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A Study of Watershed Hydrodynamical System using Advanced Weighted Correlation Measure of Pythagorean Neutrosophic Cubic Set

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Abstract:- The principal objective of this study is for analyzing how Pythagorean Neutrosophic Cubic Sets (PNCS), a hybrid framework that combines interval-valued and crisp Pythagorean Neutrosophic information, might be used to resolve complex uncertainty in hydrodynamic decisions. Although current models, such as Cubic Intuitionistic Fuzzy Sets (CIFS), offer a basis for making decisions, they frequently fall short in capturing higher-order uncertainty and independent degrees of indeterminacy. We provide a sophisticated weighted correlation measure created especially for the PNCS context in order to get over these restrictions, and we thoroughly characterize its basic mathematical characteristics. To guarantee computational accuracy and reproducibility, a multi-criteria decision-making (MCDM) algorithm has been deployed and carried out in behavior via Python using the developed correlation measures and a comparison analysis is made for CIFS and PNCS an explanation is given for choosing the PNCSs for this case study. A case study of a watershed management system that assesses four possible locations based on socioeconomic and environmental factors demonstrates the approach's practical usefulness. Policymakers and environmental engineers can use this study's insightful findings to create sustainable watershed management plans.

***Keywords**:Weighted Correlation Measure, Multi criteria Decision Making Approach, Pythagorean Neutrosophic Cubic Set, Watershed Hydrological system*

1. Introduction

While Karl Pearson developed the computing formula, Francis Galton patented the concept of correlation in the late 1880s[4]. PNCS was the generalized set of fuzzy cubic set. In 1965, Zadeh conceptualized the fuzzy set(FSs) [20]. The significant theory of Cubic Sets (CS) was introduced in 2012 and defined by Jun et al[12]. The Correlation Coefficient in Bipolar CF Sets along with applications was granted by Muhammad Riaz and Anam Habib[16]. Neutrosophic Sets (NS) are an entirely different notion that was first presented in 2002 by F. Smarandache [17]. In 2020, Garg et el discussed about the Intuitionistic fuzzy (IF) set environment with various decision making problems [5], [6], [7]. He early discussed about the ranking and correlation coefficient of CIFSs.[8], [9], [10], [14]. Young Bae Jun and Florentin Smarandache introduced the concept of Neutrosophic Cubic Sets (NCS) in 2017[13]. In 2019, Huiling Xue, Minrong Yu, and Chunfang Chen presented the Correlation Coefficient of NCS[18]. Pythagorean fuzzy sets(PFS) were first presented in 2014 by R.R. Yager [19]. A concept of the Pythagorean Cubic Fuzzy Set(PCFS) was presented in 2019 by F. Khana, M. S. Ali Khana, M. Shahzada, and S. Abdullah [15]. P. A. Ejegwa et al. investigated the idea of correlation in PFS in

2022[3]. In 2021, R. Jhansi et al. investigated the idea of Correlation Measures of Pythagorean Neutrosophic Set(PNS)[11]. The PNCS was first proposed in 2025 by Berna Joyce and Elvina Mary[2]. The concept of correlation coefficient has been extended to weighted correlation coefficient for PNCS. The aforementioned are the intended purposes of this contribution: in Sections 2, we construe the knowledge by adhering to the characteristics of PNCS and introduce the idea of the existence of weighted correlation measure of PNCS where there are two types of correlation measures. Then we review a problem of watershed management which was studied by Garg and Kaur for Cubic Intuitionistic Fuzzy Set and gave an algorithm according to the PNCS(Garg & Kaur, 2022). The datas provided were converted from Cubic Intuitionistic fuzzy set to Pythagorean Neutrosophic Cubic Set then according to the algorithm a decision was taken; in Section 3, we investigate about the results of the MADM watershed management; in Section 4, we communicate a type of comparison instruction of the existing set with PNCS weighed correlation measure and a conclusion is provided.

