

# Synergistic Impact of Dual Split Injection on the Combustion, and Emission Characteristics of Crdi Diesel Engine Fueled with Carbon Black Nanoparticle-Enriched Dairy Scum Biodiesel B20 Blend

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## Abstract:

This study investigates the combined influence of dual split injection and carbon-black nanoparticle enrichment on the performance, combustion, and emission characteristics of a single-cylinder CRDI diesel engine fueled with dairy-scum biodiesel (DSOBD) blends. Experiments were conducted at varying injection timings, pilot–main mass fractions, and loads, comparing single injection with optimized dual split injection. Results show that the optimized split injection strategy—main injection at 14° BTDC and pilot injection at 7° BTDC with a 90:10 mass fraction—achieved up to 8.6% higher brake thermal efficiency (BTE) and 6–7% lower brake specific fuel consumption (BSFC) than single injection. Carbon-black nanoparticle addition shortened ignition delay (ID) by 18%, reduced combustion duration (CD) by 12%, and increased peak pressure (PP) by 4–5%, indicating faster and more efficient combustion. At full load, the optimized split strategy reduced smoke opacity by 23.1%, HC emissions by 15–17%, CO emissions by 12–14%, and NO<sub>x</sub> emissions by 8–10% compared to single-injection biodiesel operation. These improvements are attributed to enhanced atomization, better air–fuel mixing, and controlled combustion phasing. The findings confirm that integrating carbon-black nanoparticles with dairy-scum biodiesel under optimized dual split injection delivers a balanced improvement in efficiency and emissions, offering a viable pathway for sustainable CRDI diesel engine operation.

**Keywords:** Dairy-scum biodiesel, Carbon-black nanoparticles, Dual split injection, CRDI diesel engine, Sustainable fuels, Nanofuel blends

## • Introduction

The global urgency to reduce greenhouse gas emissions and to diversify liquid-fuel supplies has intensified research into sustainable, waste-derived fuels for compression-ignition (CI) engines. Biodiesel produced from

industrial wastes couples circular-economy benefits with renewable fuel production; among these wastes, dairy-industry scum (hereafter “dairy scum”) is an attractive feedstock due to its widespread availability and appreciable lipid content that can be converted to fatty-acid methyl esters (FAMES). Dairy-scum-derived biodiesel (DSOBD) has been demonstrated to meet many key fuel-property specifications after optimized extraction and transesterification, yet its altered viscosity, volatility and ignition characteristics relative to petroleum diesel can affect spray, atomization and combustion in modern high-pressure common-rail direct-injection (CRDI) engines[1–5]. To address these combustion and emissions challenges, two complementary approaches have received broad attention: (i) addition of nano-scale fuel additives (metal-oxide and carbonaceous nanoparticles) to improve atomization, thermal conductivity and oxidation pathways; and (ii) advanced injection strategies — especially split/dual-pulse injection (pilot + main, multiple pilots, variable mass split) in CRDI systems — to shape early heat release and reduce soot formation without excessive NO<sub>x</sub> penalties. Experimental and numerical studies show that suitable nanoparticles (e.g., CeO<sub>2</sub>, Al<sub>2</sub>O<sub>3</sub>, TiO<sub>2</sub>, CuO, and carbon-based particles including carbon black) can shorten ignition delay, increase cetane-like effects, and promote soot oxidation, while split-injection strategies can smooth pressure-rise rates, reduce peak rates of heat release, and lower particulate emissions when optimally tuned [6–14]. Research combining nanoparticle-enhanced biodiesel with controlled split-injection has indicated synergistic effects: nanoparticles enhance droplet breakup and heat transfer while split injections moderate premixed combustion spikes, together yielding lower particulate mass and number emissions and improved combustion stability. These combined benefits, however, strongly depend on nanoparticle chemistry and loading, biodiesel blend ratio, and specific split-injection parameters (pilot timing, pilot/main mass fraction, injection pressure). Consequently, parametric studies are necessary for each feedstock and engine configuration to identify optimal trade-offs among brake thermal efficiency (BTE), brake specific fuel consumption (BSFC), NO<sub>x</sub>, HC, CO and particulate matter (PM) [15–22]. Specific to dairy-scum biodiesel, several recent investigations have characterized feedstock availability, lipid extraction methods, fuel-property modification, and basic engine feasibility. Results indicate that properly processed DSOBD can approach acceptable calorific value and cetane index; nevertheless, DSOBD’s performance in CRDI engines benefits from additive-enhanced atomization or injection-timing optimization to fully match diesel baseline performance, especially under transient and high-load conditions. Life-cycle and techno-economic analyses further support valorization of dairy scum for local circular-economy implementations [23–27]. Carbon-black nanoparticles present a particularly interesting carbonaceous additive for biodiesel applications. Owing to their carbonaceous nature, small particle size, and potential catalytic/oxidative surface effects, carbon black can influence soot formation pathways, alter PM size distributions, and affect spray breakup and in-cylinder heat transfer. Prior studies using carbon-black or carbonaceous additives show mixed but frequently positive outcomes for soot reduction and combustion completeness; however, dispersion stability, abrasion/wear concerns, and long-term engine effects require careful evaluation [28–31]. Despite these encouraging developments, a clear gap exists: few studies have systematically combined dairy-scum-derived nano-biodiesel blends, carbon-black nanoparticle infusion, and a controlled dual split-injection strategy in modern CRDI engines to evaluate integrated effects on combustion phasing, performance (BTE, BSFC), and regulated emissions (NO<sub>x</sub>, HC, CO, PM). Because split-injection parameters interact strongly with fuel volatility and ignition characteristics that are altered by nanoparticle addition, a dedicated experimental program is necessary to map the multidimensional design space (blend ratio, nanoparticle loading, pilot timing, pilot/main split, injection pressure) and identify robust operating points offering improved efficiency and lower particulate emissions with manageable No<sub>x</sub> [32–36].

