$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}C_2Con$, $\mathfrak{F}\mathcal{F}\mathcal{P}C_2Con$ and $\mathfrak{F}\mathcal{F}ZC_2Con$) $\Rightarrow$ $\mathfrak{F}\mathcal{F}C_4Con$ (resp. $\mathfrak{F}\mathcal{F}\delta C_4Con$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}C_4Con$, $\mathfrak{F}\mathcal{F}\mathcal{P}C_4Con$ and $\mathfrak{F}\mathcal{F}ZC_4Con$).

Example 3.3 In Example 3.1, Let $S = A_2$. If $L = A_1$ and $M = A_3$ are $\mathfrak{F}\mathcal{F}Zo$ sets, then S is

(i) $\mathfrak{F}\mathcal{F}ZC_2Con$ but not $\mathfrak{F}\mathcal{F}ZC_1Con$.

(ii) $\mathfrak{F}\mathcal{F}ZC_3Con$ but not $\mathfrak{F}\mathcal{F}ZC_1Con$.

(iii) $\mathfrak{F}\mathcal{F}ZC_4Con$ but not $\mathfrak{F}\mathcal{F}ZC_1Con$.

Example 3.4 *????* Let $X = \{a, b\}$ and the $\mathfrak{F}\mathcal{F}s$'s A_1, A_2, A_3, A_4, A_5 and A_6 are defined as
$\mu_{A_1}(a) = 0.2,\ \lambda_{A_1}(a) = 0.8,$
$\mu_{A_1}(b) = 0.4,\ \lambda_{A_1}(b) = 0.6;$
$\mu_{A_2}(a) = 0.1,\ \lambda_{A_2}(a) = 0.9,$
$\mu_{A_2}(b) = 0.3,\ \lambda_{A_2}(b) = 0.7;$
$\mu_{A_3}(a) = 1,\ \lambda_{A_3}(a) = 0,$
$\mu_{A_3}(b) = 0,\ \lambda_{A_2}(b) = 1;$
$\mu_{A_4}(a) = 0,\ \lambda_{A_4}(a) = 1,$
$\mu_{A_4}(b) = 1,\ \lambda_{A_4}(b) = 0;$
$\mu_{A_5}(a) = 0.1,\ \lambda_{A_5}(a) = 1,$
$\mu_{A_5}(b) = 0.3,\ \lambda_{A_4}(b) = 0.4;$
$\mu_{A_6}(a) = 0.2,\ \lambda_{A_6}(a) = 0.8,$
$\mu_{A_6}(b) = 0.6,\ \lambda_{A_6}(b) = 0.4;$

Let $\tau = \{0_{\mathfrak{F}},\ 1_{\mathfrak{F}},\ A_1,\ A_2\}$ be a $\mathfrak{F}\mathcal{F}ts$ on X.
Let $S = A_5$. If $L = A_3$ and $M = A_6$ are $\mathfrak{F}\mathcal{F}Mo$ sets, then S is $\mathfrak{F}\mathcal{F}MC_4Con$ but not $\mathfrak{F}\mathcal{F}MC_3Con$.

Definition 3.5 Let (X, τ) be a $\mathfrak{F}\mathcal{F}ts$ is Fermatean fuzzy (resp. δ, $\delta\mathcal{S}$, $\mathcal{P}$ and Z) C_5-disconnected (briefly, $\mathfrak{F}\mathcal{F}C_5DCon$ (resp. $\mathfrak{F}\mathcal{F}\delta C_5DCon$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}C_5DCon$, $\mathfrak{F}\mathcal{F}\mathcal{P}C_5DCon$ and $\mathfrak{F}\mathcal{F}ZC_5DCon$)) if there exists Fermatean fuzzy subset S in X which is both $\mathfrak{F}\mathcal{F}o$ (resp. $\mathfrak{F}\mathcal{F}\delta o$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$, $\mathfrak{F}\mathcal{F}\mathcal{P}o$ and $\mathfrak{F}\mathcal{F}Zo$) and $\mathfrak{F}\mathcal{F}c$ (resp. $\mathfrak{F}\mathcal{F}\delta c$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}c$, $\mathfrak{F}\mathcal{F}\mathcal{P}c$ and $\mathfrak{F}\mathcal{F}Zc$) in X, such that $S \neq 0_{\mathfrak{F}}$, $S \neq 1_{\mathfrak{F}}$. If X is not $\mathfrak{F}\mathcal{F}C_5DCon$ (resp. $\mathfrak{F}\mathcal{F}\delta C_5DCon$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}C_5DCon$, $\mathfrak{F}\mathcal{F}\mathcal{P}C_5DCon$ and $\mathfrak{F}\mathcal{F}ZC_5DCon$) then it is said to be Fermatean fuzzy (resp. δ, $\delta\mathcal{S}$, $\mathcal{P}$ and Z) C_5-connected (briefly, $\mathfrak{F}\mathcal{F}C_5Con$ (resp. $\mathfrak{F}\mathcal{F}\delta C_5Con$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}C_5Con$, $\mathfrak{F}\mathcal{F}\mathcal{P}C_5Con$ and $\mathfrak{F}\mathcal{F}ZC_5Con$)).

Example 3.5 In Example 3.1, Let $S = A_3$ is $\mathfrak{F}\mathcal{F}C_5DCon$ (resp. $\mathfrak{F}\mathcal{F}\delta C_5DCon$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}C_5DCon$, $\mathfrak{F}\mathcal{F}\mathcal{P}C_5DCon$ and $\mathfrak{F}\mathcal{F}ZC_5DCon$).

Theorem 3.1 $\mathfrak{F}\mathcal{F}C_5Con$ (resp. $\mathfrak{F}\mathcal{F}\delta C_5Con$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}C_5Con$, $\mathfrak{F}\mathcal{F}\mathcal{P}C_5Con$ and $\mathfrak{F}\mathcal{F}ZC_5Con$)-ness implies $\mathfrak{F}\mathcal{F}Con$ (resp. $\mathfrak{F}\mathcal{F}\delta Con$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Con$, $\mathfrak{F}\mathcal{F}\mathcal{P}Con$ and $\mathfrak{F}\mathcal{F}ZCon$)-ness.

Proof. Suppose that there exists non-empty $\mathfrak{F}\mathcal{F}Zo$ sets A & $B \ni A \cup B = 1_{\mathfrak{F}}$ and $A \cap B = 0_{\mathfrak{F}}$. In this case we have $\mu_A \vee \mu_B = 1$, $\lambda_A \wedge \lambda_B = 0$ and $\mu_A \wedge \mu_B = 0$, $\lambda_A \vee \lambda_B = 1$. Let $L = \{x \in U : \mu_A(x) > 0\}$, $M = \{x \in U : \mu_A(x) = 0\}$. If $x \in L$, then $\mu_A(x) > 0 \Rightarrow \mu_B(x) = 0 \Rightarrow \mu_A(x) = 1 \Rightarrow \lambda_A(x) = 0 \Rightarrow \lambda_B(x) = 1$. If $x \in M$, then $\mu_B(x) = 1 \Rightarrow \lambda_B(x) = 0$. Hence $\mu_A = \lambda_B$ and $\lambda_A = \mu_B$, that is $B = A^c$. Therefore A is both $\mathfrak{F}\mathcal{F}Zo$ and $\mathfrak{F}\mathcal{F}Zc$ set, and since $B \neq 0_{\mathfrak{F}} \Rightarrow A \neq 1_{\mathfrak{F}}$, $0_{\mathfrak{F}} \neq A \neq 1_{\mathfrak{F}}$. Thus (X, τ) is $\mathfrak{F}\mathcal{F}ZC_5DCon$. Similar situations exist in other scenarios. ■

However, as the following example demonstrates, the opposite may not be true.

Example 3.6 In Example 3.1, Let A_1 and A_3 are $\mathfrak{F}\mathcal{F}Zo$ sets. Then X is $\mathfrak{F}\mathcal{F}ZCon$ but not $\mathfrak{F}\mathcal{F}ZC_5Con$.

Theorem 3.2 Let $h_{\mathfrak{F}} : (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ be a $\mathfrak{F}\mathcal{F}Cts$ (resp. $\mathfrak{F}\mathcal{F}\delta Cts$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Cts$, $\mathfrak{F}\mathcal{F}\mathcal{P}Cts$ and $\mathfrak{F}\mathcal{F}ZCts$) surjection. If X_1 is $\mathfrak{F}\mathcal{F}Con$ (resp. $\mathfrak{F}\mathcal{F}\delta Con$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Con$, $\mathfrak{F}\mathcal{F}\mathcal{P}Con$ and $\mathfrak{F}\mathcal{F}ZCon$), then so is X_2.

