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Analysis of Diseased One Prey Two Predator Model with Prey Harvesting in Holling type -II Functional Response

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Abstract: - The article develops and studies two possible diseases that can impact the predator population in an ecological model.SIS spread horizontallywhen an infected predator came into touch with a suspicious predator. Moreover, the second (SI illeness) is vertically transferred from environmental influence due toan external source. Diseases cannot be spread from preadtor to victim through touch or predation. Holling type II and linerafunctional response are utilized to show how healthy and suspectable predators feedon each other, while linearincidence is utilized for demonstrationg the evolution of disorders. All possible equilibrium locations were examined for this model. The model's local and global dynamics are examined by numerical simulation, as wellas the parameter sensitivity analysis.

Keywords: *Functional Response, Holling type-II, Hopf bifurcation, Stability.*

1. Introduction

Mathematical models in biology are generally grouped into two major classes: ecological models, which describe interactions among species in an ecosystem, and epidemiological models, which examine the spread of diseases within animal or human populations [15, 18]. The foundational SIS framework, first analyzed in [13], marked an important step in the development of epidemiological modelling. As research progressed, it became clear that combining ecological interactions with disease transmission leads to eco-epidemiological models, which offer a more realistic understanding of population dynamics [3]. Several studies have incorporated infection into prey–predator systems, particularly by introducing disease in the prey population [4, 6, 8, 23]. Other investigations have focused on models in which the predator species may become infected [8,9,17,20]. Kadhim and Azhar [11] explored a predator population experiencing two different types of infections under linear and Holling type-II functional responses. Additional dynamical analyses involving various ecological effects can be found in [14, 16, 21, 22]. The authors of [5,12,24,25] examined stability behaviour and the existence of Hopf bifurcation in intraguild predation models with ratio-dependent responses. Several researchers [26–29] have also employed different types of functional responses in both epidemiological and ecological models.

In recent years, there has been increasing interest in prey–predator frameworks that include disease in either the prey or the predator population [1,2,7,10,19]. However, to the best of our knowledge, no work has examined a three-species prey–predator system involving two interacting predator species, Holling type-II functional responses, and infection within the predator population. Motivated by this gap, we formulate and study a non-delayed eco-epidemiological model that captures these interactions.

2. Mathematical Formulation

The model captures the interactive behaviour among the prey and predator populations. The growth of prey is influenced by birth, natural mortality, competition, and predation, while the predators grow by consuming prey and decline due to natural death and intraspecies competition.

$$\frac{dP}{dT} = \frac{lP}{1+k_1q+k_2r} - b_0P - c_0P^2 - \frac{a_1PQ}{b_1+c+1P+P^2} - \frac{a_2PR}{b_2+c_2P+P^2} - H_1\mathrm{P},$$

$$\frac{dQ}{dT} = \frac{f_1a_1PQ}{b_1+c_1P+P^2} - u_1Q^2 - v_1QR - d_1Q,$$

$$\frac{dR}{dT} = \frac{f_2a_2PR}{b_2+c_2P+P^2} - u_2R^2 - v_2QR - d_2R$$

Table 1: Description of the parameters used in the model

Parameter	Biological Representation
l	Birth rate of prey
b_0	Natural death rate of prey
c_0	Intraspecies competition rate of prey
k_1	Fear effect caused by predator Q
k_2	Fear effect caused by predator R
a_1	Attack rate of predator Q on prey P
a_2	Attack rate of predator R on prey P
b_1, b_2	Half-saturation constants for predators Q and R
c_1, c_2	Inhibitory effect constants for predators Q and R
f_1, f_2	Conversion efficiency rates
u_1, u_2; v_1, v_2	Intraspecies competition rates of predators
d_1, d_2	Natural death rates of predators

Accompanied by initial conditions $F(0) \geq 0$, $G(0) \geq 0$, and $H(0) \geq 0$,that there are sixteen parameters which can be reduced to make the model easy to deal with it by dimensionless parameters and variables to simplify the system.