Previous studies have examined waste-derived biodiesel fuels, nanoparticle additives, and advanced split-injection strategies in CRDI engines—often separately—but there is almost no research that integrates these three approaches in a single, systematic investigation. For dairy-scum biodiesel (DSOBD) in particular, existing work has been largely limited to feedstock characterization, property improvement, and baseline engine performance tests. Key gaps include the lack of detailed understanding of how carbon-black nanoparticles affect DSOBD spray behavior, ignition delay, heat release, and soot formation pathways; the absence of comprehensive mapping of the combined influence of biodiesel blend ratio, nanoparticle loading, and dual split-injection parameters (pilot timing, pilot/main mass fraction, injection pressure); and the scarcity of studies quantifying the integrated impact of these factors on combustion phasing, brake thermal efficiency (BTE), brake specific fuel consumption (BSFC),

and regulated emissions (NO<sub>x</sub>, HC, CO, PM) under CRDI operation. Without such integrated studies, it is not possible to design optimized injection strategies that fully exploit the synergy between nanoparticles and dual split injection—strategies that could offset biodiesel's altered viscosity and ignition characteristics while achieving both low emissions and high efficiency.

- **Materials and Methodology adopted**

The present segment discusses on the adoption of fuels for evaluation of CRDI engine performance. ASTM standards are adopted to evaluation of fuel physico-chemical properties.

- **Physico-chemical properties of nano-biodiesel blends**

Table 1 and Table 2 sequentially unveils the explicit Physico-chemical properties of diesel and DSOBD B20 along with ASTM standards. Table 2 showcases the Specification of carbon black (CB) nanoparticle. Nano biodiesel blends of DSOBD B20 infused with carbon black nanoparticles are shown in Fig. 1. Table 3 shows properties of these nano-fuel blends.

**Table 1:** Properties of Diesel, DSOBD B20 with ASTM Standards

Properties	Unit	Diesel	DSOBD B20	ASTM Standards
Density	kg/m <sup>3</sup> @15 °C	840	835	ASTM D4052
Kinematic Viscosity	cSt @40°C	2.54	3.9	ASTMD445
Calorific value	kJ/kg	43,000	40,658	ASTM D5865
Cloud point	°C	−2	6.35	ASTM D2500-11
Pour Point	°C	−3	4.56	ASTM D97-12
Flash point	°C	70	133	ASTMD93
Cetane Number	--	45	52.25	ASTMD613
Sulphur content	% w/w	0.009	0.035	ASTM D5453

**Table 2:** Specification of carbon black (CB) nanoparticle

Parameter	Value
Assay	99.9%
pH (±1)	8.5
Particle size	30±2 nm
Appearance (Form)	Powder
Appearance (Colour)	Black