Proof. Assume that X_2 is not $\mathfrak{F}\mathcal{F}ZCon$, then there exists nonempty $\mathfrak{F}\mathcal{F}o$ sets L & Z in $X_2 \ni L \cup M = 1_{\mathfrak{F}}$ and $L \cap M = 0_{\mathfrak{F}}$. Since $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZCts$ mapping, $A = h_{\mathfrak{F}}^{-1}(L) \neq 0_{\mathfrak{F}}$, $B = h_{\mathfrak{F}}^{-1}(M) \neq 0_{\mathfrak{F}}$, which are $\mathfrak{F}\mathcal{F}Zo$ sets in X_1 and $h_{\mathfrak{F}}^{-1}(L) \cup h_{\mathfrak{F}}^{-1}(M) = h_{\mathfrak{F}}^{-1}(1_{\mathfrak{F}}) = 1_{\mathfrak{F}}$, which implies $A \cup B = 1_{\mathfrak{F}}$. Also $h_{\mathfrak{F}}^{-1}(L) \cap h_{\mathfrak{F}}^{-1}(M) = h_{\mathfrak{F}}^{-1}(0_{\mathfrak{F}}) = 0_{\mathfrak{F}}$, which implies $A \cap B = 0_{\mathfrak{F}}$. Thus X_1 is $\mathfrak{F}\mathcal{F}ZDCon$, which is a contradiction to our hypothesis. Hence X_2 is $\mathfrak{F}\mathcal{F}ZCon$. ■

Corollary 3.1 Let $h_{\mathfrak{F}} : (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ be a $\mathfrak{F}\mathcal{F}Cts$ (resp. $\mathfrak{F}\mathcal{F}\delta Cts$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Cts$, $\mathfrak{F}\mathcal{F}\mathcal{P}Cts$ and $\mathfrak{F}\mathcal{F}ZCts$) surjection. If X_1 is $\mathfrak{F}\mathcal{F}C_5Con$ (resp. $\mathfrak{F}\mathcal{F}\delta C_5Con$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}C_5Con$, $\mathfrak{F}\mathcal{F}\mathcal{P}C_5Con$ and $\mathfrak{F}\mathcal{F}ZC_5Con$), then so is X_2.

Theorem 3.3 Let $h_{\mathfrak{F}} : (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ be a $\mathfrak{F}\mathcal{F}Irr$ (resp. $\mathfrak{F}\mathcal{F}\delta Irr$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Irr$, $\mathfrak{F}\mathcal{F}\mathcal{P}Irr$ and $\mathfrak{F}\mathcal{F}ZIrr$) surjection. If X_1 is $\mathfrak{F}\mathcal{F}Con$ (resp. $\mathfrak{F}\mathcal{F}\delta Con$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Con$, $\mathfrak{F}\mathcal{F}\mathcal{P}Con$ and $\mathfrak{F}\mathcal{F}ZCon$), then so is X_2.

Proof. Assume that X_2 is not $\mathfrak{F}\mathcal{F}ZCon$, then there exists nonempty $\mathfrak{F}\mathcal{F}Zo$ sets L & Z in $X_2 \ni L \cup M = 1_{\mathfrak{F}}$ and $L \cap M = 0_{\mathfrak{F}}$. Since $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZIrr$ mapping, $A = h_{\mathfrak{F}}^{-1}(L) \neq 0_{\mathfrak{F}}$, $B = h_{\mathfrak{F}}^{-1}(M) \neq 0_{\mathfrak{F}}$, which are $\mathfrak{F}\mathcal{F}Zo$ sets in X_1 and $h_{\mathfrak{F}}^{-1}(L) \cup h_{\mathfrak{F}}^{-1}(M) = h_{\mathfrak{F}}^{-1}(1_{\mathfrak{F}}) = 1_{\mathfrak{F}}$, which implies $A \cup B = 1_{\mathfrak{F}}$. Also $h_{\mathfrak{F}}^{-1}(L) \cap h_{\mathfrak{F}}^{-1}(M) = h_{\mathfrak{F}}^{-1}(0_{\mathfrak{F}}) = 0_{\mathfrak{F}}$, which implies $A \cap B = 0_{\mathfrak{F}}$. Thus X_1 is $\mathfrak{F}\mathcal{F}ZDCon$, which is a contradiction to our hypothesis. Hence X_2 is $\mathfrak{F}\mathcal{F}ZCon$.

Similar situations exist in other scenarios. ■

Corollary 3.2 Let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be a $\mathfrak{F}\mathcal{F}Irr$ (resp. $\mathfrak{F}\mathcal{F}\delta Irr$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Irr$, $\mathfrak{F}\mathcal{F}\mathcal{P}Irr$ and $\mathfrak{F}\mathcal{F}ZIrr$) surjection. If X_1 is $\mathfrak{F}\mathcal{F}C_5Con$ (resp. $\mathfrak{F}\mathcal{F}\delta C_5Con$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}C_5Con$, $\mathfrak{F}\mathcal{F}\mathcal{P}C_5Con$ and $\mathfrak{F}\mathcal{F}ZC_5Con$), then so is X_2.

Theorem 3.4 Let (X, τ) be a $\mathfrak{F}\mathcal{F}ts$ is $\mathfrak{F}\mathcal{F}C_5Con$ ((resp. $\mathfrak{F}\mathcal{F}\delta C_5Con$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}C_5Con$, $\mathfrak{F}\mathcal{F}\mathcal{P}C_5Con$ and $\mathfrak{F}\mathcal{F}ZC_5Con$) if and only if there exists no nonempty $\mathfrak{F}\mathcal{F}o$ (resp. $\mathfrak{F}\mathcal{F}\delta o$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$, $\mathfrak{F}\mathcal{F}\mathcal{P}o$ and $\mathfrak{F}\mathcal{F}Zo$) sets L and Z in X such that $L = M^c$.

Proof. Suppose that L and Z are $\mathfrak{F}\mathcal{F}Zo$ sets in X such that $L \neq 0_{\mathfrak{F}}$, $M \neq 0_{\mathfrak{F}}$ and $L = M^c$. Since $L = M^c$, M^c is a $\mathfrak{F}\mathcal{F}Zo$ sets and Z is $\mathfrak{F}\mathcal{F}Zc$ set and $L \neq 0_{\mathfrak{F}}$ implies $M \neq 1_{\mathfrak{F}}$. But this is a contradiction to the fact that X is $\mathfrak{F}\mathcal{F}ZC_5Con$.

Conversely, let L be a both $\mathfrak{F}\mathcal{F}Zo$ set and Z is $\mathfrak{F}\mathcal{F}Zc$ set in X such that $L \neq 0_{\mathfrak{F}}$, $L \neq 1_{\mathfrak{F}}$. Now take $L^c = M$ is a $\mathfrak{F}\mathcal{F}Zo$ set and $L \neq 1_{\mathfrak{F}}$ which implies $L^C = M \neq 0_{\mathfrak{F}}$ which is a contradiction. Hence X is $\mathfrak{F}\mathcal{F}ZC_5Con$.

Similar situations exist in other scenarios. ■

Theorem 3.5 Let (X, τ) be a $\mathfrak{F}\mathcal{F}ts$ is $\mathfrak{F}\mathcal{F}Con$ (resp. $\mathfrak{F}\mathcal{F}\delta Con$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Con$, $\mathfrak{F}\mathcal{F}\mathcal{P}Con$ and $\mathfrak{F}\mathcal{F}ZCon$) space iff there exists no non-zero $\mathfrak{F}\mathcal{F}o$ (resp. $\mathfrak{F}\mathcal{F}\delta o$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$, $\mathfrak{F}\mathcal{F}\mathcal{P}o$ and $\mathfrak{F}\mathcal{F}Zo$) set L & Z in X, $\ni L = M^c$.

Proof. Necessity: Let L & M be two $\mathfrak{F}\mathcal{F}Zo$ sets in X such that $L \neq 0_{\mathfrak{F}}$, $M \neq 0_{\mathfrak{F}}$ and $L = M^c$. Therefore M^c is a $\mathfrak{F}\mathcal{F}Zc$ set. Since $L \neq 0_{\mathfrak{F}}$, $M \neq 1_{\mathfrak{F}}$. This implies M is a proper Fermatean fuzzy subset which is both $\mathfrak{F}\mathcal{F}Zo$ set and $\mathfrak{F}\mathcal{F}Zc$ set in X. Hence X is not a $\mathfrak{F}\mathcal{F}ZCon$ space. But this is a contradiction to our hypothesis. Thus, there exist no non-zero $\mathfrak{F}\mathcal{F}Zo$ sets L & M in X, such that $L = M^c$.

Sufficiency: Let L be both $\mathfrak{F}\mathcal{F}Zo$ and $\mathfrak{F}\mathcal{F}Zc$ in U such that $L \neq 0_{\mathfrak{F}}$, $L \neq 1_{\mathfrak{F}}$. Now let $M = L^c$. Then M is a $\mathfrak{F}\mathcal{F}Zo$ set and $M \neq 1_{\mathfrak{F}}$. This implies $L^c = M \neq 0_{\mathfrak{F}}$, which is a contradiction to our hypothesis. Therefore X is $\mathfrak{F}\mathcal{F}MCon$ space.

Similar situations exist in other scenarios. ■

Theorem 3.6 Let (X, τ) be a $\mathfrak{F}\mathcal{F}ts$ is $\mathfrak{F}\mathcal{F}Con$ (resp. $\mathfrak{F}\mathcal{F}\delta Con$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Con$, $\mathfrak{F}\mathcal{F}\mathcal{P}Con$ and $\mathfrak{F}\mathcal{F}ZCon$) space iff there exists no non-zero Fermatean fuzzy subsets L & M in X, $\ni L = M^c$, $M = (\mathfrak{F}\mathcal{F}cl(L))^c$ (resp. $M = (\mathfrak{F}\mathcal{F}\delta cl(L))^c$, $M = (\mathfrak{F}\mathcal{F}\delta\mathcal{S}cl(L))^c$, $M = (\mathfrak{F}\mathcal{F}\mathcal{P}cl(L))^c$ and $M = (\mathfrak{F}\mathcal{F}Zcl(L))^c$) and $L = (\mathfrak{F}\mathcal{F}cl(M))^c$ (resp. $L = (\mathfrak{F}\mathcal{F}\delta cl(M))^c$, $L = (\mathfrak{F}\mathcal{F}\delta\mathcal{S}cl(M))^c$, $L = (\mathfrak{F}\mathcal{F}\mathcal{P}cl(M))^c$ and $L = (\mathfrak{F}\mathcal{F}Zcl(M))^c$).