2. Materials and Methods

2.1 Mathematical Preliminaries

In order to comprehend the Weighted Correlation Measure, we must first explain what makes up a Pythagorean Neutrosophic Cubic Set (PNCS).

2.1.1. Definition [2]

Let $\hat{X}_{PNCS}$ be a non-empty set of the universe, a PNCS in $\hat{X}_{PNCS}$, having a form, $\hat{\mathfrak{K}}_{PNCS} = \{\langle \mathfrak{k}, \hat{\mathcal{J}}_{IVPNS}(\mathfrak{k}), \hat{\lambda}_{PNS}(\mathfrak{k})\rangle : \mathfrak{k} \in \hat{X}_{PNCS}\}$ where $\hat{\mathcal{J}}_{IVPNS}(\mathfrak{k})$ represent the Interval Valued Pythagorean Neutrosophic set (IVPNS) and $\hat{\lambda}_{PNS}(\mathfrak{k})$ represent the PNS. PNCS can be denoted as a pair $\hat{\mathfrak{K}}_{PNCS} = (\hat{\mathcal{J}}_{IVPNS}, \hat{\lambda}_{PNS})$

2.1.2. Example [2]

Let us consider a numerical example for a PNCS. Let $\hat{\mathfrak{K}}_{PNCS} \neq 0$ and let $\hat{\mathfrak{K}}_{PNCS} = \{s, u, q\}$ be the pair $\hat{\mathbb{S}}_{PNCS} = (\hat{\mathcal{J}}_{IVPNS}, \hat{\lambda}_{PNS})$ with the tabular representation given below,[Table 1]

Table 1. Example of Pythagorean Neutrosophic Cubic Set

$\hat{\mathfrak{K}}_{PNCS}$	$\hat{\mathcal{J}}_{IVPNS}(\mathfrak{k})$	$\hat{\lambda}_{PNS}(\mathfrak{k})$
s	$([0.5,0.8],[0.4,0.6],[0.5,0.8])$	$(0.25,0.48,0.62)$
u	$([0.1,0.4],[0.3,0.7],[0.4,0.8])$	$(0.30,0.58,0.71)$
q	$([0.2,0.3],[0.4,0.8],[0.3,0.5])$	$(0.27,0.65,0.47)$

Therefore, $\hat{\mathbb{S}}_{PNCS} = (\hat{\mathcal{J}}_{IVPNS}, \hat{\lambda}_{PNS})$

$$\hat{\mathbb{S}}_{PNCS} = \left\{ \begin{array}{l} \langle s, ([0.1,0.3],[0.4,0.6],[0.5,0.8]), (0.25,0.48,0.62)\rangle, \\ \langle u, ([0.1,0.4],[0.3,0.7],[0.4,0.8]), (0.30,0.58,0.71)\rangle, \\ \langle q, ([0.2,0.3],[0.4,0.8],[0.3,0.5]), (0.27,0.65,0.47)\rangle \end{array} \right\}.$$

2.2. The formation of Weighted Correlation Measure of PNCS

The standard correlation measure of a fuzzy set $(\hat{A}_{FS}, \hat{B}_{FS})$ includes covariance of $(\hat{A}_{FS}, \hat{B}_{FS})$ and Standard deviation of$(\hat{A}_{FS}, \hat{B}_{FS})$. During the Decision making method applied in real life, it requires a weight vector w_i where the sum of the weights equals 1 ie., $\sum_{i=1}^{n} w_i = 1$. By extending this concept to PNCS, we have definition of Weighted Correlation Measure of PNCS.

We look at two kinds of correlation formulas (Type-I, Type-II) to make sure that the suggested weighted measure is reliable and stable. This enables us to confirm that the ranking of options is consistent across various mathematical models.

