

Interactive Fuzzy Goal Programming Approach For Multi-Objective Economic Emission Load Dispatch Optimization In Power Systems

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Abstract

The growing incidence of thermal power generation plants and strict environmental laws have constituted the Economic Emission Load Dispatch (EELD) issue into a challenging problem in contemporary power systems operation. This issue comprises the strategy of reducing the cost of fuel and pollutant emission at the same time and meeting the power balance within the system and the operating limits of the generator as well as the transmission losses. Due to the nonlinearity and conflict of these aims, the traditional single-objective optimization methods do not give flexible and environmentally sustainable solutions. An Interactive Fuzzy Goal Programming (IFGP) methodology is suggested in this work to be used to solve the multi-objective EELD problem. The technique converts the economic and emission targets to fuzzy target in terms of linear membership functions based on the ideal and anti-ideal solutions. This is an interactive decision-making mechanism, which enables the levels of aspiration to be updated stochastically until a satisfying compromise solution has been reached. The approach suggested is verified on three generators thermal power system where the transmission losses are modeled using B-coefficients. Extensive graphical explanations such as cost and emission change with demand of load, trade-off properties, and relative comparison with classical Economic Load Dispatch (ELD), are given. The findings indicate a high level of emission reduction by the IFGP approach at a small cost addition to fuel as opposed to traditional ELD. The suggested framework has increased

flexibility, clear sightedness, and decision support and is thus a great fit to sustainable and viable power system planning and operation.

Keywords: Economic emission load dispatch, Interactive fuzzy goal programming, Multi-objective optimization, Power system, Compromise solution.

1. Introduction

The economic emission dispatch is a power system optimization problem with the goal of attaining optimum power dispatch for generation while respecting certain restrictions. Common electric utility system is interconnected to achieve the benefits of minimum production cost, maximum reliability and better operating conditions. A properly flexibility due to strict governmental regulations on environment protection, the conventional operation at absolute minimum fuel cost cannot be the only basis for transmitting electric power. Various processes have been incorporated to solve this problem. Various types of goal programming techniques which are used to solve the problems related to environment considering optimization process was one of the first approach by Nanda et al. [1]. The cost as well as the emission level which were minimized by using ε -constrained method is done by Yokoyama et al. [2]. The decision-making analysis for different power dispatch model has been identified in 2009 by Xuebin [3]. The uncertainty and imprecision which occurs in real life problems was solved by implementing fuzzy set which was introduced by Zadeh [4]. Again, fuzzy set was used for solving multi-functional problem by Zimmermann [5]. Further, optimization of load dispatch model was done by Feng et al. [6]. Recently, parameterized t-norm process for solving structural problems was used by Dey and Roy [7]. This approach provides a most preferred compromised solution of the problem. A powerful method to find optimal solution of multi-objective problem was utilized by De and Yadav [8]. Here, interactive fuzzy goal programming technique is used to minimize power generation cost as well as emission levels of load dispatch problem and obtain a best preferred compromised result and the corresponding aspiration levels which was done by Waief Abd El-Wahed et al. [9]. The contributions of our work are as follows:

- Design of an Interactive Fuzzy Goal Programming framework to solve the multi objective Economic Emission Load Dispatching problem.
 - Preference of the decision-maker by adjusting the level of aspiration of the level to reach compromise solutions.
 - Graphical analysis of the comparison of the performance of IFGP with classical Economic Load Dispatch.
2. Unified and automatic visualization of optimization results to show economic, environmental and operational properties.

Multi-Objective Economic Emission Load Dispatch Model

The multi-objective economic emission dispatch can be formulated as follows,

The fuel cost objective is expressed as the following equation.

$$\text{Minimize } \text{Cost}(P) = \sum_{i=1}^n (a_i P_i^2 + b_i P_i + c_i) \quad (1)$$

where a_i , b_i and c_i are the coefficients of the i^{th} generator, P_i is the generated power of i^{th} power plants and n is the number of generators.

The emission objective is summarized in the following expression.

$$\text{Minimize } \text{Em}(P) = \sum_{i=1}^n (\alpha_i P_i^2 + \beta_i P_i + \gamma_i) \quad (2)$$

where α_i, β_i and γ_i are emission coefficients of the i^{th} generator and n is the number of generators.

