Investigaton on Fuzzy g̈ - Irresolute Functions

S. Vijayalakshmi[1]* S. Sahaya Arockiya Selvi[1] S. Geetha[2] A. abirami[1]

[1] *Department of Mathematics, St.Michael College of Engineering and Technology, Kalayarkoil, Sivaganga District, Tamilnadu and India.*

**Corresponding author Email: vijimuthupandimp@gmail.com*

[2] *Department of Mathematics, Seethalakshmi Achi College for Women, Pallathur Sivaganga District, Tamilnadu and India*

Abstract:- A fuzzy g̈- irresolute function is a concept from fuzzy topology using fuzzy sets. Fuzzy topological space is a set of with collection of fuzzy subsets. In this work, fuzzy g̈- irresolute functions from a fuzzy topological space X to a fuzzy topological space Y was introduced and obtained some properties of fuzzy g̈-irresolute functions which are more helpful for solving many Engineering problems.

Keywords: *Fuzzy topological space, Fuzzy g̈--irresolute functions, Fuzzy g - continuous functions, Fuzzy open sets, Fuzzy closed sets.*

Introduction

Fuzzy g̈-irresolute function is the study of a specific type of continuous mapping between fuzzy topological spaces. The field of fuzzy general topology have shown more interest in studying the concept of fuzzy generalization of continuous functions [1],[2],[3]. A week form of fuzzy continuous functions called fuzzy g-continuous function were introduced by Balasubramanian et al [2] and Balasubramanian]3]. Jayaraman et al [4],[5],[6],[7] introduced and studied another form of fuzzy generalized continuous functions called fuzzy g̈-irresolute functions. Kiran Potadar and Sadanand Patil [8] studied on closed fuzzy sets maps in fuzzy topological spaces. Kiran Potadar et al.[9] studied fuzzy g¨irresolute and fuzzy Homomorphism maps in fuzzy topological spaces. Even though some attempts were made by previous authors, there are some lacking in the study of their relation. So in this paper, fuzzy g̈-irresolute functions were studied and investigated their relations with various fuzzy generalized continuous functions and also discussed their some properties.

Preliminaries

Throughout this paper (X, τ), (Y, σ) and (Z, η) (or X, Y and Z) represent fuzzy topological spaces on which no separation axioms are assumed unless otherwise mentioned. For any fuzzy subset A of a space (X, τ), the closure of A, the interior of A and the complement of A are denoted by Cl(A), int(A) and A respectively.

The following definitions recalled which are useful in the sequel.

Definition 2.1. [10, 11] If X is a set, then any function A: X → [0, 1] (from X to the closed unit interval [0,1]) is called a fuzzy set in X.

Definition 2.2. [12] If X is a set, then A, B: X → [0, 1] are fuzzy sets in X .

(i) The complement of a fuzzy set A, denoted by A’, is defined by A’(x) = 1−A(x), for all x ∈ X.

(ii) Union of two fuzzy sets A and B, denoted by A∨B, is defined by (A∨B)(x) = max{A(x), B(x)}, for all

x ∈ X.

(iii) Intersection of two fuzzy sets A and B, denoted by (A∧B) (x) = min{A(x), B(x)} for all x ∈ X .

Definition 2.3. Let X be a set and τ be a family of fuzzy sets in X. Then τ is called a fuzzy topology if τ satisfies the following conditions:

(i) $\theta, i \in \tau$

(ii) If $A_{i \in I} \in \tau$ then $V_{i \in I} \in \tau$

(iii) If A, B $\in \tau$ then $A \wedge B \in \tau$.

The pair (X, τ) is called a fuzzy topological space (or fts). The elements of τ are called fuzzy open sets. Complements of fuzzy open sets are called fuzzy closed sets.

Definition 2.4. [10] Let A be a fuzzy set in a fts (X, τ). Then

(i) The closure of A, denoted by Cl(A) is defined by $Cl(A) = \wedge\{F: A \leq F$ and F is fuzzy closed$\}$.

(ii) The interior of A, denoted by int(A) is defined by $int(A) = \vee\{G: G \leq A$ and G is fuzzy open$\}$.

Definition 2.5. A fuzzy subset A of a space (X, τ) is called fuzzy semi - open set [13] if

$A \leq Cl(int(A))$. The complement of fuzzy semi-open set is fuzzy semi- closed. The fuzzy semi-closure [14] of a fuzzy subset A of X, denoted by SCl(A), is defined to be the intersection of all fuzzy semi-closed sets if (X, τ) containing A. It is known that SCl(A) is a fuzzy - closed set.