**Fig. 1:** Fuel mixtures with CB nanoparticles

**Table 3:** Properties of Nano-fuels with CB addition


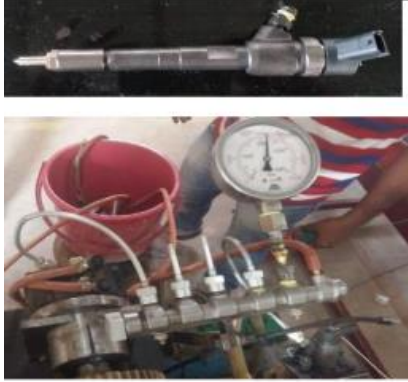
Properties	Unit	DSOBD B20 CB120	ASTM Standards
Density	kg/m <sup>3</sup> @15 °C	852	ASTM D4052
Kinematic Viscosity	cSt @40°C	4.7	ASTMD445
Calorific value	kJ/kg	41,000	ASTM D5865
Cloud point	°C	6.48	ASTM D2500-11
Pour Point	°C	4.54	ASTM D97-12
Flash point	°C	128	ASTMD93
Cetane Number	--	53.44	ASTMD613
Sulphur content	% w/w	0.038	ASTM D5453

### • Experimental setup

Fig.2 (a) unfolds four-stroke, DI diesel engine adapted for experimentation that comprises of a single-cylinder with water-cooling facility. Table 4 witness specifications of the engine.

The existing diesel engine facilitated with traditional fuel injection system is aptly refitted to run in CRDI engine mode with electronic control system (ECU) facility. Fig. 2 (b) displays the CRDI engine test facility with ECU facility as shown in Fig. 2 (c). Testing is done on an engine incorporated with toroidal reentrant combustion chamber (TRCC) ran at manufacturer specified speed of 1500 rpm using. A 7-hole CRDI injector each having 0.2 mm used is shown in Fig. 2 (d). Biodiesels blends are injected at different injection operating parameters respectively in the modified CRDI engine.



(a) Conventional CI engine test rig	(c) ECU facilitated CRDI injectors
	
(b) CRDI system integrated CI engine	CRDI injector and Pressure gauge for recording fuel IOP
<b>Fig. 2</b> Experimental test rig was depicted schematically	

**Table 4:** Engine Specifications

Make and Model	Kirloskar, TV1
Ignition System	Compression Ignition
Bore X Stroke	87.5mm X 110mm
Compression Ration	17.5 : 1
Combustion Chamber	Open Chamber (Direct Injection)
Rated Power	5.2 kW (7 HP) @1500 rpm

A piezo-electric pressure transducer (resolution: 0.1450 mV/kPa, model: HSM 111A22, make: PCB Piezotronics) used adopted for cylinder pressure measurement is seated in the cylinder head. Prior report of about the Around 100 cycles of pressure crank angle history are acquired and their ensembled values are used to calculate the HRR. The values of HRR is calculated by using first law of thermodynamics and governing equations [Hohenberg 1989, Hayes and Savage 1986]. Delta 1600S non-dispersive infrared gas analyzer is used for measuring the exhaust emissions of NO<sub>x</sub> and HC while Hartridge smoke meter is used to measure smoke emissions under steady state engine operating conditions. Differential cylinder pressure time data with crank angle and magnitudes of HRR is used to evaluate the start of the combustion phase. The stage at which 90% of the heat is released corresponds to duration of combustion. The time interval between the start of fuel injection and the start of ignition refers to ignition delay.

- Measurement of HC, CO, and NO<sub>x</sub> emissions:**

A Delta 1600 S exhaust gas analyzer, which is based on Non-Dispersive Infrared (NDIR) technology, is used to measure the exhaust emissions as shown in Fig. 3. Fig. 4 shows Hartridge Smoke meter and is used for the measurement of smoke emissions of diesel engine.