Subject to

1. Power Balance Constraints

It can be formulated as

$$\sum_{i=1}^n P_i = (P_D + P_L) \quad (3)$$

where, P_D is the power demand

$$P_L = \sum_{i=1}^n \sum_{j=1}^n P_i B_{ij} P_j + \sum_{i=1}^n B_{0i} P_i + B_{00} \quad (4)$$

where, P_D and P_L are the total system demand and line loss respectively and B_{ij}, B_{0i} and B_{00} are the loss coefficients.

2. Generator Limit Constraint

It is given as

$$P_i^{\min} \leq P_i \leq P_i^{\max}, i = 1, 2, 3, \dots, n \quad (5)$$

where, P_i is the output power of i^{th} generator and P_i^{\min} and P_i^{\max} are the minimum and maximum generated power of i^{th} generator respectively.

3. Preliminaries

Fuzzy sets were first introduced by Zadeh[4] in 1965 as a mathematical way of representing impreciseness or vagueness in everyday life.

3.1. Fuzzy set

A fuzzy set A in a universe of discourse X is defined as the following set of pairs $A = \{(x, \mu_A(x)) : x \in X\}$. Here $\mu_A : X \rightarrow [0, 1]$ is a mapping called the membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value or degree of membership of $x \in X$ in the fuzzy set A . The larger $\mu_A(x)$ is the stronger the grade of membership form in A .

3.2. Convex fuzzy set

A fuzzy set A of the universe of discourse X is convex if and only if for all x_1, x_2 in X , $\mu_A(\lambda x_1 + (1 - \lambda)x_2) \geq \min(\mu_A(x_1), \mu_A(x_2))$ when $0 \leq \lambda \leq 1$.

3.3. Normal fuzzy set

A fuzzy set A of the universe of discourse X is called a normal fuzzy set implying that there exist at least one $x \in X$ such that $\mu_A(x) = 1$.

4. Interactive Fuzzy Goal Programming Technique to Solve Multi-objective Nonlinear Programming Problem:

A Multi-Objective Non-Linear Programming (MONPL) or Vector Minimization problem (VMP) may be taken in the following form:

$$\text{Min } f(x) = [f_1(x), f_2(x), \dots, f_k(x)]^T \quad (6)$$

Subject to

$$x \in X = \{x \in R^n : g_j(x) \leq \text{or } = \text{or } \geq b_j \text{ for } j = 1, 2, 3, \dots, m\} \text{ and } l_i \leq x_i \leq u_i \text{ (} i = 1, 2, 3, \dots, n \text{)}.$$

Here $x = [x_1, x_2, \dots, x_n]^T$ is an n -dimensional vector of decision variables, $f_1(x), f_2(x), \dots, f_k(x)$ are k distinct objective functions.

The steps of hesitant fuzzy optimization technique are as follows:

Computational algorithm

Step 1: Taking the first objective function from set of k objectives of the problem and solve it as a single objective subject to the given constraints. Find value of objective functions and decision variables.

Step 2: From the result of step-1, determine the corresponding values for every objective for at each derived ideal solution. Now, we construct pay off matrix for the values of all objectives at each ideal solution as follows:

$$\begin{matrix} & f_1(x) & f_2(x) & \cdots & f_k(x) \\ \begin{matrix} x^1 \\ x^2 \\ \cdots \\ x^m \end{matrix} & \begin{pmatrix} f_1^*(x^1) & f_2^*(x^1) & \cdots & f_k^*(x^1) \\ f_1(x^2) & f_2^*(x^2) & \cdots & f_k(x^2) \\ \cdots & \cdots & \cdots & \cdots \\ f_1(x^m) & f_2(x^m) & \cdots & f_k^*(x^m) \end{pmatrix} \end{matrix} \quad (7)$$

Step 3: Here, we denote and define upper and lower bounds by $U_K^\mu = \max(Z_K(X_r))$ and $L_K^\mu = \min(Z_K(X_r))$, $1 \leq r \leq k$