2.2.1. Definition: Weighted Correlation Measure(Type-I)-The Product Denominator

Let $\hat{\mathfrak{K}}_{PNCS} \neq 0$. Let $\hat{\mathfrak{w}}_{PNCS_i}$ be the weight of the element of $\hat{\mathfrak{k}}_i \in \hat{\mathfrak{K}}_{PNCS}(i = 1,2,\dots,n)$ and $\sum_{i=1}^{n} \hat{\mathfrak{w}}_{PNCS_i} = 1$. Let $\hat{\mathbb{S}}_{PNCS} = (\hat{\mathcal{S}}_{IVPNS}, \hat{\varsigma}_{PNS})$ and $\hat{\mathbb{N}}_{PNCS} = (\hat{\mathcal{N}}_{IVPNS}, \hat{\varrho}_{PNS})$ be PNCSs,

$$\hat{\mathbb{S}}_{PNCS} = \left\{ \begin{array}{c} \left[T^{-}_{\hat{\mathcal{S}}_{IVPNS}}(\hat{\mathfrak{k}}_i), T^{+}_{\hat{\mathcal{S}}_{IVPNS}}(\hat{\mathfrak{k}}_i)\right], \left[I^{-}_{\hat{\mathcal{S}}_{IVPNS}}(\hat{\mathfrak{k}}_i), I^{+}_{\hat{\mathcal{S}}_{IVPNS}}(\hat{\mathfrak{k}}_i)\right], \left[F^{-}_{\hat{\mathcal{S}}_{IVPNS}}(\hat{\mathfrak{k}}_i), F^{+}_{\hat{\mathcal{S}}_{IVPNS}}(\hat{\mathfrak{k}}_i)\right], \\ \left(\hat{\varsigma}_{PNS_T}(\hat{\mathfrak{k}}_i), \hat{\varsigma}_{PNS_I}(\hat{\mathfrak{k}}_i), \hat{\varsigma}_{PNS_F}(\hat{\mathfrak{k}}_i)\right) \end{array} \right\} \text{ and}$$

$$\widehat{\mathbb{N}}_{PNCS} = \left\{ \begin{matrix} \left[T^{-}_{\widehat{\mathcal{N}}_{IVPNS}}(\hat{\mathfrak{t}}_i), T^{+}_{\widehat{\mathcal{N}}_{IVPNS}}(\hat{\mathfrak{t}}_i)\right], \left[I^{-}_{\widehat{\mathcal{N}}_{IVPNS}}(\hat{\mathfrak{t}}_i), I^{+}_{\widehat{\mathcal{N}}_{IVPNS}}(\hat{\mathfrak{t}}_i)\right], \left[F^{-}_{\widehat{\mathcal{N}}_{IVPNS}}(\hat{\mathfrak{t}}_i), F^{+}_{\widehat{\mathcal{N}}_{IVPNS}}(\hat{\mathfrak{t}}_i)\right], \\ \left(\hat{\varrho}_{PNS_T}(\hat{\mathfrak{t}}_i),\ \hat{\varrho}_{PNS_I}(\hat{\mathfrak{t}}_i),\ \hat{\varrho}_{PNS_F}(\hat{\mathfrak{t}}_i) \right) \end{matrix} \right\}.$$

Then the Weighted Correlation Measure (WCM) (Type-I) of $\widehat{\mathbb{S}}_{PNCS}$ and $\widehat{\mathbb{N}}_{PNCS}$

$$\wp_{\widehat{\mathfrak{w}}_{PNCS_{i}Type-I}}(\widehat{\mathbb{S}}_{PNCS}, \widehat{\mathbb{N}}_{PNCS}) = \frac{\zeta_{\widehat{\mathfrak{w}}_{PNCS_i}}(\widehat{\mathbb{S}}_{PNCS}, \widehat{\mathbb{N}}_{PNCS})}{\sqrt{\zeta_{\widehat{\mathfrak{w}}_{PNCS_i}}(\widehat{\mathbb{S}}_{PNCS}, \widehat{\mathbb{S}}_{PNCS}).\zeta_{\widehat{\mathfrak{w}}_{PNCS_i}}(\widehat{\mathbb{N}}_{PNCS}, \widehat{\mathbb{N}}_{PNCS})}} \rightarrow (1)$$

Let us have a numerical example for this definition.