Fig. 3 A DELTA 1600 S Exhaust Gas Analyzer



Fig. 4 Hartridge Smoke meter

## • Results and Discussions

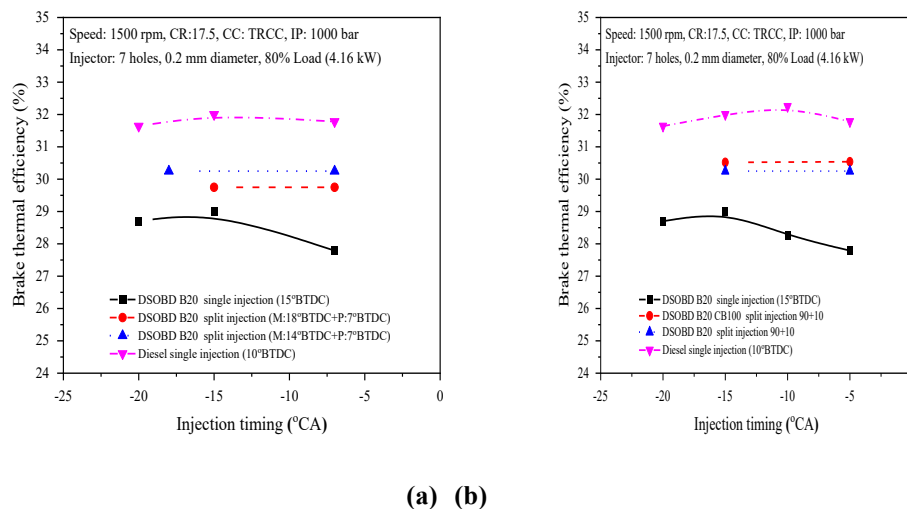
The subsequent segment of chosen task carries out trials to assess the aftereffect of split injections on the performance of CRDI engine fueled with diesel and combination of biodiesels of DSOBD B20 CB100 and DSOBD B20 blends respectively. In the preliminary phase, trials were enacted to review the potencies of discrepant injector opening pressures and injection timings in optimizing the performance of CRDI engine energized with diesel, DSOBD B20 CB100 and DSOBD B20 blends respectively. The progressive performance has been witnessed with split injection strategy synonymous to a revised injector opening pressure and injection timing of 1000 bar and 15°BTDC respectively in single injection mode. Although, an attempt to examine the impact of split injection on the DSOBD B20 CB100 and DSOBD B20 blends operated CRDI diesel engine, numerous combos of split injection were undertaken, only two optimized sets are showcased in the results and discussion section. In case of split injection, the pilot fuels were introduced at three combinations of split injection timings with main injection varied from 27°BTDC to 14°BTDC in steps of 4°BTDC respectively, while the pilot injection is varied from 9°BTDC to 5°BTDC in steps of 2°BTDC respectively. Accordingly, two main (M) and pilot (P) injection of fuel combinations of 18 °BTDC (M) and 7°BTDC (P) and 14°BTDC, and 7°BTDC were reported. Among these 14°BTDC, and 7°BTDC has resulted into improved CRDI engine performance with considerable reduction in the engine emissions.

Further mass fraction of fuel ratio for main and pilot injection were varied from 10 to 20% in steps of 10% respectively.

Split injections affect the BTE differently with decreased fuel consumption. When pilot fuel injection occurs early, then the mixture may result into too lean to burn. The vital part of these latter injections is to induce the burning effectual that ascertains crucial for supplemental oxidation of particulate matters and for more exceptional fuel utilization.

### • Brake Thermal Efficiency

Figure 5 (a) depicts the uncertainties of brake thermal efficiency with injection timing in both single and split injection modes of DSOBD B20. Split injection of biodiesels at optimized injection timings of 14°BTDC (main), and 7°BTDC (pilot) adopted with mass fraction of fuel ratio of 90:10 combination induced more elevated thermal efficiency on contrary to 80:20. Though, an enriched fuel utilization examined with chosen split injections up against single point injection, underperformance of DSOBD B20 CB100 in either version noticed in consequence of wider viscosity and poorer heating value in contrast diesel fuel operation. For single injection of DSOBD B20 CB100 maximum efficiency was observed at 15°BTDC as mentioned earlier compared to slightly retarded injection timing of 10°BTDC for diesel baseline operation shown in the figure 5 (b).