Step 4: In this step, we present uncertain and imprecise objectives of Multi-Objective problem by using the following linear membership functions $\mu_k(f_k(x))$:

$$\mu_k(f_k(x)) = \begin{cases} 1 & , \text{ if } f_k(x) \leq L_k \\ \left(\frac{(f_k(x))^t - (L_k)^t}{(U_k)^t - (L_k)^t} \right) & , \text{ if } L_k \leq f_k(x) \leq U_k \\ 0 & , \text{ if } f_k(x) \geq U_k \end{cases} \quad (8)$$

Step 5: By using the membership function and following the fuzzy decision of Bellman and Zadeh [11], multi-objective optimization problem can be written as:

$$\text{Maximize} \left(\text{Minimize} \left(\mu_k(f_k(x)) \right) \right) \quad (7)$$

Subject to $x \in X = \{x \in R^n : g_j(x) \leq \text{or} = \text{or} \geq b_j \text{ for } j = 1, 2, 3, \dots, m\}$ and $l_i \leq x_i \leq u_i \ (i = 1, 2, 3, \dots, n)$.

By introducing an auxiliary variable ϕ , problem (9) can be transformed as

$$\text{Maximize}(\phi) \quad (8)$$

Subject to $\left(\mu_k(f_k(x))\right) \geq \phi$
 $0 \leq \phi \leq 1;$

$x \in X = \{x \in R^n : g_j(x) \leq \text{or} = \text{or} \geq b_j \text{ for } j = 1, 2, 3, \dots, m\}$ and $l_i \leq x_i \leq u_i \ (i = 1, 2, 3, \dots, n)$.

To formulate problem (10) as a goal programming model, let us introduce the following positive and negative deviational variables

$$f_k(x) - d_k^+ + d_k^- = G^k \quad (9)$$

Where G^k is the aspiration level of the objective function k .

Problem (10) with these goals can be formulated as a non-linear goal programming problem as follows

$$\text{Maximize}(\phi) \quad (9)$$

Subject to $\left(\mu_k(f_k(x))\right) \geq \phi$
 $f_k(x) - d_k^+ + d_k^- = G^k$
 $d_k^+, d_k^- \geq 0;$
 $0 \leq \phi \leq 1;$

$x \in X = \{x \in R^n : g_j(x) \leq \text{or} = \text{or} \geq b_j \text{ for } j = 1, 2, 3, \dots, m\}$ and $l_i \leq x_i \leq u_i \ (i = 1, 2, 3, \dots, n)$.

The above non-linear programming problem can be easily solved by the any suitable techniques or some optimizing software packages.

The Decision Maker's role actually starts from this point by evaluating the resultant solution. In case the solution is rejected, new upper and lower bounds are determined based on the present solution to update the membership function and aspiration levels to resolve (11) and so on, until the preferred compromise solution is obtained. The DM may reject or accept the solution after comparing the solution with respect to the minimum of the ideal solution and apply the following rules to update the aspiration levels in problem (11)

- (1) If $f_k(x^*) \leq U_k$, then replace U_k by $f_k(x^*)$
- (2) If $f_k(x^*) = U_k$, then keep these aspiration levels as they are and solution terminates.

5. Computational Algorithm for Multi-Objective Economic Emission Problem Using Interactive Fuzzy Goal Programming Technique:

Step 1: Taking the first objective function from set of objectives of the problem (2) and solve it as a single objective subject to the given constraints. Find the value of objective functions and decision variables.

Step 2: Repeat the Step 1 for remaining objective functions. After that according to step 2 pay-off matrix formulated as follows:

$$\begin{array}{cc} Cost(P) & Em(P) \\ P^1 \left(\begin{array}{cc} Cost^*(P^1) & Em^*(P^1) \end{array} \right) \\ P^2 \left(\begin{array}{cc} Cost^*(P^2) & Em^*(P^2) \end{array} \right) \end{array} \quad (10)$$

The bounds are $U_C = \max \{Cost(P^{1*}), Cost(P^{2*})\}$, $L_C = \min \{Cost(P^{1*}), Cost(P^{2*})\}$ for cost function ($L_C \leq Cost(P) \leq U_C$) and the bounds of objective are $U_E = \max \{Em(P^{1*}), Em(P^{2*})\}$, $L_E = \min \{Em(P^{1*}), Em(P^{2*})\}$ for emission function $Em(P)$ (where $L_E \leq Em(P) \leq U_E$) are identified.