Definition 2.6. A fuzzy subset A of a space (X, τ) is called:

(i) a fuzzy semi-generalized closed (briefly fsg-closed) set [4] if $SCl(A) \leq U$ whenever $A \leq U$ and U is fuzzy semi open in (X, τ). The complement of fsg- closed set is called fsg-open set.

(ii) a fuzzy pre-semi-generalized closed (briefly fpsg-closed) set [12] if $SPCl(A) \leq U$ whenever $A \leq U$ and U is fuzzy semi open in (X, τ). The complement of fpsg- closed set is called fpsg-open set.

(iii) a fuzzy $\ddot{g}$-closed set [4] if $Cl(A) \leq U$ whenever $A \leq U$ and U is fsg-open in (X, τ). The complement of fuzzy $\ddot{g}$-closed set is called fuzzy $\ddot{g}$-open set.

(iv) a fuzzy $T\ddot{g}$-space [7] if every fuzzy $\ddot{g}$-closed set in it is fuzzy closed.

Definition 2.7. A fuzzy function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called fuzzy sg irresolute [16]. If the inverse image of every (Y, σ) fsg-closed (resp. fsg-open) set in (Y, σ) is fsg-closed (resp. fsg-open) in (X, τ).

Definition 2.8. A fuzzy function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called

(i) fuzzy $\ddot{g}$-continuous [5] if the inverse image of every fuzzy closed set in (y, σ) is fuzzy $\ddot{g}$-closed set in (X, τ)

(ii) fuzzy closed [15] if the image of every fuzzy closed set if X is a fuzzy closed in Y.

(iii) fuzzy open [15] if the image of every fuzzy open set of X is a fuzzy open in Y.

(iv) fuzzy $\ddot{g}$-closed map [6] if the image of every fuzzy closed set in (X, τ) is $f\ddot{g}$-closed in (Y, σ).

Proposition 2.9. If $f: (X, \tau) \rightarrow (Y, \sigma)$ is fuzzy sg-irresolute fuzzy $\ddot{g}$-closed and A is a fuzzy $\ddot{g}$-closed subset of (X, τ), then f(A) is fuzzy $\ddot{g}$-closed in (Y, σ).

Fuzzy $\ddot{g}$-irresolute functions

In this paper the following definitions were introduced

Definition 3.1. A fuzzy function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called a fuzzy $\ddot{g}$-irresolute if the inverse image of every f $\ddot{g}$-closed set in (Y, σ) is f $\ddot{g}$-closed in (X, τ).

Remark 3.2. The following examples show that the notions of fuzzy sg-irresolute and fuzzy $\ddot{g}$-irresolute are independent.

Example 3.3. Let X = Y = {a, b} and $\tau = \{0_X, \alpha, 1_X\}$ where α is a fuzzy set in X defined by $\alpha(a) = 1$, $\alpha(b) = 0$ and $\alpha = \{0_X, \beta, 1_X\}$ where β is a fuzzy set in Y defined by $\beta(a) = 0$, $\beta(b) = 0$. Then (X, τ) and (Y, σ) are fts. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be the identity fuzzy function. Then f is fuzzy $\ddot{g}$-irresolute, but it is not fuzzy sg-irresolute.

Example 3.4. Let X = Y = {a, b} and $\tau = \{0_X, \gamma, 1_X\}$ where γ is a fuzzy set in X, defined by $\gamma(a) = 0.4$, $\gamma(b) = 0.5$ and $\sigma = \{0_X, \delta, 1_X\}$, where δ is fuzzy set in Y defined by $\delta(a) = 0.6$, $\delta(b) = 0.5$. Then (X, τ) and (Y, σ) are fts. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be the identity fuzzy function. Then f is fuzzy sg-irresolute, but it is not fuzzy $\ddot{g}$-irresolute.

Proposition 3.5. A fuzzy function f: $(X, \tau) \to (Y, \sigma)$ is fuzzy $\ddot{g}$-irresolute if and only if the inverse of every $f\ddot{g}$-open set in (Y, σ) is fg-open in (X, τ).

Proof: Similar to the next proposition.

Proposition 3.6. A fuzzy function $f : (X, \tau) \to (Y, \sigma)$ is fuzzy $\ddot{g}$-continuous if and only if $f^{-1}(U)$ is fuzzy $\ddot{g}$-open in (X, τ) for every fuzzy open set U in (Y, σ).