**Fig. 5** Effect of IT on BTE for single and split injection of DSOBD B20 and DSOBD B20 CB100 at 80 and 100% load

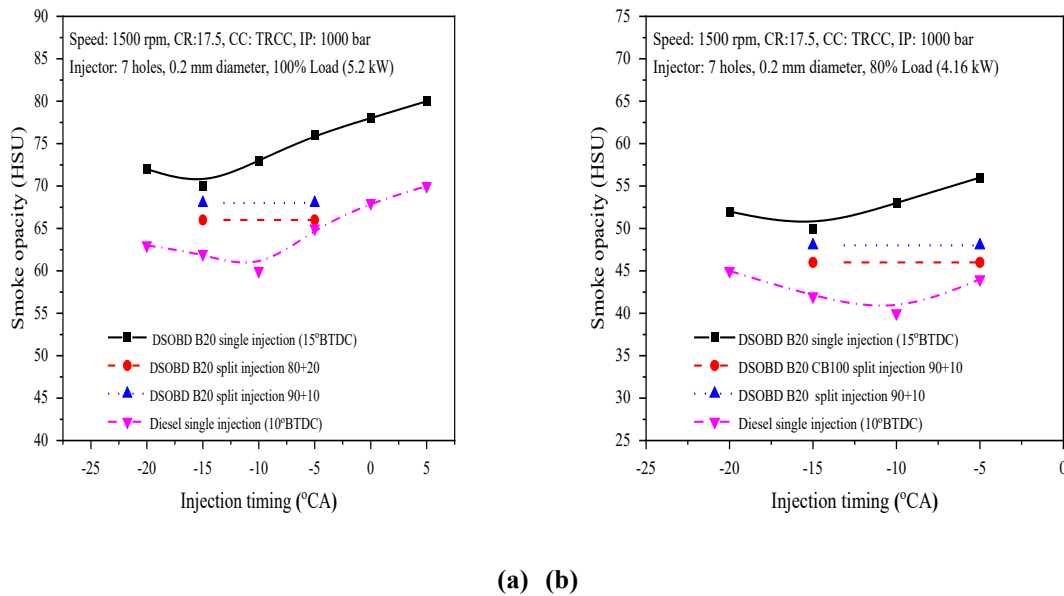
### • Emission Characteristics

Marginal aggregate of soot was perceived with split injection mode owing to more exceptional deployment of air and makes provision for the combustion to persist in due course in the expansion stroke contrast to single injection mode. The driving force behind the generation of attenuated soot is comprehensive burning of the soot formed at rich localities taken place whilst injection halted and resumed. The split injections mode adopted assist instant burning of the injected fuel and hence deny the notable formation of soot.

### • Smoke opacity

Figure 6(a) depicts the discrepancies of smoke opacity with injection timing in both single and split injection modes of DSOBD B20. The split injection aids the exhaustive burning of the unburned fuel during preceding injection and hence affirm bettered combustion that could mitigate smoke opacity.

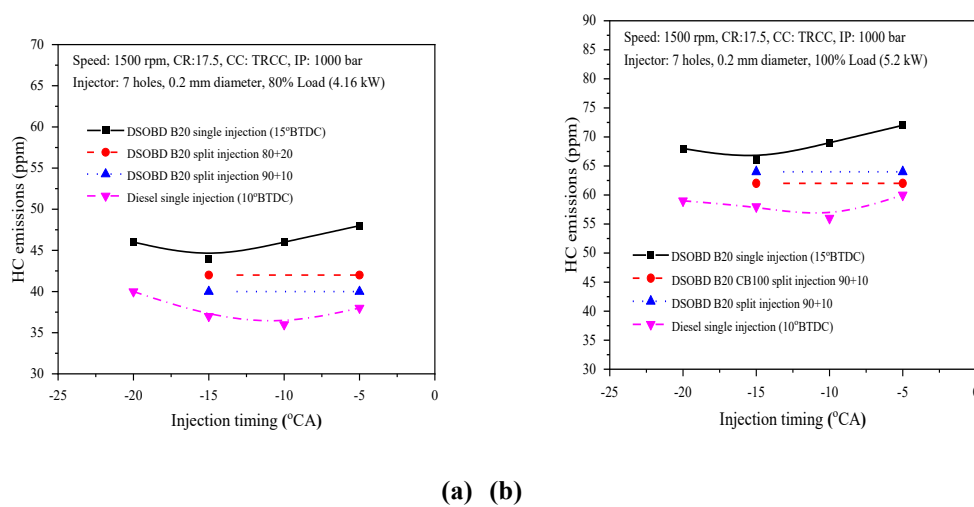
Comparatively progressive combustion allied with split injection of DSOBD B20 CB100 for mass fraction of fuel ratio of 90:10 induced nominal emissions of smoke on contrary to 80:20 as showed off in figure. An enriched fuel utilization examined with chosen split injections up against single point injection that inevitably absorb nominal fuel and accordingly attenuate smoke emissions. Underperformance of DSOBD B20 CB100 in either version noticed in consequence of wider viscosity and vaster molecular weight in contrast to diesel fuel operation. For single injection of DSOBD B20 CB100 lower smoke was observed at 15°BTDC as mentioned earlier compared to slightly retarded injection timing of 10°BTDC for diesel baseline operation depicted in the figure 6(b).



**Fig. 6** Effect of IT on Smoke opacity for single and split injection of DSOBD B20 and DSOBD B20 CB100 at 80 and 100% load

#### • HC emissions

The diversities in HC emissions with injection timing in both single and split injection modes of DSOBD B20 CB100 is unveiled in figure 7 (a and b). An enriched fuel utilization examined with chosen split injections up against single point injection due to more appropriate A:F combination creation that could mitigate the emissions of HC. Growing postponement beyond certain point leads to steep upsurge in HC release. If the pilot injection happens too early, it develops too lean to burn and consequences in an upsurge in fuel ingesting and HC releases. Comparatively enhanced combustion associated with split injection of DSOBD B20 CB100 for mass fraction of fuel ratio of 90:10 induced diminished HC emissions on contrary to 80:20 as shown in Figure. For single injection of DSOBD B20 CB100 lower HC was observed at 15°BTDC as mentioned earlier compared to slightly retarded injection timing of 10°BTDC for diesel baseline operation.