Proof. Necessity: Let L & Z be two Fermatean fuzzy subsets in X such that $L \neq 0_{\mathfrak{F}}$, $M \neq 0_{\mathfrak{F}}$ and $L = M^c$, $M = (\mathfrak{F}\mathcal{F}Zcl(L))^c$ and $L = (\mathfrak{F}\mathcal{F}Zcl(M))^c$. Since $(\mathfrak{F}\mathcal{F}Zcl(L))^c$ and $(\mathfrak{F}\mathcal{F}Zcl(M))^c$ are $\mathfrak{F}\mathcal{F}Zo$ sets in X, L and Z are $\mathfrak{F}\mathcal{F}Zo$ set in X. This implies X is not a $\mathfrak{F}\mathcal{F}ZCon$ space, which is a contradiction. Therefore, there exists no non-zero $\mathfrak{F}\mathcal{F}Zo$ set L and M in X, $\ni L = M^c$, $M = (\mathfrak{F}\mathcal{F}Zcl(L))^c$ and $L = (\mathfrak{F}\mathcal{F}Zcl(M))^c$.

Sufficiency: Let L be both $\mathfrak{F}\mathcal{F}Zo$ and $\mathfrak{F}\mathcal{F}Zc$ set in X such that $L \neq 0_{\mathfrak{F}}$, $L \neq 1_{\mathfrak{F}}$. Now by taking $M = L^c$, we obtain a contradiction to our hypothesis. Hence X is $\mathfrak{F}\mathcal{F}ZCon$ space.

Similar situations exist in other scenarios. ■

Definition 3.6 Let (X, τ) be a $\mathfrak{F}\mathcal{F}ts$ is Fermatean fuzzy (resp. δ, $\delta\mathcal{S}$, $\mathcal{P}$ and Z) strongly connected (briefly, $\mathfrak{F}\mathcal{F}StCon$ (resp. $\mathfrak{F}\mathcal{F}\delta StCon$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}StCon$, $\mathfrak{F}\mathcal{F}\mathcal{P}StCon$ and $\mathfrak{F}\mathcal{F}ZStCon$)), if there exists a nonempty $\mathfrak{F}\mathcal{F}c$ (resp. $\mathfrak{F}\mathcal{F}\delta c$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}c$, $\mathfrak{F}\mathcal{F}\mathcal{P}c$ and $\mathfrak{F}\mathcal{F}Zc$) sets A & B in $X \ni \mu_A + \mu_B \leq 1_{\mathfrak{F}}$ and $\lambda_A + \lambda_B \geq 1_{\mathfrak{F}}$.

In other words, a $\mathfrak{F}\mathcal{F}ts$ X is $\mathfrak{F}\mathcal{F}StCon$ (resp. $\mathfrak{F}\mathcal{F}\delta StCon$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}StCon$, $\mathfrak{F}\mathcal{F}\mathcal{P}StCon$ and $\mathfrak{F}\mathcal{F}ZStCon$), if there exists no nonempty $\mathfrak{F}\mathcal{F}c$ (resp. $\mathfrak{F}\mathcal{F}\delta c$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}c$, $\mathfrak{F}\mathcal{F}\mathcal{P}c$ and $\mathfrak{F}\mathcal{F}Zc$) sets A and B in X such that $A \cap B = 0_{\mathfrak{F}}$.

Example 3.7 In Example 3.4, Let A_3^c and A_4^c are $\mathfrak{F}\mathcal{F}Zc$ sets. Then X is $\mathfrak{F}\mathcal{F}ZStCon$.

Theorem 3.7 Let (X, τ) be a $\mathfrak{F}\mathcal{F}ts$ is $\mathfrak{F}\mathcal{F}StCon$ (resp. $\mathfrak{F}\mathcal{F}\delta StCon$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}StCon$, $\mathfrak{F}\mathcal{F}\mathcal{P}StCon$ and $\mathfrak{F}\mathcal{F}ZStCon$), if there exists no nonempty $\mathfrak{F}\mathcal{F}o$ (resp. $\mathfrak{F}\mathcal{F}\delta o$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$, $\mathfrak{F}\mathcal{F}\mathcal{P}o$ and $\mathfrak{F}\mathcal{F}Zo$) sets A and B in X, $A \neq 1_{\mathfrak{F}} \neq B$ such that $\mu_A + \mu_B \geq 1_{\mathfrak{F}}$.

Proof. Let A & B be $\mathfrak{F}\mathcal{F}Zo$ sets in $X \ni A \neq 1_{\mathfrak{F}} \neq B$ and $\mu_A + \mu_B \geq 1$. If we take $C = A^c$ and $D = B^c$, then C and D become $\mathfrak{F}\mathcal{F}Zc$ sets in X and $C \neq 0_{\mathfrak{F}} \neq D$, $\mu_C + \mu_D \leq 1$, a contradiction.

Conversely, use a similar technique as above.

Similar situations exist in other scenarios. ■

Theorem 3.8 Let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be a $\mathfrak{F}\mathcal{F}Cts$ (resp. $\mathfrak{F}\mathcal{F}\delta Cts$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Cts$, $\mathfrak{F}\mathcal{F}\mathcal{P}Cts$ and $\mathfrak{F}\mathcal{F}ZCts$) surjection. If X_1 is $\mathfrak{F}\mathcal{F}StCon$ (resp. $\mathfrak{F}\mathcal{F}\delta StCon$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}StCon$, $\mathfrak{F}\mathcal{F}\mathcal{P}StCon$ and $\mathfrak{F}\mathcal{F}ZStCon$), then so is X_2.

Proof. Assume that X_2 is not $\mathfrak{F}\mathcal{F}StCon$, then there exists nonempty $\mathfrak{F}\mathcal{F}c$ sets L and M in X_2 such that $L \neq 0_{\mathfrak{F}}$, $M \neq 0_{\mathfrak{F}}$ and $L \cap M = 0_{\mathfrak{F}}$. Since $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}Cts$ mapping, $A = h_{\mathfrak{F}}^{-1}(L) \neq 0_{\mathfrak{F}}$, $B = h_{\mathfrak{F}}^{-1}(M) \neq 0_{\mathfrak{F}}$, which are $\mathfrak{F}\mathcal{F}c$ sets in X_1 and $h_{\mathfrak{F}}^{-1}(L) \cap h_{\mathfrak{F}}^{-1}(M) = h_{\mathfrak{F}}^{-1}(0_{\mathfrak{F}}) = 0_{\mathfrak{F}}$, which implies $A \cap B = 0_{\mathfrak{F}}$. Thus X_1 is not a $\mathfrak{F}\mathcal{F}StCon$, which is a contradiction to our hypothesis. Hence X_2 is $\mathfrak{F}\mathcal{F}StCon$.

■

Theorem 3.9 Let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be a $\mathfrak{F}\mathcal{F}Irr$ (resp. $\mathfrak{F}\mathcal{F}\delta Irr$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Irr$, $\mathfrak{F}\mathcal{F}\mathcal{P}Irr$ and $\mathfrak{F}\mathcal{F}ZIrr$) surjection. If X_1 is $\mathfrak{F}\mathcal{F}StCon$ (resp. $\mathfrak{F}\mathcal{F}\delta StCon$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}StCon$, $\mathfrak{F}\mathcal{F}\mathcal{P}StCon$ and $\mathfrak{F}\mathcal{F}ZStCon$), then so is X_2.

Proof. Assume that X_2 is not $\mathfrak{F}\mathcal{F}ZStCon$, then there exists nonempty $\mathfrak{F}\mathcal{F}Zc$ sets L and M in X_2 such that $L \neq 0_{\mathfrak{F}}$, $M \neq 0_{\mathfrak{F}}$ and $L \cap M = 0_{\mathfrak{F}}$. Since $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZIrr$ mapping, $A = h_{\mathfrak{F}}^{-1}(L) \neq 0_{\mathfrak{F}}$, $B = h_{\mathfrak{F}}^{-1}(M) \neq 0_{\mathfrak{F}}$, which are $\mathfrak{F}\mathcal{F}Zc$ sets in X_1 and $h_{\mathfrak{F}}^{-1}(L) \cap h_{\mathfrak{F}}^{-1}(M) = h_{\mathfrak{F}}^{-1}(0_{\mathfrak{F}}) = 0_{\mathfrak{F}}$, which implies $A \cap B = 0_{\mathfrak{F}}$. Thus X_1 is not a $\mathfrak{F}\mathcal{F}ZStCon$, which is a contradiction to our hypothesis. Hence X_2 is $\mathfrak{F}\mathcal{F}ZStCon$.