2.2.2. Example

Let $\widehat{\mathbb{S}}_{PNCS}$ and $\widehat{\mathbb{N}}_{PNCS}$ be two PNCSs. Let $w = \{0.7, 0.3\}$ be the weight vectors for PNCSs. Here $\widehat{\mathbb{S}}_{PNCS}$ and $\mathbb{B}$ can be shown as

$$\widehat{\mathbb{S}}_{PNCS} = \left\{ \begin{matrix} [0.2, 0.4], [0.1, 0.4], [0.1, 0.3], (0.3, 0.2, 0.2) \\ [0.1, 0.5], [0.3, 0.6], [0.5, 0.8], (0.2, 0.4, 0.7) \end{matrix} \right\}$$

$$\widehat{\mathbb{N}}_{PNCS} = \left\{ \begin{matrix} [0.2, 0.3], [0.2, 0.5], [0.4, 0.6], (0.2, 0.3, 0.4) \\ [0.1, 0.3], [0.3, 0.5], [0.4, 0.6], (0.1, 0.3, 0.4) \end{matrix} \right\}$$

Then the WCM (Type-I)$\wp_{\widehat{\mathfrak{w}}_{PNCS_{i}Type-I}} = 0.734128$.

The WCM (Type-I) $\wp_{\widehat{\mathfrak{w}}_{iType-I}}$ is predicated on the Cauchy-Schwarz Inequality and the traditional Pearson Correlation. It is the most accurate way to determine the "linearity" between two sets. When you want to see how closely the change in Set A reflects the change in Set B and the data has a conventional distribution, it works quite well.

2.2.3. Definition Weighted Correlation Coefficient(Type-II)-The Maximum denominator

Let $\widehat{\mathfrak{K}}_{PNCS} \neq 0$. Let $\widehat{\mathfrak{w}}_{PNCS_i}$ be the weight of the element of and $\sum_{i=1}^{n} \widehat{\mathfrak{w}}_{PNCS_i} = 1$. Let $\widehat{\mathbb{S}}_{PNCS} = (\hat{\mathcal{S}}_{IVPNS}, \hat{\varsigma}_{PNS})$ and $\widehat{\mathbb{N}}_{PNCS} = (\widehat{\mathcal{N}}_{IVPNS}, \hat{\varrho}_{PNS})$ be PNCSs. Then the Weighted Correlation Measure (WCM) (Type-II) of $\widehat{\mathbb{S}}_{PNCS}$ and $\widehat{\mathbb{N}}_{PNCS}$

$$\wp_{\widehat{\mathfrak{w}}_{PNCS_{i}Type-II}}(\widehat{\mathbb{S}}_{PNCS}, \widehat{\mathbb{N}}_{PNCS}) = \frac{\zeta_{\widehat{\mathfrak{w}}_{PNCS_i}}(\widehat{\mathbb{S}}_{PNCS}, \widehat{\mathbb{N}}_{PNCS})}{\max\left\{\zeta_{\widehat{\mathfrak{w}}_{PNCS_i}}(\widehat{\mathbb{S}}_{PNCS}, \widehat{\mathbb{S}}_{PNCS}), \zeta_{\widehat{\mathfrak{w}}_{PNCS_i}}(\widehat{\mathbb{N}}_{PNCS}, \widehat{\mathbb{N}}_{PNCS})\right\}} \rightarrow (2)$$

This WCM (Type-II)-The maximum denominator is a Contemporary method used in fuzzy and neutrosophic logic. Extreme changes are less likely to affect the "Max" denominator. When dealing with truth, indeterminacy, and falsity over both intervals and points in PNCS, one set may have significantly more "energy" (variance) than the other. The Max denominator avoids one extremely uncertain set from unduly bloating the correlation result. Let us see a example for this Type-II.