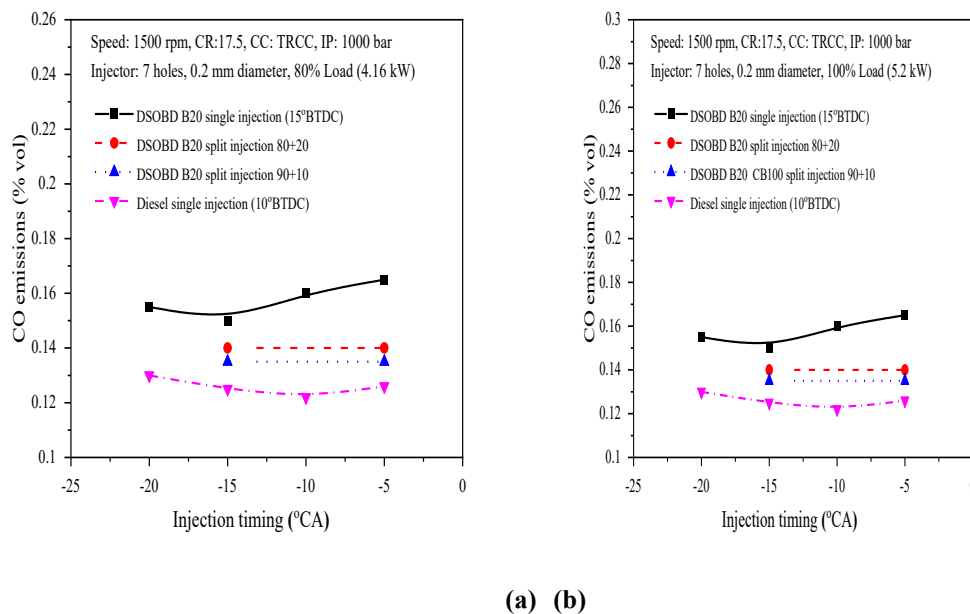


**Fig. 7** Effect of IT on HC for single and split injection of DSOBD B20 CB100 at 80 and 100% load

## • CO emissions

Figure 8(a) shows the variation of CO releases with injection timing for both single and split injections of DSOBD B20. HC and CO releases are the consequence of imperfect burning. CO discharge presented alike performance as likened to HC discharge. However, split injection exhibited marginal fuel consumption up against single point injection due to more appropriate A:F combination creation that could mitigate the emissions of CO.

Comparatively progressive combustion allied with split injection of DSOBD B20 CB100 for mass fraction of fuel ratio of 90:10 resulted into lower CO emissions compared to 80:20 as shown in Figure. For single injection of DSOBD B20 CB100 lower HC was observed at 15°BTDC as mentioned earlier compared to slightly retarded injection timing of 10°BTDC for diesel baseline operation shown in the figure 8 (b).

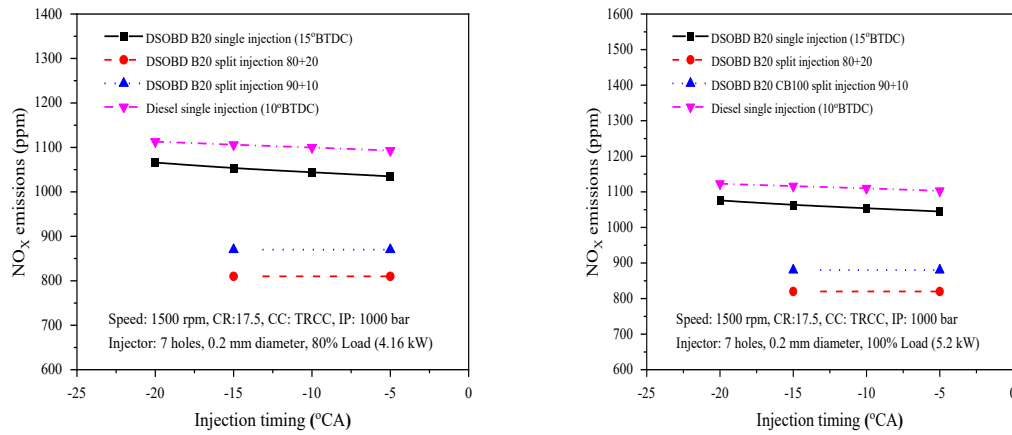


**Fig. 8** Effect of IT on CO for single and split injection of DSOBD B20 and DSOBD B20 CB100 at 80 and 100% load

## • NO<sub>x</sub> emissions

Split injection facilitate NO<sub>x</sub> reduction as low temperature combustion prevails inside the engine cylinder up against single point injection. The latter involve higher in-cylinder pressures and temperatures with combusting immense portion of the fuel in the premixed combustion phase.