Step 3: Since the objective functions are of minimization types and the satisfaction level of experts or decision makers increases if the value of the objective function tends towards its lower bound. Thus the truth hesitant membership, indeterminacy hesitant membership and falsity membership functions of the lower bound can be represented as follows:

The hesitant-membership functions for $Cost(P)$:

$$\mu_C(Cost(P)) = \begin{cases} 1 & \text{if } Cost(P) \leq L_C \\ \left(\frac{(U_C)^t - (Cost(P))^t}{(U_C)^t - (L_C)^t} \right) & \text{if } L_C \leq Cost(P) \leq U_C \\ 0 & \text{if } Cost(P) \geq U_C \end{cases} \quad (11)$$

The membership functions for $Em(P_i)$:

$$\mu_E(Em(P)) = \begin{cases} 1 & \text{if } Em(P) \leq L_E \\ \left(\frac{(U_E)^t - (Em(P))^t}{(U_E)^t - (L_E)^t} \right) & \text{if } L_E \leq Em(P) \leq U_E \\ 0 & \text{if } Em(P) \geq U_E \end{cases} \quad (12)$$

Step 4: Now the interactive fuzzy goal programming technique for Multi-Objective Economic Emission Load Dispatch Optimization with the help of auxiliary parameters model can be transformed into the following form,

$$Max(\phi) \quad (13)$$

Subject to $\mu_C(Cost(P)) \geq \phi$; $\mu_E(Em(P)) \geq \phi$;

$$Cost(P) - d_1^+ + d_1^- = G_C$$

$$Em(P) - d_2^+ + d_2^- = G_E$$

$$0 \leq \phi \leq 1$$

$$d_1^+, d_1^-, d_2^+, d_2^- \geq 0$$

$$\begin{aligned}
\sum_{i=1}^n P_i - (P_D + P_L) &= 0 \\
P_i^{\min} &\leq P_i \leq P_i^{\max} \\
Cost(P) &= \sum_{i=1}^n (a_i P_i^2 + b_i P_i + c_i) \\
Em(P) &= \sum_{i=1}^n (\alpha_i P_i^2 + \beta_i P_i + \gamma_i)
\end{aligned} \tag{14}$$

Step 5: The above non-linear programming problem can be easily solving an appropriate mathematical programming algorithm.

5. Numerical Illustration:

In this section a system consisting of 3 thermal units [10] is considered. The cost coefficient, emission coefficient and generating limits of the 3-generator system are given as follows:

$$P_D = 700.$$

Table 1: Input data for cost coefficient, generating limits of the 3-generator system

Unit	a_i	b_i	c_i	P^{\min}	P^{\max}
1	0.03546	38.30553	1243.53110	35	210
2	0.02111	36.32782	1658.56960	130	325
3	0.01799	38.27041	1356.65920	125	315

Table 2: Input data for emission coefficient of the 3-generator system

Unit	α_i	β_i	γ_i
1	0.00683	-0.54551	40.26690
2	0.00461	-0.51160	42.89553
3	0.00461	-0.51160	42.89553

B- Coefficients of 3-generator system are as follows

$$B_{ij} = \begin{bmatrix} 0.000071 & 0.000030 & 0.000025 \\ 0.000030 & 0.000069 & 0.000032 \\ 0.000025 & 0.000032 & 0.000080 \end{bmatrix}$$

where B_{0i} and B_{00} are considered as zero.

Solution: According to step 2 pay off matrix is formulated as follows:

$$\begin{array}{c} \text{Cost}(P) \quad \text{Em}(P) \\ P^1 \begin{bmatrix} 35424.44 & 660.7492 \end{bmatrix} \\ P^2 \begin{bmatrix} 35473.32 & 651.4851 \end{bmatrix} \end{array}$$

Here $U_C = 35473.32$, $L_C = 35424.44$, $U_E = 660.7492$, $L_E = 651.4851$.