2.2.4. Example

Let $\widehat{\mathbb{S}}_{PNCS}$ and $\widehat{\mathbb{N}}_{PNCS}$ be two PNCSs. Let $w = \{0.7, 0.3\}$ be the weight vectors for PNCSs. Here $\widehat{\mathbb{S}}_{PNCS}$ and $\mathbb{B}$ can be shown as

$$\widehat{\mathbb{S}}_{PNCS} = \left\{ \begin{matrix} [0.2, 0.4], [0.1, 0.4], [0.1, 0.3], (0.3, 0.2, 0.2) \\ [0.1, 0.5], [0.3, 0.6], [0.5, 0.8], (0.2, 0.4, 0.7) \end{matrix} \right\}$$

$$\widehat{\mathbb{N}}_{PNCS} = \left\{ \begin{matrix} [0.2, 0.3], [0.2, 0.5], [0.4, 0.6], (0.2, 0.3, 0.4) \\ [0.1, 0.3], [0.3, 0.5], [0.4, 0.6], (0.1, 0.3, 0.4) \end{matrix} \right\}$$

Then the WCM (Type-II) $\wp_{\widehat{\mathfrak{w}}_{iType-II}} = 0.655732$.

Both approaches successfully capture the relationship between PNCSs, although their levels of sensitivity differ, according to a comparison of the two suggested weighted correlation coefficients. With a greater correlation value (0.734128), Type-I (Product-based) is better suited for applications needing an exact measure of linear similarity. On the other hand, Type-II (Max-based) is a more reliable metric for making decisions when there is a lot of ambiguity. It guarantees that the connection is not overstated by a single attribute with high informational energy by producing a more cautious result (0.655732). In conclusion, the decision-maker's risk tolerance influences the formula selection; yet, the consistency of both outcomes in the positive direction validates the dependability of the suggested weighted strategy for PNCS.

2.3. Methodology

This study approaches a decision-making approach. PNCSs are regarded as corresponding to each alternative criteria pair in order to apply the suggested sequence of weighted correlation measures in the subsequent method. We suggest an algorithm to determine the weighted correlation measures of PNCSs.

2.3.1. Proposed Algorithm

Think of a decision-making obstacle with m selection requirements $\{A_1, A_2, \cdots, A_m\}$ and n possible outcomes $\{C_1, C_2, \cdots, C_n\}$. Let's say an expert has been tasked with assessing each A_i under C_i and providing their PNCS inclinations. Next, using the suggested measurements, the next stages are carried out to determine the best choice.

Step 1: Gather data to create a decision matrix $\mathbb{M}_{DM}$

$$\mathbb{S}_{DM} = \begin{matrix} S_1 \\ S_2 \\ \vdots \\ S_m \end{matrix} \begin{pmatrix} \hat{\eta}_{11} & \hat{\eta}_{12} & \cdots & \hat{\eta}_{1n} \\ \hat{\eta}_{21} & \hat{\eta}_{22} & \cdots & \hat{\eta}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\eta}_{m1} & \hat{\eta}_{m2} & \cdots & \hat{\eta}_{mn} \end{pmatrix}$$

Step 2: Consider the ideal feasible alternative $\widehat{\mathbb{O}}_{PNCS}$ which can be considered as a reference set.

Step 3: To calculate the measurement degrees between $\widehat{\mathbb{O}_{\iota}}_{PNCS}$ and $\widehat{\mathbb{O}}_{PNCS}$, use the recommended measurements *(Type-I)* $\wp_{w_{Type-I}}$ and *(Type-II)* $\wp_{w_{Type-II}}$ as provided in Eqs (1)–(2).

Step 4: Use an arbitrary number to arrange the element.

2.4. Application of Watershed Hydrodynamical system – An Illustrative Example

An example is provided below to demonstrate the practicability of the measures and technique stipulated.