Figure 9 (a) depicts the discrepancies of NO<sub>x</sub> emissions with injection timing in both single and split injection modes of DSOBD B20. As mentioned earlier, massive NO<sub>x</sub> induced in either version with an advancement in the injection timings. Drop in NO<sub>x</sub> emissions recorded with DSOBD B20 CB100 could be due to adulterated premixed combustion in either version in contrast to diesel fuel operation. Marginal aggregate of soot was perceived with split injection mode owing to more exceptional deployment of air and makes provision for the combustion to persist in due course in the expansion stroke contrast to single injection mode. Pulsed injection concurrently delivers diminished particulate emissions and pulls down NO<sub>x</sub> levels emanating from constrained pressure rise. For single injection of DSOBD B20 CB100 lower NO<sub>x</sub> was observed at 15°BTDC as mentioned earlier compared to slightly retarded injection timing of 10°BTDC for diesel baseline operation shown in the figure 9 (b).



(a) (b)

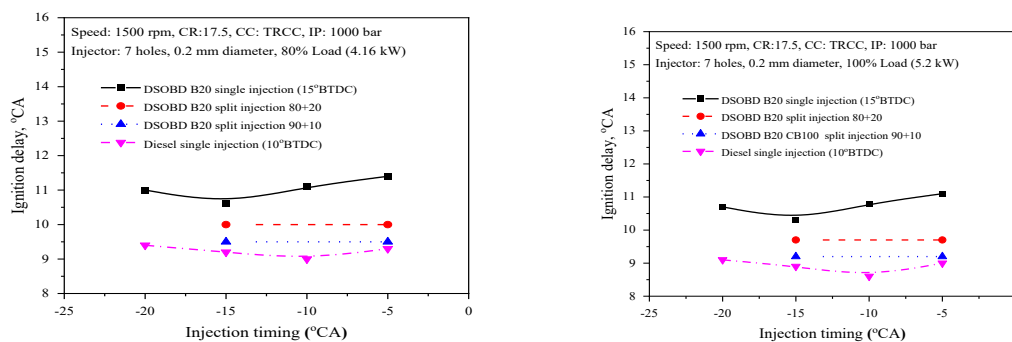
**Fig. 9** Effect of IT on NO<sub>x</sub> for single and split injection of DSOBD B20 and DSOBD B20 CB100 at 80 and 100% load

### • Combustion Characteristics

Split injection imparts more exceptional exploitation of air and makes provision for the combustion to persist in due course in the expansion stroke contrast to single injection mode and consequently adulterated degree of in-cylinder pressures and temperatures are exhibited.

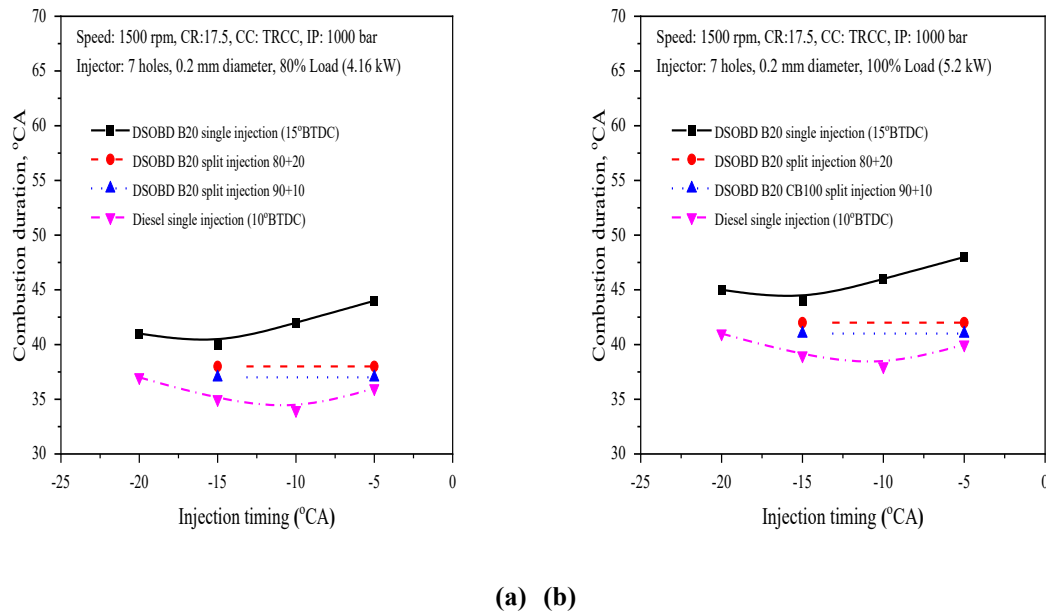
Figure 10-12 (a & b) shows the variation of combustion parameters ID, CD, and PP with injection timing in both single and split injection modes of DSOBD B20. Single injection of DSOBD B20 CB100 showed poor combustion characteristics such as increased ID, CD and lower peak pressures compared to when using optimized split injection timings. Comparatively enhanced combustion associated with split injection of DSOBD B20 CB100 for mass fraction of fuel ratio of 90:10 resulted into lower ID, CD and higher peak pressures compared to 80:20 as shown in Figure. Better BTE and immense peak pressures achieved due to combusting immense portion of the fuel in the premixed combustion phase and henceforth eventuates results into lowermost values of CD, ID and immense peak pressures respectively.