$$t = lT, p = \frac{c_0}{l}p, q = \frac{a_1c_0}{l^3}q, r = \frac{a_2c_0^2}{l^3}R, m_1 = \frac{k_1l^3}{a_1c_0^2}, m_2 = \frac{k_2l^3}{a_2c_0^2}, m_3 = \frac{b_0}{l},$$

$$m_4 = \frac{b_1c_0^2}{l^2}, m_5 = \frac{c_1c_0}{l}, m_6 = \frac{b_2c_0^2}{l^2}, m_7 = \frac{c_2c_0}{l},$$

$$m_8 = \frac{h_1}{l}, m_9 = \frac{f_1a_1c_0}{l^2},$$

$$m_{10} = \frac{u_1 l^2}{a_1 c_0^2}, m_{11} = \frac{v_1 l^2}{a_2 c_0^2}, m_{12} = \frac{d_1}{l}, m_{13} = \frac{f_2 a_2 c_0}{l^2}, m_{14} = \frac{u_2 l^2}{a_2 c_0^2}, m_{15} = \frac{v_2 l^2}{a_1 c_0^2}, m_{16} = \frac{d_2}{l}.$$

By accordance with the following dimensionless system

$$\frac{dp}{dt} = \frac{p}{1 + m_1 q + m_2 r} - m_3 p - p^2 - \frac{pq}{m_4 + m_5 p + p^2} - \frac{pr}{m_6 + m_7 p + p^2} - m_8 p,$$
$$\frac{dq}{dt} = \frac{m_9 pq}{m_4 + m_5 p + p^2} - m_{10} q^2 - m_{11} qr - m_{12} q$$
$$\frac{dr}{dt} = \frac{m_{13} pr}{m_6 + m_7 p + p^2} - m_{14} r^2 - m_{15} qr - m_{16} r$$

With $p\ (0) \geq 0,\ q0) \geq 0$, and $r\ (0) \geq 0$. Note that there is reduced in number of parameters from seventeen in the system to fifteen in the system. It is easy to exam about all the functions of the system are continuous and have continuous partial derivatives on the following positive three-dimensional space R^+: $p\ (0) \geq 0, q\ (0) \geq 0, r\ (0) \geq 0$. So, the solution of the system exists and unique. Moreover, with the non-negative initial conditions all the solution of the system uniformly bound as illustrated in the following theorem.

3. Positivity and Boundedness

In this section we discuss the positivity and bounded solution of the system (2.2)

Positivity and Boundedness of Solutions

THEOREM 3.1 In the region $\Lambda \subseteq R^+_3$ all solutions of the system remain positive and uniformly bounded, Were

$$[\Lambda = (p, q, r) \in R_3^+ : 0 \leq p \leq (1 - m_3), p + \frac{q}{mq} + \frac{r}{m_{13}} \leq \frac{2(1 - m_3 - m_8)(1 - m_3)}{\mu}]$$

And

$$\mu = min 1 - m_3 - m_8, m_{12}, m_{16}$$

Proof:

From the first equation of the system, we have

$$\frac{dp}{dt} \leq (1 - m_3)p - p^2 = (1 - m_3)p\left(1 - \frac{p}{(1 - m_3)}\right)$$

This implies that

$$\lim_{t \to \infty} supp(t) \leq (1 - m_3)$$

Let

$$w = p + \frac{p}{m_9} + \frac{r}{m_{13}}$$

Differentiating w with respect to t gives

$$\frac{dw}{dt} = \frac{p}{1 + m_1 q + m_2 r} - m_3 p - p^2 - \frac{pq}{m_4 + m_5 p + p^2} - \frac{pr}{m_6 + m_7 p + p^2} - m_8 p + \frac{pq}{m_4 + m_5 p + p^2} - \frac{m_{10}}{m_9} q^2 - \frac{m_{11}}{m_9} qr - \frac{m_{12}}{m_9} q + \frac{pr}{m_6 + m_7 p + p^2} - \frac{m_{14}}{m_{13}} r^2 - \frac{m_{15}}{m_{13}} qr - \frac{m_{16}}{m_{13}} r$$

Simplifying, we obtain

$$\frac{dw}{dt} \leq (1 - m_3 - m_8)p - \frac{m_{12}}{m_9}q - \frac{m_{16}}{m_{13}}r$$

Using the definition of μ, we further derive

$$\frac{dw}{dt} \leq 2(1 - m_3 - m_8)p - \mu\left(p + \frac{q}{m_9} + \frac{r}{m_{13}}\right)$$

Since biologically $1 - m_3 - m_8 > 0$, it follows that

$$\frac{dw}{dt} + \mu w \leq 2(1 - m_3 - m_8)(1 - m_3)$$

Applying Gron wall's inequality yields

$$w(t) \leq \frac{2(1 - m_3 - m_8)(1 - m_3)}{\mu} \ as\ t \to \infty$$

Thus p(t), q(t), r(t) remains positive and uniformly bounded in Λ.