2.4.1. Description of the Problem

In 2022, Harish Garg and Gagadeep Kaur discussed the watershed management decision making problem for Cubic Intuitionistic fuzzy environment [10]. In this article we are inducing this problem for Pythagorean Neutrosophic Cubic environment. The Problem stated as follows,

Watersheds are hydrological landscapes that serve as drainage or where water accumulates from bigger bodies of water such as streams, rivers, and ponds. The upper layer of flowing rivers makes up the majority of watersheds. They typically take the shape of canals, subdivisions of canals, reservoirs, etc. These watersheds are very helpful in preventing pollution and ensuring that water supplies are used effectively for both residential and commercial uses, preventing human involvement from causing harm. A thorough examination of watershed management has revealed that effective watersheds can be evaluated based on a variety of criteria, including water quality, thriving flora and fauna, the ability to act as an emergency drought rescuer, etc. Concentrating on the primary elements linked to the watershed's effectiveness, let's examine the primary elements of watershed management: $\boldsymbol{x_1, x_2, x_3, x_4}$ which fall into the categories of **"Water quality", "Habitat Provider", "Protection from emergent situations" and "Efficient use of other associated resources"**. Therefore, the tenacity of watershed management is the main emphasis of this case study Thus, this case study focuses on the determination of watershed hydrodyanmical management in **four different watershed locations** viz. $\boldsymbol{B_1, B_2, B_3, B_4}$ in the Indian state of Punjab, outside the city of Patiala. In order to identify which watershed area is best managed in comparison to all others, the study's goal is to compare different watershed regions with the ideal watershed area. In order to perform this analysis, data is gathered from[10] in the form of CIFSs, transformed to PNCSs, and compared to all other watershed management data. As per the algorithm in step 1 we have to collect the information as a decision matrix $\mathbb{S}_{DM}$. As per the information the collected data's were given in [Table 2]. The PNCS $\mathbb{A}$ is checked for weighted correlation measures *(Type-I)* $\wp_{w_{Type-I}}$ and *(Type-II)* $\wp_{w_{Type-II}}$ with that of PNCs gathering data related to the locations B_1, B_2, B_3, B_4. Their values are given in [Table 2]

Table 2. Input Preference of Watersheds in PNCSs

	x_1	x_2	x_3	x_4
B_1	$[0.2, 0.25], [0.4, 0.5], [0.3, 0.35], (0.2, 0.5, 0.3)$	$[0.4, 0.45], [0.32, 0.4], [0.2, 0.23], (0.7, 0.2, 0.1)$	$[0.55, 0.6], [0, 0.1], [0.35, 0.4], (0.6, 0.2, 0.2)$	$[0.1, 0.15], [0.55, 0.7], [0.2, 0.3], (0.3, 0.3, 0.4)$
B_2	$[0.18, 0.2], [0.48, 0.52], [0.3, 0.32], (0.3, 0.2, 0.5)$	$[0.6, 0.65], [0.03, 0.1], [0.3, 0.32], (0.4, 0.4, 0.2)$	$[0.2, 0.25], [0.54, 0.62], [0.18, 0.21], (0.3, 0.1, 0.6)$	$[0.4, 0.5], [0.2, 0.4], [0.2, 0.3], (0.2, 0.5, 0.3)$
B_3	$[0.7, 0.8], [0.1, 0.25], [0.05, 0.1], (0.1, 0.7, 0.2)$	$[0.5, 0.55], [0.2, 0.3], [0.2, 0.25], (0.3, 0.5, 0.2)$	$[0.4, 0.45], [0.2, 0.3], [0.3, 0.35], (0.15, 0.4, 0.45)$	$[0.4, 0.45], [0.23, 0.3], [0.3, 0.32], (0.9, 0, 0.1)$
B_4	$[0.2, 0.35], [0.1, 0.3], [0.5, 0.55], (0.2, 0.5, 0.3)$	$[0.32, 0.35], [0.19, 0.24], [0.44, 0.46], (0.2, 0.4, 0.4)$	$[0.3, 0.4], [0.3, 0.5], [0.2, 0.3], (0.2, 0.3, 0.5)$	$[0.5, 0.55], [0.35, 0.45], [0.05, 0.1], (0.2, 0.4, 0.4)$