Similar situations exist in other scenarios. ■

Remark 3.1 $\mathfrak{F}\mathcal{F}StCon$ (resp. $\mathfrak{F}\mathcal{F}\delta StCon$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}StCon$, $\mathfrak{F}\mathcal{F}\mathcal{P}StCon$ and $\mathfrak{F}\mathcal{F}ZStCon$) and $\mathfrak{F}\mathcal{F}C_5Con$ (resp. $\mathfrak{F}\mathcal{F}\delta C_5Con$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}C_5Con$, $\mathfrak{F}\mathcal{F}\mathcal{P}C_5Con$ and $\mathfrak{F}\mathcal{F}ZC_5Con$) are independent.

Example 3.8 In Example 3.4, Let A_3^c and A_4^c are $\mathfrak{F}\mathcal{F}Zo$ sets. Then X is $\mathfrak{F}\mathcal{F}ZStCon$ but not $\mathfrak{F}\mathcal{F}ZC_5Con$.

Example 3.9 In Example 3.1, Let A_2 and A_3 are $\mathfrak{F}\mathcal{F}Zo$ sets. Then X is $\mathfrak{F}\mathcal{F}ZC_5Con$ but not $\mathfrak{F}\mathcal{F}ZStCon$.

4 Fermatean fuzzy (resp. δ, $\delta\mathcal{S}$, $\mathcal{P}$ and Z)- separated sets

This section provides an overview of the concept of Fermatean fuzzy (resp. δ, $\delta\mathcal{S}$, $\mathcal{P}$ and Z)- separated sets in Fermatean fuzzy topological spaces. Also, we study some of the main results depending on Fermatean fuzzy (resp. δ, $\delta\mathcal{S}$, $\mathcal{P}$ and Z)- separated sets.

Definition 4.1 Let (X, τ) be a $\mathfrak{F}\mathcal{F}ts$. If A & B are non-zero Fermatean fuzzy subsets in X. Then A & B are said to be Fermatean

(i) fuzzy nano (resp. δ, $\delta\mathcal{S}$, $\mathcal{P}$ and Z) weakly separated (briefly, $\mathfrak{F}\mathcal{F}WSep$ (resp. $\mathfrak{F}\mathcal{F}\delta WSep$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}WSep$, $\mathfrak{F}\mathcal{F}\mathcal{P}WSep$ and $\mathfrak{F}\mathcal{F}ZWSep$)) if $\mathfrak{F}\mathcal{F}cl(A) \subseteq B^c$ (resp. $\mathfrak{F}\mathcal{F}\delta cl(A) \subseteq B^c$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}cl(A) \subseteq B^c$, $\mathfrak{F}\mathcal{F}\mathcal{P}cl(A) \subseteq B^c$ and $\mathfrak{F}\mathcal{F}Zcl(A) \subseteq B^c$) and $\mathfrak{F}\mathcal{F}cl(B) \subseteq A^c$ (resp. $\mathfrak{F}\mathcal{F}\delta cl(B) \subseteq A^c$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}cl(B) \subseteq A^c$, $\mathfrak{F}\mathcal{F}\mathcal{P}cl(B) \subseteq A^c$ and $\mathfrak{F}\mathcal{F}Zcl(B) \subseteq A^c$).

(ii) fuzzy nano (resp. δ, $\delta\mathcal{S}$, $\mathcal{P}$ and Z)-separated (briefly, $\mathfrak{F}\mathcal{F}Sep$ (resp. $\mathfrak{F}\mathcal{F}\delta Sep$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Sep$, $\mathfrak{F}\mathcal{F}\mathcal{P}Sep$ and $\mathfrak{F}\mathcal{F}ZSep$)) if $\mathfrak{F}\mathcal{F}cl(A) \cap B = A \cap \mathfrak{F}\mathcal{F}cl(B) = 0_{\mathfrak{F}}$ (resp. $\mathfrak{F}\mathcal{F}\delta cl(A) \cap B = A \cap \mathfrak{F}\mathcal{F}\delta cl(B) = 0_{\mathfrak{F}}$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}cl(A) \cap B = A \cap \mathfrak{F}\mathcal{F}\delta\mathcal{S}cl(B) = 0_{\mathfrak{F}}$, $\mathfrak{F}\mathcal{F}\mathcal{P}cl(A) \cap B = A \cap \mathfrak{F}\mathcal{F}\mathcal{P}cl(B) = 0_{\mathfrak{F}}$ and $\mathfrak{F}\mathcal{F}Zcl(A) \cap B = A \cap \mathfrak{F}\mathcal{F}Zcl(B) = 0_{\mathfrak{F}}$).

Remark 4.1 Any two disjoint non-empty $\mathfrak{F}\mathcal{F}c$ (resp. $\mathfrak{F}\mathcal{F}\delta c$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}c$, $\mathfrak{F}\mathcal{F}\mathcal{P}c$ and $\mathfrak{F}\mathcal{F}Zc$) sets are $\mathfrak{F}\mathcal{F}Sep$ (resp. $\mathfrak{F}\mathcal{F}\delta Sep$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Sep$, $\mathfrak{F}\mathcal{F}\mathcal{P}Sep$ and $\mathfrak{F}\mathcal{F}ZSep$).

Proof. Suppose A and B are disjoint non-empty $\mathfrak{F}\mathcal{F}Zc$ sets. Then $\mathfrak{F}\mathcal{F}Zcl(A) \cap B = A \cap \mathfrak{F}\mathcal{F}Zcl(B) = A \cap B = 0_{\mathfrak{F}}$. This shows that A and B are $\mathfrak{F}\mathcal{F}ZSep$.

Similar situations exist in other scenarios. ■

Theorem 4.1 Let (X, τ) be a $\mathfrak{F}\mathcal{F}ts$. If A & B are non-zero Fermatean fuzzy subsets in X.

(i) If A & B are $\mathfrak{F}\mathcal{F}Sep$ (resp. $\mathfrak{F}\mathcal{F}\delta Sep$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Sep$, $\mathfrak{F}\mathcal{F}\mathcal{P}Sep$ and $\mathfrak{F}\mathcal{F}ZSep$) and $C \subseteq A$, $D \subseteq B$, then C and D are also $\mathfrak{F}\mathcal{F}Sep$ (resp. $\mathfrak{F}\mathcal{F}\delta Sep$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Sep$, $\mathfrak{F}\mathcal{F}\mathcal{P}Sep$ and $\mathfrak{F}\mathcal{F}ZSep$).

(ii) If A and B are both $\mathfrak{F}\mathcal{F}o$ (resp. $\mathfrak{F}\mathcal{F}\delta o$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$, $\mathfrak{F}\mathcal{F}\mathcal{P}o$ and $\mathfrak{F}\mathcal{F}Zo$) sets and if $H = A \cap B^c$ and $G = B \cap A^c$, then H and G are $\mathfrak{F}\mathcal{F}Sep$ (resp. $\mathfrak{F}\mathcal{F}\delta Sep$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Sep$, $\mathfrak{F}\mathcal{F}\mathcal{P}Sep$ and $\mathfrak{F}\mathcal{F}ZSep$).

Proof. (i) Let A & B be $\mathfrak{F}\mathcal{F}ZSep$ sets in $\mathfrak{F}\mathcal{F}ts$ X. Then $\mathfrak{F}\mathcal{F}Zcl(A) \cap B = 0_{\mathfrak{F}} = A \cap \mathfrak{F}\mathcal{F}Zcl(B)$. Since $C \subseteq A$ and $D \subseteq B$, then $\mathfrak{F}\mathcal{F}Zcl(C) \subseteq \mathfrak{F}\mathcal{F}Zcl(A)$ and $\mathfrak{F}\mathcal{F}Zcl(D) \subseteq \mathfrak{F}\mathcal{F}Zcl(B)$. This implies that, $\mathfrak{F}\mathcal{F}Zcl(C) \cap D \subseteq \mathfrak{F}\mathcal{F}Zcl(A) \cap B = 0_{\mathfrak{F}}$ and hence $\mathfrak{F}\mathcal{F}Zcl(C) \cap D = 0_{\mathfrak{F}}$. Similarly $\mathfrak{F}\mathcal{F}Zcl(D) \cap C \subseteq \mathfrak{F}\mathcal{F}Zcl(B) \cap A = 0_{\mathfrak{F}}$ and hence $\mathfrak{F}\mathcal{F}Zcl(D) \cap C = 0_{\mathfrak{F}}$. Therefore C and D are $\mathfrak{F}\mathcal{F}ZSep$.

(ii) Let A and B both $\mathfrak{F}\mathcal{F}Zo$ subsets in X. Then A^c and B^c are $\mathfrak{F}\mathcal{F}Zc$ sets. Since $H \subseteq B^c$, then $\mathfrak{F}\mathcal{F}Zcl(H) \subseteq \mathfrak{F}\mathcal{F}Zcl(B^c) = B^c$ and so $\mathfrak{F}\mathcal{F}Mcl(H) \cap B = 0_{\mathfrak{F}}$. Since $G \subseteq B$, then $\mathfrak{F}\mathcal{F}Zcl(H) \cap G \subseteq \mathfrak{F}\mathcal{F}Zcl(H) \cap B = 0_{\mathfrak{F}}$. Thus, $\mathfrak{F}\mathcal{F}Zcl(H) \cap G = 0_{\mathfrak{F}}$. Similarly, $\mathfrak{F}\mathcal{F}Zcl(G) \cap H = 0_{\mathfrak{F}}$. Hence H and G are $\mathfrak{F}\mathcal{F}ZSep$.