4. The Existence of Equilibrium Points

In the below study, it appears there are at most in system six equilibrium points which will be studied of the stability at each of these points, explicit computation appears as follows:

- The equilibrium point is trivial as $E_0(0, 0, 0)$.
- $E_1(\hat{p}, 0, 0), \hat{p} = 1 - m_3 - m_8$ which exist under the survival condition $1 - m_3 - m_8 > 0$.
- $E_2 = (\hat{p}, \hat{q}, 0), \hat{q} = \frac{1}{m_{10}}\left[\frac{m_9\hat{p}}{m_4+m_5\hat{p}+\hat{p}^2} - m_{12}\right]$ *where* $\hat{p}$ is the positive root of the equation.

$$N_1p^7 + N_2p^6 + N_3p^5 + N_4p^4 + N_5p^3 + N_6p^2 + N_7p + N_8 = 0$$

Were

There is atleast one positive $m_{12} < \frac{m_9\hat{p}}{m_4+m_5\hat{p}+\hat{p}^2}$,

$N_1 > 0, N_8 < 0$,

Or $N_1 < 0, N_8 > 0$

- First predator free equilibrium is denoted by $E_3 = (\hat{p}, 0, \hat{r})$ where $\hat{r}$ is given by

  $\hat{r} = \frac{1}{m_{14}}\left[\frac{m_{13}\hat{p}}{m_6+m_7\hat{p}+\hat{p}^2} - m_{16}\right]$,

  $\hat{p}$ is positive root,

  $D_1p^7 + D_2p^6 + D_3p^5 + D_4p^4 + D_5p^3 + D_6p^2 + D_7p + D_8 = 0,$

  $m_{16} < \frac{m_{13}\hat{p}}{m_6+m_7\hat{p}+\hat{p}^2}, D_1 > 0, D_8 < 0,$

  $D_1 < 0, D_8 > 0,$

- $E_4 = (p^*, q^*, z^*), g_1 = \frac{1}{1+m_1q+m_2r} - m_3 - p - \frac{q}{m_4+m_5p+p^2} - \frac{r}{m_6+m_7p+p^2} - m_8 = 0,$

  $g_2 = \frac{m_9p}{m_4+m_5p+p^2} - m_{10}q - m_{11}r - m_{12} = 0,$

  $g_3 = \frac{m_{13}p}{m_6+m_7p+p^2} - m_{14}r - m_{15}q - m_{16} = 0,$

  $r = \frac{m_{13}p-(m_6+m_7p+p^2)(m_{15}q+m_{16})}{m_{14}(m_6+m_7p+p^2)},$

  Substitute the equations

$$h_1(p,q) = \frac{1}{1+m_1q+m_2\left[\frac{m_{13}p-(m_6+m_7p+p^2)(m_{15}q+m_{16})}{m_{14}(m_6+m_7p+p^2)}\right]} - m_3 - p - \frac{q}{m_4+m_5p+p^2} - \frac{m_{13}p-(m_6+m_7p+p^2)(m_{15}q+m_{15})}{m_{14}(m_6+m_7p+p^2)}$$

$$p_8 = 0$$

$$h_2(p,q) = \frac{m_9p}{m_4+m_5p+p^2} - m_{10}q - m_{12} - m_{11}\left[\frac{m_{13}p-(m_6+m_7p+p^2)(m_{15}q+m_{16})}{m_{14}(m_6+m_7p+p^2)}\right] = 0$$