Proof. Let f: $(X, \tau) \to (Y, \sigma)$ be fuzzy $\ddot{g}$-continuous and U be fuzzy open set in (Y, σ). Then U^c is fuzzy closed in (Y, σ) and since f is fuzzy $\ddot{g}$-continuous,

$f^{-1}(U^c)$ is fuzzy $\ddot{g}$-closed in (X, τ). But $f^{-1}(U^c) = (f^{-1}(U))^c$ and so $f^{-1}(U)$ is fuzzy $\ddot{g}$-open in (X, τ). Conversely,

Assume that $f^{-1}(U)$ is $f\ddot{g}$-open in (X, τ) for each fuzzy open set U in (Y, σ).

Let F be a fuzzy closed set in (Y, σ). Then F^c is fuzzy open in (Y, σ) and by assumption, $f^{-1}(F^c)$ is $f\ddot{g}$-open in (X, τ). Since $f^{-1}(F^c) = (f^{-1}(F))^c$, we have $f^{-1}(F)$ is $f\ddot{g}$-closed in (X, τ) and so f is fuzzy $\ddot{g}$-continuous.

Proposition 3.7. If a fuzzy function f: $(X, \tau) \to (Y, \sigma)$ is fuzzy $\ddot{g}$- irresolute, then it is fuzzy $\ddot{g}$-continuous but not conversely.

Example 3.8. Let $X = Y = \{a, b\}$ and $\tau = \{0_X, \alpha, \beta, 1_X\}$ where α is a fuzzy set in X defined by $\alpha(a) = 0.5, \alpha(b) = 0$ and $\sigma = \{0_X, \beta, 1_X\}$ where β is a fuzzy set in Y defined by $\beta(a) = 1, \beta(b) = 0$. Then (X, τ) and (Y, σ) are fts. Let f: $(X, \tau) \to (Y, \sigma)$ be the identity fuzzy function. Then f is fuzzy $\ddot{g}$-continuous, but it is not fuzzy $\ddot{g}$-irresolute.

Proposition 3.9. Let (X, τ) be any fts, (Y, σ) be a fuzzy $T\ddot{g}$-space and $f : (X, \tau) \to (Y, \sigma)$ be a fuzzy function. Then the following are equivalent.

1. f is fuzzy $\ddot{g}$-irresolute.
2. f is fuzzy $\ddot{g}$-continuous.

Proof. (1) $\Rightarrow$ (2) follows from Proposition (3.7).

(2) $\Rightarrow$ (1). Let F be a $f\ddot{g}$-closed set in (Y, σ). Since (Y, σ) is a fuzzy $T\ddot{g}$-space, F is a fuzzy closed set in (Y, σ) and by hypothesis, $f^{-1}(F)$ is $f\ddot{g}$-closed in (X, τ).

Therefore, f is fuzzy $\ddot{g}$-irresolute.

Definition 3.10. A identity fuzzy function $f : (X, \tau) \to (Y, \sigma)$ is called fuzzy pre-sg-open if $f(U)$ is fsg-open in (Y, σ), for each fsg-open set U in (X, τ).

Proposition 3.11. If f: $(X, \tau) \to (Y, \sigma)$ is bijective fuzzy pre-sg-open and fuzzy $\ddot{g}$-continuous then f is fuzzy $\ddot{g}$-irresolute.

Proof. Let A be $f\ddot{g}$-closed set in (Y, σ). Let U be any fst-open set in (X, τ) such that $f^{-1}(A) \leq U$. Then $A \leq f(U)$.

Since A is $f\ddot{g}$-closed and $f(U)$ is fsg-open in (Y, σ). $Cl(A) \leq f(U)$ holds and hence $f^{-1}(Cl(A)) \leq U$.

Since f is fuzzy $\ddot{g}$-continuous and $Cl(A)$ is fuzzy closed in (Y, σ). $f^{-1}(Cl(A))$ is $f\ddot{g}$-closed and hence $Cl(f^{-1}(Cl(A))) \leq U$ and so $Cl(f^{-1}(A)) \leq U$.

Therefore, $f^{-1}(A)$ is $f\ddot{g}$-closed in (X, τ) and hence f is fuzzy $\ddot{g}$-irresolute. The following examples show that no assumption of Proposition (3.9) can be removed.

Example 3.12. The identity fuzzy function is fuzzy g- continuous and fuzzy bijective but not fuzzy pre-sg-open and so f is not fuzzy g̈-irresolute.