$L_C \leq \text{Cost}(P) \leq U_C$, $L_E \leq \text{Em}(P) \leq U_E$. Here linear membership functions for the objective functions $\text{Cost}(P)$ and $\text{Em}(P)$ are defined as follows

$$\mu_C(\text{Cost}(P)) = \begin{cases} 1 & \text{if } \text{Cost}(P) \leq 35424.44 \\ \left(\frac{(35473.32)^t - (\text{Cost}(P))^t}{(35473.32)^t - (35424.44)^t} \right) & \text{if } 35424.44 \leq \text{Cost}(P) \leq 35473.32 \\ 0 & \text{if } \text{Cost}(P) \geq 35473.32 \end{cases}$$

$$\mu_E(\text{Em}(P)) = \begin{cases} 1 & \text{if } \text{Em}(P) \leq 651.4851 \\ \left(\frac{(660.7492)^t - (\text{Em}(P))^t}{(660.7492)^t - (651.4851)^t} \right) & \text{if } 651.4851 \leq \text{Em}(P) \leq 660.7492 \\ 0 & \text{if } \text{Em}(P) \geq 660.7492 \end{cases}$$

Table 3: Solution based on proposed algorithm taking $t = 2$

P_1	P_2	P_3	$\text{Cost}(P)$	$\text{Em}(P)$
170.0901	279.3704	274.0703	35436.66	653.8065

6. Results and Discussions:

All graphical results are generated using a unified Python-based post-processing framework, enabling automatic visualization of optimization outputs.

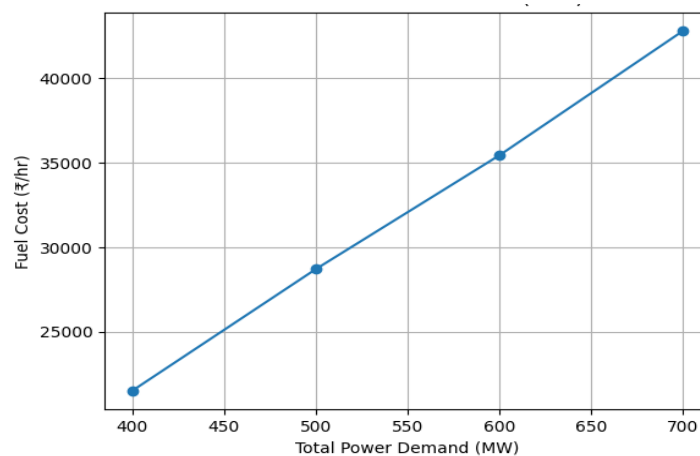


Figure 1: Fuel Cost vs Total Power Demand (IFGP)

Figure 1 shows how the total fuel cost changes with respect to the total power demand as calculated with Interactive Fuzzy Goal Programming (IFGP) proposed methodology. The fuel cost monotonically increases with the load demand, as this is anticipated since there is an increase in the outputs of the generators and the transmission losses. This value should indicate the economic performance and scalability of the given approach and prove that IFGP can generate stable and viable solutions in a large variety of operating conditions.

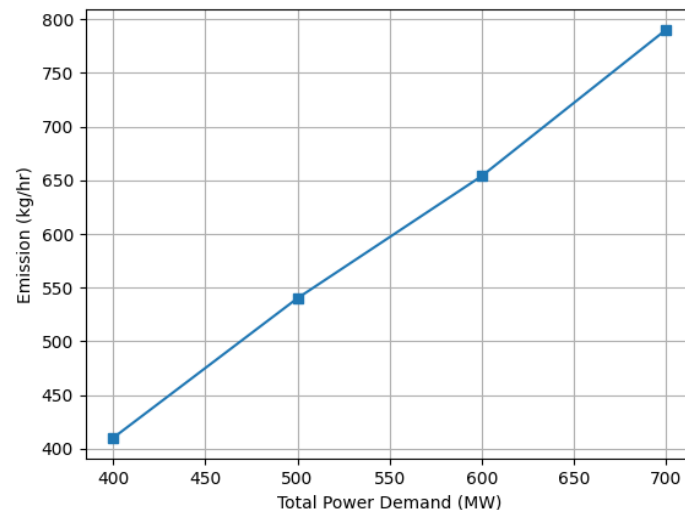


Figure 2: Emission vs Total Power Demand (IFGP)

The figure 2 demonstrates the dependence between the total emission level and power demand when using the IFGP-based dispatch strategy. The demand also leads to the increase in emissions as the higher the levels of generation the more fuel is needed. The rate of increase is however kept within check because of the minimization of emission goal imprinted within the IFGP framework. Such a figure points to the environmental concern of the suggested approach in relation to conventional dispatch methods that focus on costs only.