$$h_1(0,y) = E_1q^2 + E_2q + E_3 = 0$$
$$h_2(0,y) = C_1q + C_2 = 0$$
$$E_1 = m_6^2(m_6m_{14} - m_4m_{15})(m_1m_{14} - m_2m_{15})$$
$$E_2 = m_6^2m_{14}^2(1 + m_1m_4m_3 + m_1m_4m_8) - m_4m_6^2m_{14}(m_{15} + m_1m_{15} + m_1) - m_2m_4m_6^3m_{14}m_{15}(m_3 + m_8) - m_2mm_6^3m_{14}m_16 + 2m_2m_4m_6^2m_{15}m_{16}$$
$$E_3 = -m_4m_6^3m_{14}^2(1 - m_3 - m_8) - m_4m_6^2m_{14}m_{16}[1 + m_2m_6(m_3 + m_8)] + m_2m_4m_6^2m_{16}^2$$
$$C_1 = m_4m_6(m_{10}m_{14} - m_{11}m_{15})$$
$$C_2 = m_4m_6[m_{14}(m_{12} + m_{13}) - m_{11}m_{16}]$$
$$q_1 = \frac{-E_1 - \sqrt{E_2^2 - 4E_1E_3}}{2E_1} > 0$$
$$q_2 = \frac{-E_2 + \sqrt{E_2^2 - 4E_1E_3}}{2E_1} < 0$$
$$q_3 = -\frac{C_2}{C_1} > 0$$

The following conditions are satisfied

$E_1 > 0$

$E_3 > 0$

$C_1 > 0, C_2 < 0$

or

$C_1 < 0, C_2 > 0$

$q_1 > q_3$

$$\frac{dp}{dq} = -\frac{\frac{\partial h_2}{\partial q}}{\frac{\partial h_2}{\partial p}} > 0$$

$$m_{13}p^* > (m_6 + m_7p^* + p^{*2})(m_{15}q^* + m_{16})$$

By finding the jacobian matrix

$$\begin{matrix} p\frac{\partial g_1}{\partial p} + g_1 & p\frac{\partial g_1}{\partial q} & p\frac{\partial g_1}{\partial r} \\ q\frac{\partial g_2}{\partial p} & q\frac{\partial g_2}{\partial q} + g_2 & q\frac{\partial g_2}{\partial r} \\ r\frac{\partial g_3}{\partial p} & r\frac{\partial g_3}{\partial q} & r\frac{\partial g_3}{\partial r} + g_3 \end{matrix}$$

$$\frac{\partial g_1}{\partial p} = -1 + \frac{q(m_5 + 2p)}{B_2^2} + \frac{r(m_7 + 2p)}{B_3^2}$$

$$\frac{\partial g_1}{\partial q} = -\frac{m_1}{B_1^2} - \frac{1}{B_2}$$

$$\frac{\partial g_1}{\partial r} = -\frac{m_2}{B_1^2} - \frac{1}{B_3}$$

$$\frac{\partial g_2}{\partial p} = \frac{m_9(m_4 - p^2)}{B_2^2}$$

$$\frac{\partial g_2}{\partial q} = -m_{10}$$

$$\frac{\partial g_2}{\partial r} = -m_{11}$$

$$\frac{\partial g_3}{\partial p} = \frac{m_{13}(m_6 - p^2)}{B_3^2}$$

$$\frac{\partial g_3}{\partial q} = -m_{15}$$

$$\frac{\partial g_3}{\partial r} = -m_{14}$$

$$J(E0) = \begin{matrix} 1 - m_3 - m_8 & 0 & 0 \\ 0 & -m_{12} & 0 \\ 0 & 0 & -m_{16} \end{matrix}$$

$J(E_0)$ can be written as

$$\lambda_1 = 1 - m_3 + m_8$$

$$\lambda_2 = -m_{12} < 0$$

$$\lambda_3 = -m_{16} < 0$$

The above is locally asymptotically stable when it follows the condition

$$1 < m_3 + m_8$$

$$\begin{bmatrix} \hat{p} & -m_1\hat{p} + \dfrac{\hat{p}}{m_4 + m_5\hat{p} + \widehat{p^2}} & -\dfrac{\hat{p}}{m_6 + m_7\hat{p} + \widehat{p^2}} \\ 0 & \dfrac{m_9\hat{p}}{m_4 + m_5\hat{p} + \widehat{p^2}} - m_{12} & 0 \\ 0 & 0 & \dfrac{m_{13}\hat{p}}{m_6 + m_7\hat{p} + \widehat{p^2}} - m_{16} \end{bmatrix}$$

$$\lambda_{11} = -\hat{p}$$

$$\lambda_{12} = \frac{m_9\hat{p}}{m_4 + m_5\hat{p} + \widehat{p^2}} - m_{12}$$

$$\lambda_{13} = \frac{m_{13}\hat{p}}{m_6 + m_7\hat{p} + \widehat{p^2}} - m_{16}$$