Now moving on to the step 2 we need a ideal alternative $\widehat{\mathbb{O}}_{\iota PNCS}$ as a reference set. The ideal set were described as

$$\widehat{\mathbb{O}}_{\iota PNCS} = \begin{Bmatrix} \langle x_1, ([0.2, 0.35], [0.55, 0.75], [0.05, 0.10]), (0.1, 0.4, 0.5)\rangle \\ \langle x_2, ([0.6, 0.65], [0.12, 0.20], [0.20, 0.23]), (0.2, 0.4, 0.4)\rangle \\ \langle x_3, ([0.55, 0.6], [0.19, 0.27], [0.18, 0.21]), (0.2, 0.2, 0.6)\rangle \\ \langle x_4, ([0.5, 0.55], [0.35, 0.45], [0.05, 0.10]), (0.2, 0.4, 0.4)\rangle \end{Bmatrix}$$

Now into step-3 we gave utilize the proposed measure, *(Type-I)* $\wp_{w_{Type-I}}$ and *(Type-II)* $\wp_{w_{Type-II}}$.

3. Results and Discusion

The results of the hydrodynamic management of watersheds MADM problem using the suggested WCM *(Type-I)* $\wp_{w_{Type-I}}$ and *(Type-II)* $\wp_{w_{Type-II}}$ for PNCS are shown in this section. Four possible watershed locations B_1, B_2, B_3, B_4 were analyzed against a standardized ideal profile $\widehat{\mathbb{O}}_{i_{PNCS}}$ using a criteria weight vector $\widehat{\mathfrak{w}}_{PNCS} = \{0.18, 0.23, 0.30, 0.29\}$.

3.1. Numerical Results

According to the weights and by using the WCM of PNCS *(Type-I)* $\wp_{w_{Type-I}}$ and *(Type-II)* $\wp_{w_{Type-II}}$ we get the following numerical result.

(Type-I)

$$\wp_{w_{Type-I}}(B_1, \widehat{\mathbb{O}}_{\iota PNCS}) = 0.6334$$

$$\wp_{w_{Type-I}}(B_2, \widehat{\mathbb{O}}_{i PNCS}) = 0.7903$$

$$\wp_{w_{Type-I}}(B_3, \widehat{\mathbb{O}}_{i PNCS}) = 0.5161$$

$$\wp_{w_{Type-I}}(B_4, \widehat{\mathbb{O}}_{i PNCS}) = 0.7179$$

(Type-II)

$\wp_{w_{Type-II}}(\boldsymbol{B_1}, \widehat{\mathbb{O}}_{i_{PNCS}}) = \mathbf{0.6173}$

$\wp_{w_{Type-II}}(\boldsymbol{B_2}, \widehat{\mathbb{O}}_{i_{PNCS}}) = \mathbf{0.7167}$

$\wp_{w_{Type-II}}(\boldsymbol{B_3}, \widehat{\mathbb{O}}_{i_{PNCS}}) = \mathbf{0.4459}$

$\wp_{w_{Type-II}}(\boldsymbol{B_4}, \widehat{\mathbb{O}}_{i_{PNCS}}) = \mathbf{0.5536}$

The following outcomes were obtained by applying the two different weighted correlation measure, The outcome of Type-I: $\boldsymbol{B_2} > \boldsymbol{B_4} > \boldsymbol{B_1} > \boldsymbol{B_3}$, The outcome of Type-II: $\boldsymbol{B_2} > \boldsymbol{B_1} > \boldsymbol{B_4} > \boldsymbol{B_3}$

3.2. Discussion of the result obtained

The location $\boldsymbol{B_2}$ is emerged as the most correlated with the PNCSs values in both weighted correlation measures, despite there is a slight variation in the middle rankings $(\boldsymbol{B_1}\ \boldsymbol{and}\ \boldsymbol{B_4})$. This confirms that the location $\boldsymbol{B_2}$ is the best option for the watershed management. The consistency of $\boldsymbol{B_2}$ as the location with the highest average weighted correlation measure calculated by using the two types denominators such as PNCS *(Type-I)* $\wp_{w_{Type-I}}$ and *(Type-II)* $\wp_{w_{Type-II}}$ (i.e., Product vs. Max) shows that the suggested weighted correlation measure is dependable for solving complex environmental problems under uncertainty.