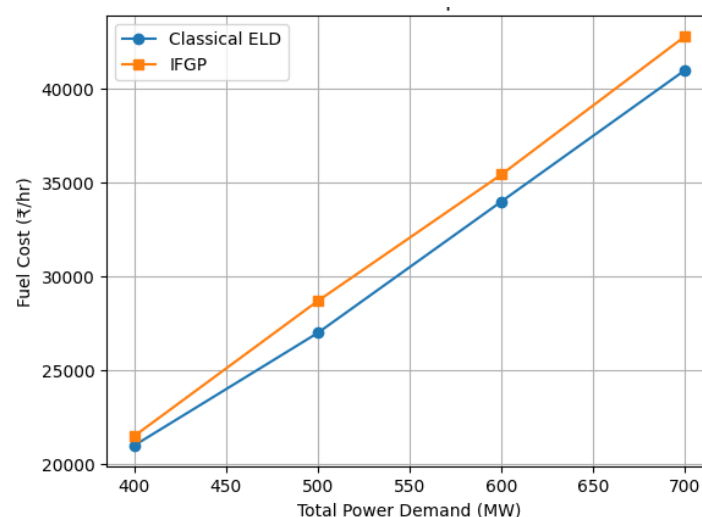


Figure 3: Fuel Cost Comparison between IFGP and Classical ELD

The comparison of fuel cost obtained by IFGP and that obtained by classical Economic Load Dispatch (ELD) are plotted in Figure 3 at varying levels of power demand. It is noted that IFGP leads to a small increment in the fuel cost compared to classical ELD. This growth is anticipated since the IFGP will both factor in emission reduction and the cost minimization. The figure is a clear indication of the cost-environment trade-off, which proves the multi-objective character of the proposed approach.

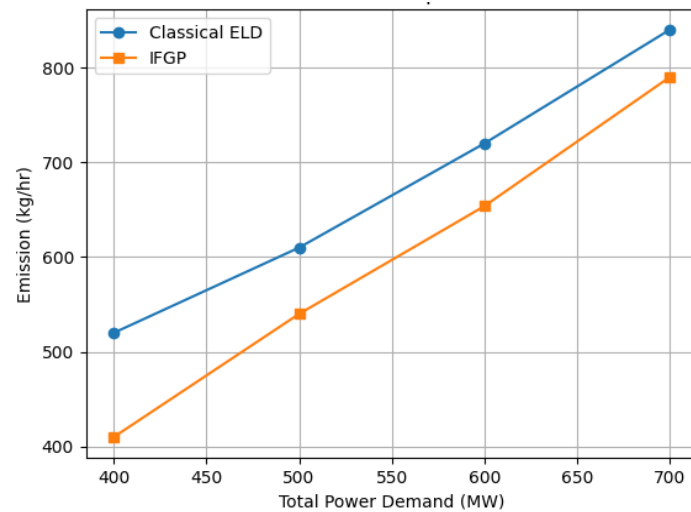


Figure 4: Emission Comparison between IFGP and Classical ELD

Figure 4 shows a comparison between the level of emissions obtained with the aid of IFGP and classical ELD. It is noticeable that IFGP produces a considerable reduction in emissions produced in all load requirements as compared to classical ELD. This has been improved, which proves that the proposed approach is efficient in prioritizing environmental restrictions without compromising the decent economic performance. The figure largely upholds the excellence of IFGP to operate the power system in a sustainable manner.