Similar situations exist in other scenarios. ■

Theorem 4.2 Let (X, τ) be a $\mathfrak{F}\mathcal{F}ts$. If A & B are non-zero Fermatean fuzzy subsets in X are $\mathfrak{F}\mathcal{F}Sep$ (resp. $\mathfrak{F}\mathcal{F}\delta Sep$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Sep$, $\mathfrak{F}\mathcal{F}\mathcal{P}Sep$ and $\mathfrak{F}\mathcal{F}ZSep$) if and only if there exist L and M in $\mathfrak{F}\mathcal{F}o$ (resp. $\mathfrak{F}\mathcal{F}\delta o$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$, $\mathfrak{F}\mathcal{F}\mathcal{P}o$ and $\mathfrak{F}\mathcal{F}Zo$) set in $X \ni A \subseteq L$, $B \subseteq M$ and $A \cap M = 0_{\mathfrak{F}}$ and $B \cap L = 0_{\mathfrak{F}}$.

Proof. Let A and B be $\mathfrak{F}\mathcal{F}ZSep$. Then $A \cap \mathfrak{F}\mathcal{F}Zcl(B) = 0_{\mathfrak{F}} = \mathfrak{F}\mathcal{F}Zcl(A) \cap B$. Take $M = (\mathfrak{F}\mathcal{F}Zcl(A))^c$ and $L = (\mathfrak{F}\mathcal{F}Zcl(B))^c$. Then L and M are $\mathfrak{F}\mathcal{F}Zo$ sets $\ni A \subseteq L$, $B \subseteq M$ and $A \cap M = 0_{\mathfrak{F}}$ and $B \cap L = 0_{\mathfrak{F}}$.

Conversely let L and M be $\mathfrak{F}\mathcal{F}Zo$ sets such that $A \subseteq L$, $B \subseteq M$ and $A \cap M = 0_{\mathfrak{F}}$, $B \cap L = 0_{\mathfrak{F}}$. Then $A \subseteq M^c$ and $B \subseteq L^c$ and M^c and L^c are $\mathfrak{F}\mathcal{F}Zc$ sets. This implies, $\mathfrak{F}\mathcal{F}Zcl(A) \subseteq \mathfrak{F}\mathcal{F}Zcl(M^c) = M^c \subseteq B^c$ and $\mathfrak{F}\mathcal{F}Zcl(B) \subseteq \mathfrak{F}\mathcal{F}Zcl(L^c) = L^c \subseteq A^c$. That is, $\mathfrak{F}\mathcal{F}Zcl(A) \subseteq B^c$ and $\mathfrak{F}\mathcal{F}Zcl(B) \subseteq A^c$. Therefore $A \cap \mathfrak{F}\mathcal{F}Zcl(B) = 0_{\mathfrak{F}} = \mathfrak{F}\mathcal{F}Zcl(A) \cap B$. Hence A and B are $\mathfrak{F}\mathcal{F}ZSep$.

Similar situations exist in other scenarios. ■

Proposition 4.1 Each two $\mathfrak{F}\mathcal{F}Sep$ (resp. $\mathfrak{F}\mathcal{F}\delta Sep$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Sep$, $\mathfrak{F}\mathcal{F}\mathcal{P}Sep$ and $\mathfrak{F}\mathcal{F}ZSep$) sets are always disjoint.

Proof. Let A and B be $\mathfrak{F}\mathcal{F}ZSep$. Then $A \cap \mathfrak{F}\mathcal{F}Zcl(B) = 0_{\mathfrak{F}} = \mathfrak{F}\mathcal{F}Zcl(A) \cap B$. Now, $A \cap B \subseteq A \cap \mathfrak{F}\mathcal{F}Zcl(B) = 0_{\mathfrak{F}}$. Therefore $A \cap B = 0_{\mathfrak{F}}$ and hence A and B are disjoint.

Similar situations exist in other scenarios. ■

Theorem 4.3 Let (X, τ) be a $\mathfrak{F}\mathcal{F}ts$. Then X is $\mathfrak{F}\mathcal{F}Con$ (resp. $\mathfrak{F}\mathcal{F}\delta Con$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Con$, $\mathfrak{F}\mathcal{F}\mathcal{P}Con$ and $\mathfrak{F}\mathcal{F}ZCon$) iff $1_{\mathfrak{F}} \neq A \cup B$, where A and B are $\mathfrak{F}\mathcal{F}Sep$ (resp. $\mathfrak{F}\mathcal{F}\delta Sep$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Sep$, $\mathfrak{F}\mathcal{F}\mathcal{P}Sep$ and $\mathfrak{F}\mathcal{F}ZSep$) sets.

Proof. Assume that, X is a $\mathfrak{F}\mathcal{F}ZCon$ space. Suppose $1_{\mathfrak{F}} = A \cup B$, where A & B are $\mathfrak{F}\mathcal{F}ZSep$ sets. Then $\mathfrak{F}\mathcal{F}Zcl(A) \cap B = A \cap \mathfrak{F}\mathcal{F}Zcl(B) = 0_{\mathfrak{F}}$. Since $A \subseteq \mathfrak{F}\mathcal{F}Zcl(A)$, we have $A \cap B \subseteq \mathfrak{F}\mathcal{F}Zcl(A) \cap B = 0_{\mathfrak{F}}$. Therefore $\mathfrak{F}\mathcal{F}Zcl(A) \subseteq B^c = A$ and $\mathfrak{F}\mathcal{F}Zcl(B) \subseteq A^c = B$. Hence $A = \mathfrak{F}\mathcal{F}Zcl(A)$ and $B = \mathfrak{F}\mathcal{F}Zcl(B)$. Therefore A and B are $\mathfrak{F}\mathcal{F}Zc$ sets and hence $A = B^c$ and $B = A^c$ are disjoint $\mathfrak{F}\mathcal{F}Zo$ sets. Thus $A \neq 0_{\mathfrak{F}}$, $B \neq 0_{\mathfrak{F}} \ni A \cup B = 1_{\mathfrak{F}}$ and $A \cap B = 0_{\mathfrak{F}}$, A and B are $\mathfrak{F}\mathcal{F}Zo$ sets. That is X is not $\mathfrak{F}\mathcal{F}ZCon$, which is a contradiction to X is a $\mathfrak{F}\mathcal{F}ZCon$ space. Hence $1_{\mathfrak{F}}$ is not the union of any two $\mathfrak{F}\mathcal{F}ZSep$ sets.

Conversely, assume that $1_{\mathfrak{F}}$ is not the union of any two $\mathfrak{F}\mathcal{F}ZSep$ sets. Suppose X is not $\mathfrak{F}\mathcal{F}ZCon$. Then $1_{\mathfrak{F}} = A \cup B$, where $A \neq 0_{\mathfrak{F}}$, $B \neq 0_{\mathfrak{F}} \ni$ and $A \cap B = 0_{\mathfrak{F}}$, A and B are $\mathfrak{F}\mathcal{F}Zo$ sets in X. Since $A \subseteq B^c$ and $B \subseteq A^c$, $\mathfrak{F}\mathcal{F}Zcl(A) \cap B \subseteq B^c \cap B = 0_{\mathfrak{F}}$ and $A \cap \mathfrak{F}\mathcal{F}Zcl(B) \subseteq A \cap A^c = 0_{\mathfrak{F}}$. That is A and B are $\mathfrak{F}\mathcal{F}ZSep$ sets. This is a contradiction. Therefore X is $\mathfrak{F}\mathcal{F}ZCon$.

Similar situations exist in other scenarios. ■

Definition 4.2 Let (X, τ) be a $\mathfrak{F}\mathcal{F}ts$. Let S be a Fermatean fuzzy subset of X. Then Fermatean fuzzy

(i) δ (resp. $\delta\mathcal{S}$, $\mathcal{P}$ and Z) regular open (briefly, $\mathfrak{F}\mathcal{F}\delta ro$ (resp. $\mathfrak{F}\mathcal{F}\delta\mathcal{S}ro$, $\mathfrak{F}\mathcal{F}\mathcal{P}ro$ and $\mathfrak{F}\mathcal{F}Zro$)) set if $S = \mathfrak{F}\mathcal{F}\delta int(\mathfrak{F}\mathcal{F}\delta cl(S))$ (resp. $S = \mathfrak{F}\mathcal{F}\delta\mathcal{S}int(\mathfrak{F}\mathcal{F}\delta\mathcal{S}cl(S))$, $S = \mathfrak{F}\mathcal{F}\mathcal{P}int(\mathfrak{F}\mathcal{F}\mathcal{P}cl(S))$ & $S = \mathfrak{F}\mathcal{F}Zint\ (\mathfrak{F}\mathcal{F}Zcl(S))$).