For single injection of DSOBD B20 CB100 lowermost ID and CD and extensive peak pressures was observed at 15°BTDC as mentioned earlier compared to slightly retarded injection timing of 10°BTDC for diesel baseline operation.

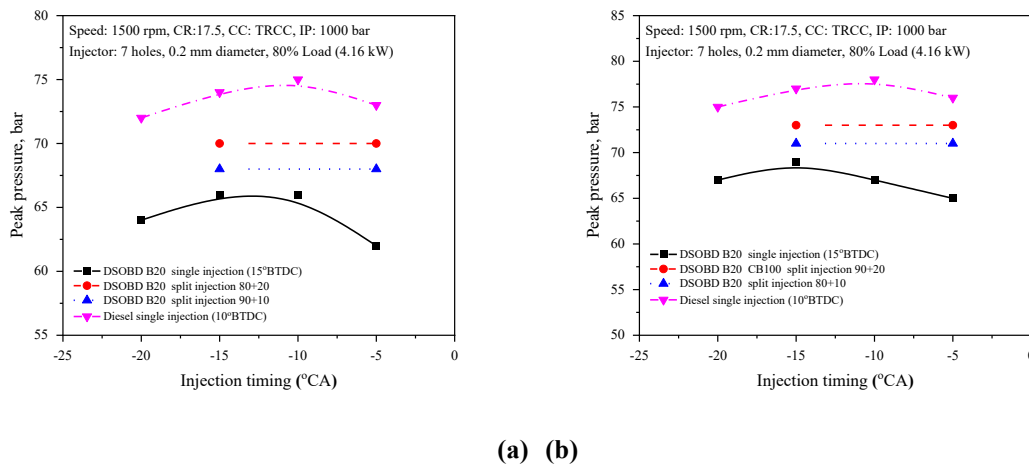


(a) (b)

**Fig. 10** Effect of IT on ignition delay for single and split injection of DSOBD B20 CB100 at 80 and 100% load



**Fig. 11** Effect of IT on combustion duration for single and split injection of DSOBD B20 and DSOBD B20 CB100 at 80 and 100% load



**Fig. 12** Effect of IT on peak pressure for single and split injection of DSOBD B20 and DSOBD B20 CB100 at 80 and 100% load

### Conclusions:

The experimental investigation confirms that integrating carbon-black nanoparticle-enhanced dairy-scum biodiesel (DSOBD) with optimized dual split injection significantly improves engine performance and emissions:

1. Dual split injection at 14° BTDC (main) and 7° BTDC (pilot) with a 90:10 pilot–main split increased brake thermal efficiency (BTE) by up to 8.6% and reduced brake specific fuel consumption (BSFC) by approximately 6–7% compared to single injection of DSOBD blends.
2. Carbon-black nanoparticle doping shortened ignition delay (ID) by 18% and combustion duration (CD) by 12%, while increasing peak pressure (PP) by 4–5%, indicating faster and more complete combustion.

3. Optimized split injection reduced smoke opacity by 23.1%, HC emissions by 15–17%, CO emissions by 12–14%, and NO<sub>x</sub> emissions by 8–10% compared to single-injection biodiesel operation. These improvements are attributed to enhanced mixture preparation, sustained combustion during the expansion stroke, and moderated in-cylinder temperatures.
4. The combination of nanoparticle enrichment and split injection delivered complementary benefits—nanoparticles improved spray atomization and ignition quality, while split injection effectively managed combustion phasing and soot oxidation.

Overall, the integration of carbon-black nanoparticle-enhanced dairy-scum biodiesel with optimized dual split injection provides a balanced improvement in efficiency and emissions, positioning it as a viable pathway toward cleaner and more efficient CRDI diesel engine operation.

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