The above-mentioned values is negative, then it is said to be locally asymptotically stable if the below condition satisfies

$$\frac{m_9\hat{p}}{m_4 + m_5\hat{p} + \widehat{p^2}} < m_{12} \frac{m_{13}\hat{p}}{m_6 + m_7\hat{p} + \widehat{p^2}} < m_{16}$$

The Jacobian matrix can be written as

$$J(E_2) = [a_{ij}^{\vee}]$$

Where

$$a_{11}^{\vee} = p^{\vee}\left[-1 + \frac{q^{\vee}(m_5 + 2p)^{\vee}}{m_4 + m_5p^{\vee} + \hat{p}}\right]$$

$$a_{12}^{\vee} = \frac{-m_1p^{\vee}}{B_1^{2\vee}}$$

$$a_{13}^{\vee} = \frac{-m_2p^{\vee}}{B_1^{2\vee}} - \frac{p^{\vee}}{m_6 + m_7p^{\vee} + \widehat{p^2}}$$

$$a_{21}^{\vee} = \frac{m_9(m_4 - p^{\vee})^2q^{\vee}}{m_4 + m_5p^{\vee} + \widehat{p^2}}$$

$$a_{22}^{\vee} = -m_{10}q^{\vee}$$

$$a_{23}^{\vee} = -m_{11}q^{\vee}$$

$$a_{31}^{\vee} = a_{32}^{\vee} = 0$$

$$a_{33}^{\vee} = \frac{m_{13}p^{\vee}}{m_6 + m_7\hat{p} + \widehat{p^2}} - m_{16}$$

With

$$B_1^{\vee} = 1 + m_1\hat{y}$$

$$B_2^{\vee} = m_4 + m_5p^{\vee} + p^{\vee 2}$$

$$B_3^{\vee} = m_6 + m_7p^{\vee} + p^{\vee 2}$$

The characteristic equation of $J(E_2)$ is

$$(\lambda^2 - T_{r_1}\lambda + Det_1)(a_{33}^{\vee} - \lambda) = 0$$

i.e.,

$$\lambda^2 - (a_{11}^{\vee} + a_{22}^{\vee})\lambda + a_{11}^{\vee}a_{22}^{\vee} - a_{12}^{\vee}a_{21}^{\vee}$$

If the condition

$$\frac{m_{13}p^{\vee}}{B_3 \vee} < m_{15}q^{\vee} + m_{16}$$

$$\frac{q^{\vee}(m_5 + 2p^{\vee})}{B_2^{2\vee}} < 1$$

$$p^{\vee 2} < m_4$$

These are the activated eigen values of $J(E_2)$ and root of the characteristic equation arrives negative real parts.

Similarly, jaccobian matrix can be written as

$J(E_3) = [\overline{a_{ij}}]$

Then the $J(E3)$ equation can be written as

$$(\lambda^2 - Tr_2\lambda + Det_2)(\overline{a_{22}} - \lambda) = 0$$

i.e.,

$$\left(\lambda^2 - (\overline{a_{11}} + \overline{a_{33}})\lambda + \overline{a_{(11)}}\,\overline{a_{(33)}} - \overline{a_{(13)}}\,\overline{a_{(31)}}\right)(\overline{a_{22}} - \lambda) = 0$$

If the condition

$$\frac{m_9\bar{p}}{m_4 + m_5\bar{p} + \overline{p^2}} < m_{11}\bar{r} + m_{12}$$

$$\frac{\bar{r}(m_7 + 2p^2)}{m_6 + m_7\bar{p} + \overline{p^2}} < 1$$

$$p^{\vee}2 < m_6$$

Arrives this shows that the eigen values of $J(E_3)$ and roots of equation have a negative real part, now the jaccobian matrix of interior equilibrium point can be represented as