(ii) δ (resp. $\delta\mathcal{S}$, $\mathcal{P}$ and Z) regular closed (briefly, $\mathfrak{F}\mathcal{F}\delta rc$ (resp. $\mathfrak{F}\mathcal{F}\delta\mathcal{S}rc$, $\mathfrak{F}\mathcal{F}\mathcal{P}rc$ and $\mathfrak{F}\mathcal{F}Zrc$)) set if $S = \mathfrak{F}\mathcal{F}\delta cl(\mathfrak{F}\mathcal{F}\delta int(S))$ (resp. $S = \mathfrak{F}\mathcal{F}\delta\mathcal{S}cl\ (\mathfrak{F}\mathcal{F}\delta\mathcal{S}int(S))$, $S = \mathfrak{F}\mathcal{F}\mathcal{P}cl(\mathfrak{F}\mathcal{F}\mathcal{P}int(S))$ & $S = \mathfrak{F}\mathcal{F}Zcl(\mathfrak{F}\mathcal{F}Zint(S))$).

(iii) The complement of $\mathfrak{F}\mathcal{F}\delta ro$ (resp. $\mathfrak{F}\mathcal{F}\delta\mathcal{S}ro$, $\mathfrak{F}\mathcal{F}\mathcal{P}ro$ and $\mathfrak{F}\mathcal{F}Zro$) set is $\mathfrak{F}\mathcal{F}\delta rc$ (resp. $\mathfrak{F}\mathcal{F}\delta\mathcal{S}rc$, $\mathfrak{F}\mathcal{F}\mathcal{P}rc$ and $\mathfrak{F}\mathcal{F}Zrc$) set.

Proposition 4.2 Let (X, τ) be a $\mathfrak{F}\mathcal{F}ts$.

(i) Every $\mathfrak{F}\mathcal{F}\delta ro$ (resp. $\mathfrak{F}\mathcal{F}\delta\mathcal{S}ro$, $\mathfrak{F}\mathcal{F}\mathcal{P}ro$ and $\mathfrak{F}\mathcal{F}Zro$) set is $\mathfrak{F}\mathcal{F}\delta o$ (resp. $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$, $\mathfrak{F}\mathcal{F}\mathcal{P}o$ and $\mathfrak{F}\mathcal{F}Zo$).

(ii) Every $\mathfrak{F}\mathcal{F}\delta rc$ (resp. $\mathfrak{F}\mathcal{F}\delta\mathcal{S}rc$, $\mathfrak{F}\mathcal{F}\mathcal{P}rc$ and $\mathfrak{F}\mathcal{F}Zrc$) set is $\mathfrak{F}\mathcal{F}\delta c$ (resp. $\mathfrak{F}\mathcal{F}\delta\mathcal{S}c$, $\mathfrak{F}\mathcal{F}\mathcal{P}c$ and $\mathfrak{F}\mathcal{F}Zc$).

Proof. Let A be a $\mathfrak{F}\mathcal{F}Zro$ in X. Then $A = \mathfrak{F}\mathcal{F}Zint(\mathfrak{F}\mathcal{F}Zcl(A))$. Clearly, $\mathfrak{F}\mathcal{F}Zint(\mathfrak{F}\mathcal{F}Zcl(A))$ is $\mathfrak{F}\mathcal{F}Zo$. Therefore, A is $\mathfrak{F}\mathcal{F}Zo$ set. Other cases are similar. (ii) Similar proof of (i). ■

Definition 4.3 Let (X, τ) be a $\mathfrak{F}\mathcal{F}ts$. Then X is Fermatean fuzzy (resp. δ, $\delta\mathcal{S}$, $\mathcal{P}$ and Z) super disconnected (briefly, $\mathfrak{F}\mathcal{F}superDCon$ (resp. $\mathfrak{F}\mathcal{F}\delta superDCon$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}superDCon$, $\mathfrak{F}\mathcal{F}\mathcal{P}superDCon$ and $\mathfrak{F}\mathcal{F}ZsuperDCon$)) if there exists a $\mathfrak{F}\mathcal{F}ro$ (resp. $\mathfrak{F}\mathcal{F}\delta ro$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}ro$, $\mathfrak{F}\mathcal{F}\mathcal{P}ro$ and $\mathfrak{F}\mathcal{F}Zro$) set A in X such that $A \neq 0_{\mathfrak{F}}$ and $A \neq 1_{\mathfrak{F}}$.

A $\mathfrak{F}\mathcal{F}ts$ X is called Fermatean fuzzy (resp. δ, $\delta\mathcal{S}$, $\mathcal{P}$ and Z) super connected (briefly, $\mathfrak{F}\mathcal{F}superCon$ (resp. $\mathfrak{F}\mathcal{F}\delta superCon$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}superCon$, $\mathfrak{F}\mathcal{F}\mathcal{P}superCon$ and $\mathfrak{F}\mathcal{F}ZsuperCon$)) if X is not $\mathfrak{F}\mathcal{F}superDCon$ (resp. $\mathfrak{F}\mathcal{F}\delta superDCon$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}superDCon$, $\mathfrak{F}\mathcal{F}\mathcal{P}superDCon$ and $\mathfrak{F}\mathcal{F}ZsuperDCon$).

Example 4.1 In Example 3.1, Let

(i) $S = A_1$ is $\mathfrak{F}\mathcal{F}superDCon$ (resp. $\mathfrak{F}\mathcal{F}\delta superDCon$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}superDCon$ and $\mathfrak{F}\mathcal{F}\mathcal{P}superDCon$).

(ii) $S = A_1^c$ is $\mathfrak{F}\mathcal{F}ZsuperDCon$.

Example 4.2 In Example 3.1, Let

(i) $S = \left\{\left\langle \frac{s_1,s_4}{0.1}\right\rangle, \left\langle \frac{s_2}{0.2}\right\rangle, \left\langle \frac{s_3}{0.3}\right\rangle\right\}$ is $\mathfrak{F}\mathcal{F}Z superDCon$ (resp. $\mathfrak{F}\mathcal{F}\mathcal{P}superDCon$).

(ii) $S = \left\{\left\langle \frac{s_1,s_4}{0.2}\right\rangle, \left\langle \frac{s_2}{0.3}\right\rangle, \left\langle \frac{s_3}{0.5}\right\rangle\right\}$ is $\mathfrak{F}\mathcal{F}\delta\mathcal{S}superDCon$.

Theorem 4.4 Let (X,τ) be a $\mathfrak{F}\mathcal{F}ts$, the equivalents are as follows:

(i) X is $\mathfrak{F}\mathcal{F}superCon$ (resp. $\mathfrak{F}\mathcal{F}\delta superCon$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}superCon$, $\mathfrak{F}\mathcal{F}\mathcal{P}superCon$ and $\mathfrak{F}\mathcal{F}Z superCon$).

(ii) For each $\mathfrak{F}\mathcal{F}o$ (resp. $\mathfrak{F}\mathcal{F}\delta o$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$, $\mathfrak{F}\mathcal{F}\mathcal{P}o$ and $\mathfrak{F}\mathcal{F}Zo$) set $L \neq 0_{\mathfrak{F}}$ in X, we have $\mathfrak{F}\mathcal{F}cl(L) = 1_{\mathfrak{F}}$ (resp. $\mathfrak{F}\mathcal{F}\delta cl(L) = 1_{\mathfrak{F}}$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}cl(L) = 1_{\mathfrak{F}}$, $\mathfrak{F}\mathcal{F}\mathcal{P}cl(L) = 1_{\mathfrak{F}}$ and $\mathfrak{F}\mathcal{F}Zcl(L) = 1_{\mathfrak{F}}$).

(iii) For each $\mathfrak{F}\mathcal{F}c$ (resp. $\mathfrak{F}\mathcal{F}\delta c$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}c$, $\mathfrak{F}\mathcal{F}\mathcal{P}c$ and $\mathfrak{F}\mathcal{F}Zc$) set $L \neq 1_{\mathfrak{F}}$ in X, we have $\mathfrak{F}\mathcal{F}int(L) = 0_{\mathfrak{F}}$ (resp. $\mathfrak{F}\mathcal{F}\delta int(L) = 0_{\mathfrak{F}}$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}int(L) = 0_{\mathfrak{F}}$, $\mathfrak{F}\mathcal{F}\mathcal{P}int(L) = 0_{\mathfrak{F}}$ and $\mathfrak{F}\mathcal{F}Zint(L) = 0_{\mathfrak{F}}$).

(iv) There exists no $\mathfrak{F}\mathcal{F}o$ (resp. $\mathfrak{F}\mathcal{F}\delta o$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$, $\mathfrak{F}\mathcal{F}\mathcal{P}o$ and $\mathfrak{F}\mathcal{F}Zo$) subsets L and Z in X, such that $L \neq 0_{\mathfrak{F}}$, $M \neq 0_{\mathfrak{F}}$ and $L \subseteq M^c$.

(v) There exists no $\mathfrak{F}\mathcal{F}o$ (resp. $\mathfrak{F}\mathcal{F}\delta o$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$, $\mathfrak{F}\mathcal{F}\mathcal{P}o$ and $\mathfrak{F}\mathcal{F}Zo$) subsets L and Z in X, such that $L \neq 0_{\mathfrak{F}}$, $M \neq 0_{\mathfrak{F}}$, $M = (\mathfrak{F}\mathcal{F}cl(L))^c$ (resp. $M = (\mathfrak{F}\mathcal{F}\delta cl(L))^c$, $M = (\mathfrak{F}\mathcal{F}\delta\mathcal{S}cl(L))^c$, $M = (\mathfrak{F}\mathcal{F}\mathcal{P}cl(L))^c$ and $M = (\mathfrak{F}\mathcal{F}Zcl(L))^c$) and $L = (\mathfrak{F}\mathcal{F}cl(M))^c$ (resp. $L = (\mathfrak{F}\mathcal{F}\delta cl(M))^c$, $L = (\mathfrak{F}\mathcal{F}\delta\mathcal{S}cl(M))^c$, $L = (\mathfrak{F}\mathcal{F}\mathcal{P}cl(M))^c$ and $L = (\mathfrak{F}\mathcal{F}Zcl(M))^c$).