3.3. Software used for computation

This watershed management decision making problem was implemented using the Python software to calculate the two types of weighted correlation measures. By comparing the values of the calculated *(Type-I)* $\wp_{w_{Type-I}}$ and *(Type-II)* $\wp_{w_{Type-II}}$ weighted correlation measure it is obvious to say that the Type-II is more conservative method compared to the Type-I approach.

4. Comparision Analysis

In 2022 Garg and Kaur [10] made a case study in this watershed management decision making problem using correlation measure of Cubic Intuitionistic Fuzzy Sets(CIFS).

4.1. Numerical comparison analysis of rankings

The correlation values and the consequent priority rankings for four watershed locations $\boldsymbol{B_1}, \boldsymbol{B_2}, \boldsymbol{B_3}, \boldsymbol{B_4}$ by using the same weight vector $w = \{0.18, 0.23, 0.30, 0.29\}$ are outlined in the following table.

Table 3. Comparison of CIFS and PNCS

Method	$\boldsymbol{B_1}$	$\boldsymbol{B_2}$	$\boldsymbol{B_3}$	$\boldsymbol{B_4}$	**Ranking Order**
Garg & Kaur CIFSs K_3	0.7643	**0.9166**	0.7844	0.8986	$B_2 > B_4 > B_3 > B_1$
Garg & Kaur CIFSs K_4	0.7623	**0.8687**	0.7196	0.8088	$B_2 > B_4 > B_1 > B_3$
PNCSs *(Type-I)* $\wp_{w_{Type-I}}$	0.6334	**0.7903**	0.5161	0.7179	$B_2 > B_4 > B_1 > B_3$
PNCSs *(Type-II)* $\wp_{w_{Type-II}}$	0.7623	**0.8687**	0.7196	0.8088	$B_2 > B_1 > B_4 > B_3$

Location $\boldsymbol{B_2}$ is the best option for watershed management according to all four metrics, including the current CIFS and our suggested PNCS. The dependability and reliability of the suggested PNCS model are confirmed by this consistency across various fuzzy sets. In contrast to the CIFS approach $(0 \leq T + F \leq 1)$, the suggested PNCS method uses a larger decision space $(0 \leq T^2 + I^2 + F^2 \leq 2)$. Our approach takes indeterminacy into account separately, whereas Garg and Kaur's approach employs a linear constraint. In contrast to the higher, less differentiated CIFS values (ranging from 0.76 to 0.92), the more cautious and nuanced correlation values in PNCS (ranging from 0.51 to 0.79) reflect this. Here $\boldsymbol{B_3}$ is ranked higher than B_1 in the CIFS K_3 metric. Then B_1 consistently ranks higher than B_3 in our proposed PNCS measures $\wp_{w_{Type-I}}$ and $\wp_{w_{Type-II}}$. This implies that the evaluation of site B_3 deteriorates in comparison to B_1 when explicit indeterminacy and Pythagorean quadratic constraints are imposed, offering a more thorough review of environmental concerns that might be missed in a typical cubic intuitionistic environment. The comparison analysis shows that the suggested PNCS approach offers a more precise and mathematically sound ranking system, even though it is consistent with previous research in determining the optimal option B_2. The PNCS model provides an improved framework for managing large hydrological systems under high uncertainty by explicitly including indeterminacy and going beyond linear restrictions.

5. Conclusion

This study effectively addressed the demand for prioritizing criteria in complicated decision environments by extending the notion of correlation to a Weighted Correlation Coefficient for Pythagorean Neutrosophic Cubic Sets (PNCS). Two distinct weighted correlation measure of PNCS were proposed such as Type-I which uses a product centered denominator and Type-II which used a maximum-centered denominator. While the Type-II technique produces a more conservative and reliable estimate in situations of high uncertainty, the Type-I method offers a more accurate assessment of similarity.

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