$E_4 = (p^*, q^*, r^*)$

$J(E_4) = [a_{ij}]$

Were,

$$a_{11} = p^*\left(-1 + \frac{(m_5 + 2p^*)q^*}{B_2^{2*}} + \frac{(m_7 + 2p^*)r^*}{B_3^{2*}}\right)$$

$$a_{12} = p^*\left(\frac{m_1}{B_1^{2*}} + \frac{1}{B_2^*}\right)$$

$$a_{13} = -p^*\left(\frac{m_2}{B_1^*} + \frac{1}{B_3^*}\right)$$

$$a_{21} = q^*\frac{m_9(m_4 - p^{*2})}{B_2^{2*}}$$

$$a_{22} = -m_{10}q^*$$

$$a_{23} = -m_{11}q^*$$

$$a_{31} = r^* \frac{m_{13}(m_6 - p^{*2})}{B_3^{*2}}$$

$$a_{32} = -m_{15} r^*$$

$$a_{33} = -m_{14} r^*$$

Were

$$B_1^* = 1 + m_1 q^* + m_2 r^*$$

$$B_2^* = m_4 + m_5 p^* + p^{*2}$$

$$B_3^* = m_6 + m_7 p^* + p^{*2}$$

The below theorem excute the local stability criterion of interior equilibrium point. Assume that system has an interior equilibrium point, it is locally asymptotically stable if the following conditions are met

$$\frac{(m_5 + 2p^*)q^*}{B_2^{*2}} + \frac{(m_7 + 2p^*)r^*}{B_3^{*2}} < 1$$

$$p^{*2} < \min\{m_4, m_6\}$$

$$m_{11} m_{15} < m_{10} m_{14}$$

$$\frac{m_9 m_{15}(m_4 - p^{*2})}{m_{10} m_{13} B_2^{*2}} < \frac{m_6 - p^{*2}}{B_3^{2*}} < \frac{m_9 m_{14}(m_3 - p^{*2})}{m_{11} m_{13} B_2^{*2}}$$

$$m_{11} q^* \frac{m_{13}(m_6 - p^{2*})}{B_3^{2*}} < m_{15} p^* \left(\frac{m_2}{B_1^{2*}} + \frac{1}{B_3^*} \right)$$

5. ensitivity Analysis:

The outcomes of deterministic model systems are governed by the input parameters of model system, which may show some uncertainty in their selection or determination. We employed a local sensitivity analysis to evaluate the impact of uncertainty and the sensitivity of the outputs of numerical simulations to variations in each parameter of the system using the method of partial rank correlation coefficients (PRCC) and Latin hypercube sampling. The parameters with significant impact on the outcomes of numerical simulations are determined by sensitivity analysis. To generate the LHS matrices. We assume that all the model parameters are uniformly distributed. Notice that the PRCC value lies between -1 and 1. Negative (positive) values represent a negative (positive) correlation of the model outcomes with its parameter. A negative (positive) correlation indicates that a negative (positive) change the parameter will decrease (increase) the model output. Bigger absolute value of the PRCC represents the larger correlation of the parameter with the outcome. The PRCC values are represented by bar graphs in Figure 5.

6. Hopf Bifurcation

THEOREM 6.1 (Hopf bifurcation at the coexistence equilibrium) Consider system (2.2) and let $E^*(h_1) = (P^*, Q^*, R^*)$ be a coexistence equilibrium depending smoothly on the harvesting parameter h_1. Let the characteristic polynomial of the Jacobian matrix $J^* = J(P^*, Q^*, R^*)$ at E^* be

$$\lambda^3 + A_1(h_1)\lambda^2 + A_2(h_1)\lambda + A_3(h_1) = 0,$$

where $A_1(h_1)$, $A_2(h_1)$ and $A_3(h_1)$ are defined

$A_1 = -\mathrm{tr}(J^*)$, A_2 = sum of the principal 2×2 minors of J^*, $A_3 = -\det(J^*)$.

Assume that there exists $h_1^* > 0$ such that

$$A_1(h_1^*) > 0,$$

$$A_2(h_1^*) > 0,$$

$$A_3(h_1^*) > 0,$$

$$A_1(h_1^*)A_2(h_1^*) = A_3(h_1^*)$$

and the transversality condition

$$\left|\frac{d}{dh_1}\big(A_1(h_1)A_2(h_1) - A_3(h_1)\big)\right|_{h_1=h_1^*} \neq 0$$

Then, as h_1 passes through h_1^*,system(2.2) undergoes a Hopf bifurcation h_1^*,system(2.2) undergoes a Hopf bifurcation at the equilibrium $E^*(h_1^*)$, and a family of nontrivial periodic solutions bifurcates from E^*.