(vi) There exists no $\mathfrak{F}\mathcal{F}c$ (resp. $\mathfrak{F}\mathcal{F}\delta c$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}c$, $\mathfrak{F}\mathcal{F}\mathcal{P}c$ and $\mathfrak{F}\mathcal{F}Zc$) subsets L and M in X such that $L \neq 1_{\mathfrak{F}}$, $M \neq 1_{\mathfrak{F}}$, $M = (\mathfrak{F}\mathcal{F}cl(L))^c$ (resp. $M = (\mathfrak{F}\mathcal{F}\delta cl(L))^c$, $M = (\mathfrak{F}\mathcal{F}\delta\mathcal{S}cl(L))^c$, $M = (\mathfrak{F}\mathcal{F}\mathcal{P}cl(L))^c$ and $M = (\mathfrak{F}\mathcal{F}Zcl(L))^c$) and $L = (\mathfrak{F}\mathcal{F}cl(M))^c$ (resp. $L = (\mathfrak{F}\mathcal{F}\delta cl(M))^c$, $L = (\mathfrak{F}\mathcal{F}\delta\mathcal{S}cl(M))^c$, $L = (\mathfrak{F}\mathcal{F}\mathcal{P}cl(M))^c$ and $L = (\mathfrak{F}\mathcal{F}Zcl(M))^c$).

Proof. (i) $\Rightarrow$ (ii) Assume that there exists a $\mathfrak{F}\mathcal{F}Zo$ set $A \neq 0_{\mathfrak{F}} \ni \mathfrak{F}\mathcal{F}Zcl(A) \neq 1_{\mathfrak{F}}$. Now take $B = \mathfrak{F}\mathcal{F}Zint(\mathfrak{F}\mathcal{F}Zcl(A))$. Then B is proper $\mathfrak{F}\mathcal{F}Zro$ set in X which contradicts that X is $\mathfrak{F}\mathcal{F}Z superCon$-ness.

(ii) $\Rightarrow$ (iii) Let $A \neq 1_{\mathfrak{F}}$ be a $\mathfrak{F}\mathcal{F}Zc$ set in X. If $B = A^c$, then B is $\mathfrak{F}\mathcal{F}Zo$ set in X and $B \neq 0_{\mathfrak{F}}$. Hence $\mathfrak{F}\mathcal{F}Zcl(A) = 1_{\mathfrak{F}}$, $(\mathfrak{F}\mathcal{F}Zcl(B))^c = 0_{\mathfrak{F}} \Rightarrow \mathfrak{F}\mathcal{F}Zint(B^c) = 0_{\mathfrak{F}} \Rightarrow \mathfrak{F}\mathcal{F}Zint(A) = 0_{\mathfrak{F}}$.

(iii) $\Rightarrow$ (iv) Let A and B be $\mathfrak{F}\mathcal{F}Zo$ sets in $X \ni A \neq 0_{\mathfrak{F}} \neq B$ and $A \subseteq B^c$. Since B^c is $\mathfrak{F}\mathcal{F}Zc$ set in X and $B \neq 0_{\mathfrak{F}}$ implies $B^c \neq 1_{\mathfrak{F}}$, we obtain $\mathfrak{F}\mathcal{F}Zint(B^c) = 0_{\mathfrak{F}}$. But, from $A \subseteq B^c$, $0_{\mathfrak{F}} \neq A = \mathfrak{F}\mathcal{F}Zint(A) \subseteq \mathfrak{F}\mathcal{F}Zint(B^c) = 0_{\mathfrak{F}}$, which is a contradiction.

(iv) $\Rightarrow$ (i) Let $0_{\mathfrak{F}} \neq A \neq 1_{\mathfrak{F}}$ be $\mathfrak{F}\mathcal{F}Zro$ set in X. If we take $B = (\mathfrak{F}\mathcal{F}Zcl(A))^c$, we get $B \neq 0_{\mathfrak{F}}$. Otherwise, we have $B \neq 0_{\mathfrak{F}}$ implies $(\mathfrak{F}\mathcal{F}Zcl(A))^c = 0_{\mathfrak{F}}$. That implies $\mathfrak{F}\mathcal{F}Zcl(A) = 1_{\mathfrak{F}}$. That shows $A = \mathfrak{F}\mathcal{F}Zint(\mathfrak{F}\mathcal{F}Zcl(A)) = \mathfrak{F}\mathcal{F}Zint(1_{\mathfrak{F}}) = 1_{\mathfrak{F}}$. But this is to a contradiction to $A \neq 1_{\mathfrak{F}}$.

Further, $A \subseteq B^c$, this is also a contradiction.

(i) $\Rightarrow$ (v) Let A and B be $\mathfrak{F}\mathcal{F}Zo$ sets in $X \ni A \neq 0_{\mathfrak{F}} \neq B$ and $B = (\mathfrak{F}\mathcal{F}Zcl(A))^c$, $A = (\mathfrak{F}\mathcal{F}Zint(B))^c$. Now $\mathfrak{F}\mathcal{F}Zint(\mathfrak{F}\mathcal{F}Zcl(A)) = \mathfrak{F}\mathcal{F}Zint(B^c) = (\mathfrak{F}\mathcal{F}Zcl(B))^c = A$ and $A \neq 0_{\mathfrak{F}}$, $A \neq 1_{\mathfrak{F}}$. Suppose not, if $A = 1_{\mathfrak{F}}$, then $1_{\mathfrak{F}} = (\mathfrak{F}\mathcal{F}Zcl(B))^c$ implies $0 = \mathfrak{F}\mathcal{F}Zcl(B) \Rightarrow B = 0$. This is a contradiction.

(v) $\Rightarrow$ (i) Let A be $\mathfrak{F}\mathcal{F}Zo$ set in $X \ni A = \mathfrak{F}\mathcal{F}Zint(\mathfrak{F}\mathcal{F}Zcl(A))$, $0_{\mathfrak{F}} \neq A \neq 1_{\mathfrak{F}}$. Now $B = (\mathfrak{F}\mathcal{F}Zcl(A))^c$ and $(\mathfrak{F}\mathcal{F}Zcl(B))^c = (\mathfrak{F}\mathcal{F}Zcl(\mathfrak{F}\mathcal{F}Zcl(A))^c)^c = \mathfrak{F}\mathcal{F}Zint(\mathfrak{F}\mathcal{F}Zcl(A)) = A$. This is a contradiction.

(v) $\Rightarrow$ (vi) Let A and B be $\mathfrak{F}\mathcal{F}Zc$ set in X such that $A \neq 1_{\mathfrak{F}} \neq B$. $B = (\mathfrak{F}\mathcal{F}Zint(A))^c$, $A = (\mathfrak{F}\mathcal{F}Zint(B))^c$. Taking $C = A^c$ and $D = B^c$, C and D become $\mathfrak{F}\mathcal{F}Zo$ set in X and $C \neq 0_{\mathfrak{F}} \neq D$, $(\mathfrak{F}\mathcal{F}Zcl(C))^c = (\mathfrak{F}\mathcal{F}Zcl(A^c))^c = ((\mathfrak{F}\mathcal{F}Zint(A))^c)^c = \mathfrak{F}\mathcal{F}Zint(A) = B^c = D$ and similarly $(\mathfrak{F}\mathcal{F}Zcl(D))^c = C$. But this is a contradiction.

(vi) $\Rightarrow$ (v) Similar as in above. ■

5 Fermatean fuzzy (resp. δ, $\delta\mathcal{S}$, $\mathcal{P}$ and Z)-compact spaces

In this section, we present Fermatean fuzzy (resp. δ, $\delta\mathcal{S}$, $\mathcal{P}$ and Z)- compact spaces and examine some of their fundamental properties using Fermatean fuzzy (resp. δ, $\delta\mathcal{S}$, $\mathcal{P}$ and Z)-open sets.

Definition 5.1 Let (X,τ) be a $\mathfrak{F}\mathcal{F}ts$. A collection B of $\mathfrak{F}\mathcal{F}o$ (resp. $\mathfrak{F}\mathcal{F}\delta o$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$, $\mathfrak{F}\mathcal{F}\mathcal{P}o$ and $\mathfrak{F}\mathcal{F}Zo$) sets in X is called a Fermatean fuzzy (resp. δ, $\delta\mathcal{S}$, $\mathcal{P}$ and Z)-open cover (briefly, $\mathfrak{F}\mathcal{F}OCov$ (resp. $\mathfrak{F}\mathcal{F}\delta OCov$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}OCov$, $\mathfrak{F}\mathcal{F}\mathcal{P}OCov$ and $\mathfrak{F}\mathcal{F}ZOCov$)) of a subset B of X if $B \subseteq \bigcup\{L_\alpha : L_\alpha \in B\}$.