Example 3.13. Let $X = Y = \{a, b\}$ and $\tau = \{0_X, \gamma, 1_X\}$ where γ is a fuzzy set in X defined by $\gamma(a) = 0.6, \gamma(b) = 0.5$ and $\sigma = \{0_X, \delta, 1_X\}$ where δ is a fuzzy set in Y defined by $\delta(a) = 0.4, \delta(b) = 0.5$. Then (X, τ) and (Y, σ) are fts. Let $f: (X, \tau) \to (Y, \sigma)$ be the identity fuzzy function. Then f is fuzzy bijective and fuzzy pre-sg-open but not fuzzy g̈-continuous and so f is not fuzzy g̈-irresolute.

Proposition 3.14. If $f: (X, \tau) \to (Y, \sigma)$ is bijective fuzzy closed and fuzzy sg-irresolute. Then the inverse function $f: (X, \tau) \to (Y, \sigma)$ is fuzzy g̈-irresolute.

Proof. Let A be $f: (X, \tau) \to (Y, \sigma)$ fg-closed in (X, τ). Let $(f^{-1})^{-1}(A) = f(A) \leq U$, where U is fsg-open in (Y, σ). Then $A \leq f^{-1}(U)$ holds.

Since $f^{-1}(U)$ is fsg-open in (X, τ) and A is fg̈-closed in (X, τ), $Cl(A) \leq f^{-1}(U)$ and hence $f(Cl(A)) \leq U$. Since f is fuzzy closed and Cl(A) is fuzzy closed in (X, τ), $f(Cl(A))$ is fuzzy closed in (X, τ) and so $f(Cl(A))$ is fuzzy g̈-closed in (Y, σ). Therefore, $Cl(f(Cl(A))) \leq U$ and hence $Cl(f(A)) \leq U$. Thus f(A) is fg̈-closed in (Y, σ) and so f^{-1} is fuzzy g̈-irresolute.

Proposition 3.15. Let any fuzzy function $f : (X, \tau) \to (Y, \sigma)$ be a fuzzy function. Then the following statements are equivalent.

1. $f^{-1} : (X, \tau) \to (Y, \sigma)$ is fuzzy g̈-irresolute.
2. f is fuzzy $\ddot{g}^*$-open function.
3. f is fuzzy $\ddot{g}^*$-closed function.

Proof. (1) $\Rightarrow$(2). Let U be an fuzzy open set of (X, τ), By assumption $(f^{-1})^{-1}(U) = f(U)$ is fuzzy $\ddot{g}^*$-open in (Y, σ) and so f is fuzzy $\ddot{g}^*$-open.

(2) $\Rightarrow$(3). Let F be a fuzzy closed set of (X, τ). Then F^c is fuzzy open set in (X, τ). By assumption, $f(F^c)$ is fuzzy $\ddot{g}^*$-open in (Y, σ). That is $f(F^c) = (f(F))^c$ is fuzzy $\ddot{g}^*$-open in (Y, σ). Therefore f(F) is fuzzy $\ddot{g}^*$-closed in (Y, σ). Hence f is fuzzy $\ddot{g}^*$-closed.

(3) $\Rightarrow$ (1). Let F be a fuzzy closed set of (X, τ). By assumption, f(F) is Fuzzy $\ddot{g}^*$-closed in (Y, σ). But $f(F) = (f^{-1})^{-1}(F)$ and therefore f^{-1} is fuzzy $\ddot{g}^*$-continuous

Proposition 3.16. If $f : (X, \tau) \to (Y, \sigma)$ is fuzzy sg-irresolute and fg̈-closed, then it is an fuzzy g̈-closed function.

Proof. Let U be a fsg-open set in (Y, σ) such that $f(A) \leq U$. Since f is fuzzy sg-irresolute, $f^{-1}(U)$ is a fsg-open set containing A. Hence $Cl(A) \leq f^{-1}(U)$ as A is fuzzy $\ddot{g}^*$-closed in (X, τ). Since f is fg̈-closed, $f(Cl(A))$ is an fg̈-closed set contained inthe fsg-open set U, which implies that $Cl(f(Cl(A))) \leq U$ and hence $Cl(f(A)) \leq U$. Therefore f(A) is an fg̈-closed set in (Y, σ)

Conclusion

Through this paper, fuzzy g̈-irresolute function was introduced and studied some of their properties and relations in fuzzy topological spaces with suitable examples. This work will more helpful for solving Engineering problems, Partide Physics, Computer aided Geometric design of Engine components, computational topology and quantum physics

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