7. Numerical Simulation

In this part, we give the numerical simulation for the system (2.2) using MATLAB Ode45 software with 500 step size and, set of parameter values $p_1 = 0.2, p_2 = 0.2, p_3 = 0.01, p_4 = 0.3, p_5 = 0.2, p_6 = 0.3, p_7 = 0.2, p_8 = 0.1, p_9 = 0.7$, and $p_{10} = 0.2, p_{11} = 0.1, p_{12} = 0.1, p_{13} = 0.7, p_{14} = 0.2, p_{15} = 0.1, p_{16} = 0.1$ The rate of the u_1 does not affect the dynamics of the system (2.2). So, we fix the value for u_1 as 0.5, 0.7, 0.9. The effect of changing the conversion rate u_2 of vulnerable predator from prey on the dynamical changes of the system (ref eqn2) is investigated by setting $h_2 = 0.6, 0.8$ and leaving the other parameters the same as given above. Then, the solutions diagram of the system are plotted in the Figure. All prey species start to decline, and all predator species start to increase when the exchange rate between prey and vulnerable predator grows, yet the system remains at an asymptotic stable coexistence point. Now, under the impact of changing the infection rate of u_3, we study the proposed system numerically with $u_3 = 0.3, 0.4$ and then time evaluation plots are given in Figure 1. Again, the system has an asymptotically stable equilibrium point. According to Figure 2, the value of the prey species and susceptible predator species decreases as the infection rates increase, while the prey species and infected predator species can begin to increase. For the small rate of recovering as $u_4 = 0.1$ then the system (2.2) is locally asymptotically stable, but for the small increment of $u_4 = 0.3$ the prey will die out and survival susceptible to predator and infected predator only. Then, the rate at which the effect $u_5 = u_8 = 0.3, 0.8$ and the results which have been shown in Figure 3. For the rate of $u_6 = u_9 = 0.2, 0.4$ then the system (2.2) shows that the prey species are extinct and longtime survival only other two species, which is projected in Figure 3. Then the value of $u_7 = u_{10} = 0.6, 0.8$

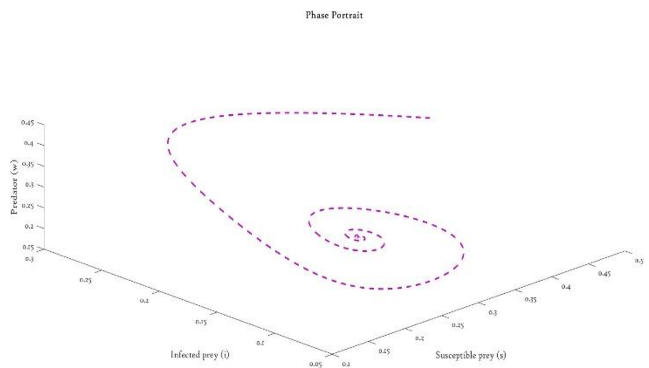

Figure 1: Phase portrait of the system (2,2)