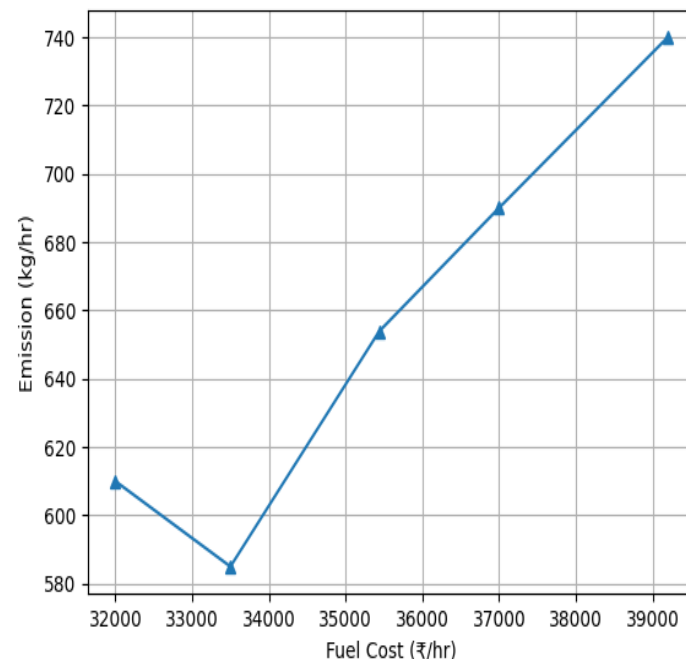


Figure 5: Fuel Cost–Emission Trade-off Curve

As shown by Figure 5, the relationship between the fuel cost and the emission is indicated in a trade off by changing the level of aspiration in the interactive fuzzy goal programming process. Every spot in the curve is the trade off between economic and environmental goals. The nonlinear tendency proves the problematic character of the objectives and the possibility of IFGP to produce Pareto-optimal compromise solutions, which enables decision-makers to choose solutions depending on their preference.

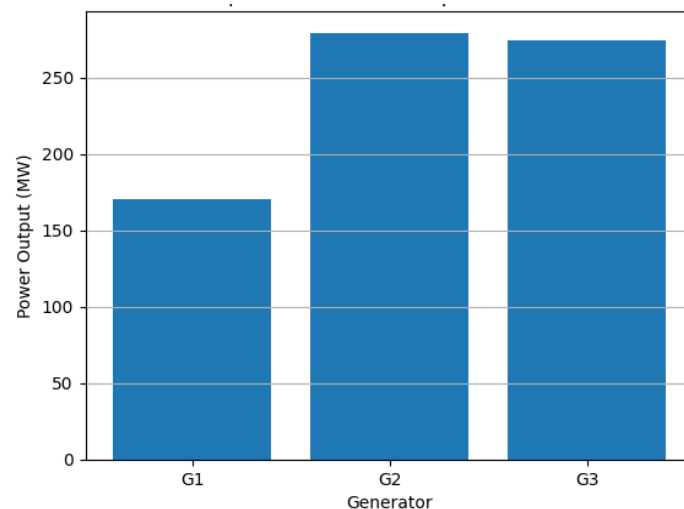


Figure 6: Optimal Generator Power Dispatch (IFGP)

The optimum power output of each generator as a result of the final IFGP compromise solution is presented in Figure 6. This figure indicates that there is a dissemination of power generation amongst the units without interfering with their lowest and highest generation limit. This will verify the practicability and functionality of the suggested strategy and will grant an idea about the pattern of dispatch, which is achieved in the course of the process of cost-emission balance.

7. Conclusion:

This paper introduces an efficient and adaptable solution methodology to multi objective Economic Emission Load Dispatch problem based on Interactive Fuzzy Goal Programming approach. The proposed approach enables the reduction of fuel cost and emission levels at the same time by operating within the practical constraints of the system, which is why the conflict between economic efficiency and environmental sustainability is inherent. The use of fuzzy membership functions can capture the imprecise goals, whereas the interactive mechanism can be used so that the decision-makers can narrow the aspiration levels and orient the solution to a desirable compromise. The high-level of robustness and viability of the suggested technique is supported by numerical findings of a three-generator thermal power system, including transmission losses. A comparative study with classical Economic Load Dispatch indicates that IFGP can provide significant reduction of emission at a very small increment in fuel cost, thus making it superior when it comes to the operation of the power system with regard to environmental consciousness. The scalability, trade-off behaviour as well as the optimality of power sharing among generators are also confirmed through the graphical evaluations. In contrast to traditional optimization models, the presented IFGP framework provides a higher level of transparency and flexibility because it explicitly takes into consideration the human preferences in decision-making. The given approach can be easily extrapolated to bigger power systems and more complicated situations of dispatch due to its simplicity, effectiveness, and practical relevance. The future study can involve combining renewable energy sources, valve-point effects, and state-of-art metaheuristic methods to improve the accuracy of the solution and computational efficiency.

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