Definition 5.2 Let (X,τ) be a $\mathfrak{F}\mathcal{F}ts$, then X is said to be fuzzy nano (resp. δ, $\delta\mathcal{S}$, $\mathcal{P}$ and Z)- compact (briefly, $\mathfrak{F}\mathcal{F}Comp$ (resp. $\mathfrak{F}\mathcal{F}\delta Comp$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Comp$, $\mathfrak{F}\mathcal{F}\mathcal{P}Comp$ and $\mathfrak{F}\mathcal{F}ZComp$)) if every $\mathfrak{F}\mathcal{F}OCov$ (resp. $\mathfrak{F}\mathcal{F}\delta OCov$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}OCov$, $\mathfrak{F}\mathcal{F}\mathcal{P}OCov$ and $\mathfrak{F}\mathcal{F}ZOCov$) of X has a finite subcover.

Definition 5.3 Let (X,τ) be a $\mathfrak{F}\mathcal{F}ts$. A subset A of X is said to be $\mathfrak{F}\mathcal{F}Comp$ (resp. $\mathfrak{F}\mathcal{F}\delta Comp$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Comp$, $\mathfrak{F}\mathcal{F}\mathcal{P}Comp$ and $\mathfrak{F}\mathcal{F}ZComp$) relative to X if every $\mathfrak{F}\mathcal{F}OCov$ (resp. $\mathfrak{F}\mathcal{F}\delta OCov$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}OCov$, $\mathfrak{F}\mathcal{F}\mathcal{P}OCov$ and $\mathfrak{F}\mathcal{F}ZOCov$) of X has a finite subcover.

Theorem 5.1 Let (X, τ) be a $\mathfrak{F}\mathcal{F}ts$.

(i) Every $\mathfrak{F}\mathcal{F}\delta Comp$ is a $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Comp$.

(ii) Every $\mathfrak{F}\mathcal{F}\delta Comp$ is a $\mathfrak{F}\mathcal{F}Comp$.

(iii) Every $\mathfrak{F}\mathcal{F}Comp$ is a $\mathfrak{F}\mathcal{F}\mathcal{P}Comp$.

(iv) Every $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Comp$ is a $\mathfrak{F}\mathcal{F}ZComp$.

(v) Every $\mathfrak{F}\mathcal{F}\mathcal{P}Comp$ is a $\mathfrak{F}\mathcal{F}ZComp$.

Proof. (iv) Let X be $\mathfrak{F}\mathcal{F}ZComp$. Suppose X is not $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Comp$. Then there exists a $\mathfrak{F}\mathcal{F}\delta\mathcal{S}OCov$ B of X has no finite subcover. Since every $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$ set is $\mathfrak{F}\mathcal{F}Zo$ set, then we have $\mathfrak{F}\mathcal{F}ZOCov$ B of X, which has no finite subcover. This is a contradiction to X is $\mathfrak{F}\mathcal{F}ZComp$. Hence X is $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Comp$.

Other cases are true. ∎

Theorem 5.2 A $\mathfrak{F}\mathcal{F}c$ (resp. $\mathfrak{F}\mathcal{F}\delta c$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}c$, $\mathfrak{F}\mathcal{F}\mathcal{P}c$ and $\mathfrak{F}\mathcal{F}Zc$) subset of a $\mathfrak{F}\mathcal{F}Comp$ (resp. $\mathfrak{F}\mathcal{F}\delta Comp$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Comp$, $\mathfrak{F}\mathcal{F}\mathcal{P}Comp$ and $\mathfrak{F}\mathcal{F}ZComp$) space X is $\mathfrak{F}\mathcal{F}Comp$ (resp. $\mathfrak{F}\mathcal{F}\delta Comp$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Comp$, $\mathfrak{F}\mathcal{F}\mathcal{P}Comp$ and $\mathfrak{F}\mathcal{F}ZComp$) relative to X.

Proof. Let A be a $\mathfrak{F}\mathcal{F}Zc$ subset of a $\mathfrak{F}\mathcal{F}ZComp$ space X. Then A^c is $\mathfrak{F}\mathcal{F}Zo$ in X. Let $B = \{A_i : i \in I\}$ be a $\mathfrak{F}\mathcal{F}ZOCov$ of A. Then $B \cup \{A^c\}$ is a $\mathfrak{F}\mathcal{F}ZOCov$ of X. Since X is $\mathfrak{F}\mathcal{F}ZComp$, it has a finite subcover say $\{P_1, P_2, \cdots P_P, A^c\}$. Then $\{P_1, P_2, \dots, P_P\}$ is a finite $\mathfrak{F}\mathcal{F}ZOCov$. Thus A is $\mathfrak{F}\mathcal{F}ZComp$ relative to X.

Similar situations exist in other scenarios. ∎

Theorem 5.3 Let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be a $\mathfrak{F}\mathcal{F}Cts$ (resp. $\mathfrak{F}\mathcal{F}\delta Cts$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Cts$, $\mathfrak{F}\mathcal{F}\mathcal{P}Cts$ and $\mathfrak{F}\mathcal{F}ZCts$) surjection and X_1 be $\mathfrak{F}\mathcal{F}Comp$ (resp. $\mathfrak{F}\mathcal{F}\delta Comp$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Comp$, $\mathfrak{F}\mathcal{F}\mathcal{P}Comp$ and $\mathfrak{F}\mathcal{F}ZComp$). Then X_2 is $\mathfrak{F}\mathcal{F}Comp$.

Proof. Let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be a $\mathfrak{F}\mathcal{F}ZCts$ surjection and X_1 be $\mathfrak{F}\mathcal{F}ZComp$. Let $\{M_\alpha\}$ be a $\mathfrak{F}\mathcal{F}ZOCov$ for X_2. Since $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZCts$, $\{h_{\mathfrak{F}}^{-1}(M_\alpha)\}$ is a $\mathfrak{F}\mathcal{F}ZOCov$ of X_1. Since X_1 is $\mathfrak{F}\mathcal{F}ZComp$, $\{h_{\mathfrak{F}}^{-1}(M_\alpha)\}$ contains a finite subcover, namely $\{h_{\mathfrak{F}}^{-1}(M_{\alpha_1}), h_{\mathfrak{F}}^{-1}(M_{\alpha_2}), \dots, h_{\mathfrak{F}}^{-1}(M_{\alpha_P})\}$. Since $h_{\mathfrak{F}}$ is surjection, $\{M_{\alpha_1}, M_{\alpha_2}, \dots, M_{\alpha_P}\}$ is a finite subcover for X_2. Thus X_2 is $\mathfrak{F}\mathcal{F}Comp$.

Similar situations exist in other scenarios. ∎

Theorem 5.4 Let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be a $\mathfrak{F}\mathcal{F}O$ (resp. $\mathfrak{F}\mathcal{F}\delta O$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}O$, $\mathfrak{F}\mathcal{F}\mathcal{P}O$ and $\mathfrak{F}\mathcal{F}Zo$) function and X_2 be $\mathfrak{F}\mathcal{F}Comp$ (resp. $\mathfrak{F}\mathcal{F}\delta Comp$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Comp$, $\mathfrak{F}\mathcal{F}\mathcal{P}Comp$ and $\mathfrak{F}\mathcal{F}ZComp$). Then X_1 is $\mathfrak{F}\mathcal{F}Comp$.

Proof. Let $h_{\mathfrak{F}} : X_1 \to X_2$ be a $\mathfrak{F}\mathcal{F}ZO$ function and X_2 be $\mathfrak{F}\mathcal{F}ZComp$. Let $\{M_\alpha\}$ be a $\mathfrak{F}\mathcal{F}ZOCov$ for X_1. Since $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}Zo$, $\{h_{\mathfrak{F}}(M_\alpha)$ is a $\mathfrak{F}\mathcal{F}ZOCov$ of X_2. Since X_2 is $\mathfrak{F}\mathcal{F}ZComp$, $\{h_{\mathfrak{F}}(M_\alpha)\}$ contains a finite sub $\mathfrak{F}\mathcal{F}ZOCov$, namely $\{h_{\mathfrak{F}}(M_{\alpha_1}), h_{\mathfrak{F}}(M_{\alpha_2}), \dots, h_{\mathfrak{F}}(M_{\alpha_P})\}$. Then $\{M_{\alpha_1}, M_{\alpha_2}, \dots, M_{\alpha_P}\}$ is a finite subcover for X_1. Thus X_1 is $\mathfrak{F}\mathcal{F}Comp$. ∎

6 Conclusion

In this paper we have introduced the concept of Fermatean fuzzy (resp. δ, $\delta\mathcal{S}$, $\mathcal{P}$ and Z) connected & respective disconnectedness and Fermatean fuzzy (resp. δ, $\delta\mathcal{S}$, $\mathcal{P}$ and Z)-strongly connected and respective strongly disconnectedness in Fermatean fuzzy topological spaces. Also we studied some of their properties in $\mathfrak{F}\mathcal{F}ts$. Further, we have introduced the concepts of Fermatean fuzzy (resp. δ, $\delta\mathcal{S}$, $\mathcal{P}$ and Z) separated sets and their corresponding super connectedness, as well as Fermatean fuzzy (resp. δ, $\delta\mathcal{S}$, $\mathcal{P}$ and Z) compactness in Fermatean fuzzy topological spaces. We have also explored several fundamental properties of these concepts within $\mathfrak{F}\mathcal{F}ts$ (Fermatean fuzzy topological spaces).

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