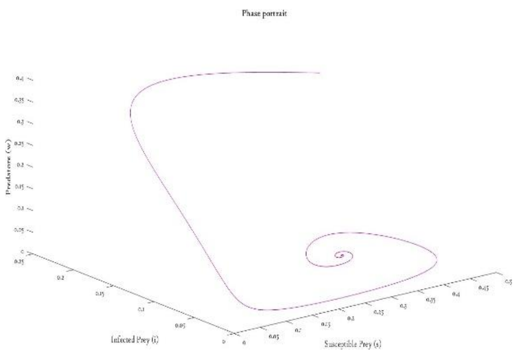

Figure 2: Phase portrait of harvesting species

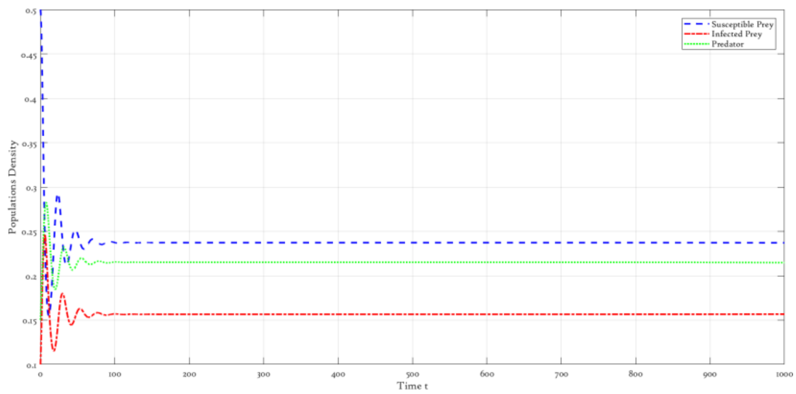

Figure 3: Time Series of the system (2.2)

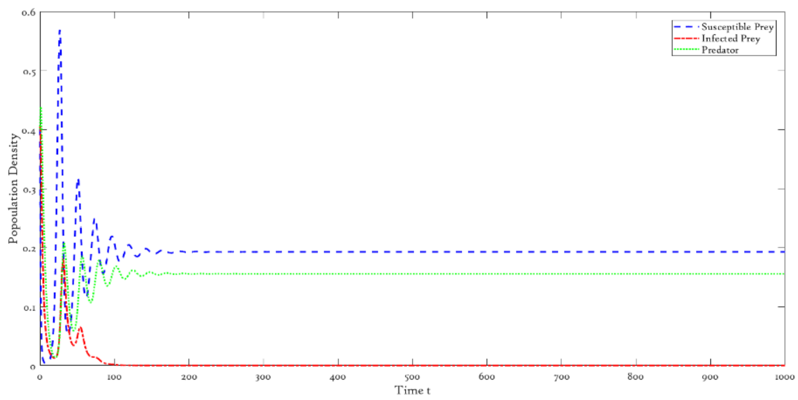

Figure 4: Time Series of harvesting species

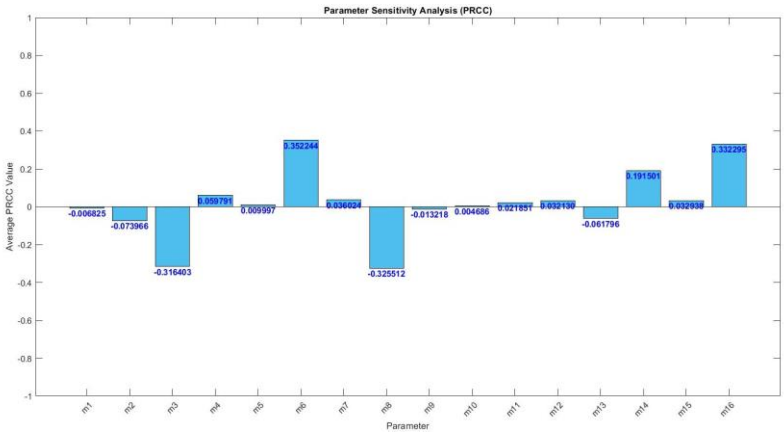

Figure 5: Sensitivity Analysis Bar Graph

8. Conclusion

We designed and evaluated the Holling type II functional response in the sick predator-prey system in this work. The model is composed of three separate, nonlinear differential equations that represent the behavior of three distinct populations: prey F, predators G that are prone to becoming ill, and predators H that are already ill. To assess the role of sensitivity and uncertainty of the numerical simulations with respect to variations in each parameter of the model system, we have also employed a local sensitivity analysis using PRCC. To validate our analytical results and understand the impact of changing the infection rates u_3, u_7 and recovery rates u_4, u_8 on the system's dynamical changes (2.2), the system 2.2) was numerically examined for the same set of initial conditions and various parameters, yielding the following results:

1. There is no periodic oscillation existing in system 2.2 through a set of fictitious parameters as in numerical simulation.
2. Changing the parameters u_i, $i = 2, 3, \cdots, 10$ and leaving the rest of the parameter values unchanged from the numerical simulation section has no impact on the dynamical character of the

system (2.2), and the solution trajectories are getting closer to the point of interior equilibrium.

3. One of the most important outcomes is that, when both diseases are present at the same time, the ecosystem cannot be destroyed.

4. The following parameters u_4, u_5, u_6, u_8 and u_9 plays a crucial role in the proposed diseased predator-prey system.

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