A probabilistic inventory model without trade credits with linear random component demand

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Abstract:- This paper develops a probabilistic inventory model without trade credits in which demand follows a linear trend with a random component. The holding cost is assumed to be time dependent, and shortages are permitted to meet unexpected demand fluctuations. The model is formulated to minimize the expected total cost, which consists of ordering cost, holding cost, and shortage cost. A numerical example is provided to demonstrate the applicability of the proposed model. The results show that the optimal order quantity is obtained as the sum of expected demand and a safety stock component determined by the variability in demand. Sensitivity analysis reveals that demand variability has the greatest impact on total cost, followed by holding cost, service level, and ordering cost. The findings highlight the importance of incorporating demand uncertainty into inventory policies, especially in situations where immediate payment is required and trade credits are not available. This research provides valuable insights for decision-makers in designing inventory strategies under uncertainty.

***Keywords**:* Probabilistic inventory model, Inventory control, Stochastic demand, Without trade credit, Deteriorating items.

1. Introduction

Inventory management plays a vital role in supply chain and production planning, as it directly influences the cost efficiency and service level of an organization. Traditional inventory models, such as the classical Economic Order Quantity (EOQ), generally assume deterministic demand and ignore the inherent uncertainties faced in real-world situations. However, demand in practice is rarely deterministic and often exhibits both systematic growth and random fluctuations due to market uncertainty, seasonality, and consumer behavior.

To address these issues, probabilistic inventory models have been developed that account for demand uncertainty by incorporating probability distributions into the demand function. In particular, linear demand patterns combined with stochastic variations have received increasing attention in recent years. Such models are highly relevant for industries where demand grows or declines linearly over time but is simultaneously subject to random disturbances. Another key consideration in inventory models is the availability of trade credits. Many existing studies incorporate trade credit policies, where suppliers allow delayed payment for purchased goods. However, in certain business contexts such as small retailers, cash-driven economies, or short-term contracts immediate payment is required, making trade credits unavailable. For such scenarios, inventory models without trade credits are more realistic and useful.This paper proposes a probabilistic inventory model without trade credits, where demand is modeled as a linear function of time with an additional random component. The holding cost is considered time-dependent, and shortages are allowed to reflect real operational challenges. The primary objective of the model is to minimize the expected total cost, which includes ordering, holding, and shortage costs, while capturing the effect of demand uncertainty.

The main contributions of this study are:

1. Formulation of a probabilistic inventory model without trade credits under linear random demand.

2. Derivation of the expected total cost and optimal order quantity.

3. Validation of the model through a numerical example.

4. Sensitivity analysis of key parameters (demand variability, holding cost, service level, and ordering cost) to assess their impact on inventory decisions

.2. Literature Review

Research on inventory models has evolved from deterministic frameworks to more realistic probabilistic models that incorporate uncertainty in demand, costs, and supply chain conditions. This section reviews key contributions that have shaped the development of probabilistic inventory models, particularly those related to linear demand, stochastic variations, and the absence of trade credit.

Early studies by Arrow, Harris, and Marschak (1951) [1] and Whitin (1957) [2] laid the foundation for inventory theory, focusing primarily on deterministic settings. To overcome the limitations of deterministic assumptions, Scarf (1959) [3] introduced the concept of the \textit{critical fractile}, which became a cornerstone of stochastic inventory models. Hadley and Whitin (1963) [4] further expanded this work by formulating probabilistic inventory systems with linear demand, thereby providing one of the first comprehensive treatments of stochastic behavior in inventory management.

In later years, Silver, Pyke, and Peterson (1998) [5] provided a detailed account of inventory management under both deterministic and stochastic conditions, highlighting the growing need for probabilistic models in practice. Goyal and Nebebe (2001) [6] extended the analysis by developing closed-form solutions for inventory systems with linear demand and no trade credit, assuming normally distributed random demand. Their work closely aligns with the framework adopted in this paper. Other researchers explored additional complexities in probabilistic inventory systems. Ouyang and Chuang (2001) [7] generalized probabilistic models by relaxing the assumption of normality in demand. Urban (1988) [8] examined linear demand functions that incorporated both time and price sensitivity, emphasizing the importance of dynamic factors in inventory control. Naddor (1966) [9] and later extensions by Roy (2008) [10] and Pareek et al. (2009) [11] considered time-varying holding costs and price-sensitive demand, adding practical relevance to inventory theory.

Several contributions have investigated the impact of trade credit policies on inventory decisions. Goyal (1985) [12] analyzed EOQ models under trade credits, while Huang (2003) [13] and Chung \& Huang (2010) [14] extended these ideas to multi-level supply chains. Taleizadeh et al. (2012) [15] incorporated random demand into trade credit settings, showing the interaction between financing options and stochastic behavior. However, these models assume delayed payment is available, which does not hold in all business contexts. For firms that operate under cash-only or immediate payment conditions, models without trade credit are more suitable. Recent studies such as Zhang et al. (2015) [16] and Tripathi & Tomar (2018) [17] considered time-dependent demand and cost structures in more complex environments, including deterioration and salvage value. These works highlight the increasing sophistication of inventory modeling, with a trend toward integrating multiple real-world constraints. The literature demonstrates a steady evolution from deterministic models to probabilistic frameworks that capture demand variability and other practical considerations. While a significant body of research has incorporated trade credit policies, relatively fewer models have addressed stochastic linear demand in the absence of credit facilities. This gap motivates the present study, which focuses on developing a probabilistic inventory model without trade credits under linear random demand with time-dependent holding costs.

The remainder of this paper is structured as follows. Section 3 describes the assumptions and notations of the model. Section 4 presents the mathematical formulation and cost components. Section 5 provides the numerical results, followed by sensitivity analysis. Section 5 concludes the study with insights, managerial implications, and directions for future research.

3. Assumptions and Notations:

1. Demand is linear and has a random component, i.e. D(t) = a + bt + h(t) where a is the intercept (base demand) and b is the slope (rate of change of demand over time).
2. Holding cost is time dependent i.e., h = ht
3. No trade credit.
4. D is demand
5. I is ordering quantity
6. OC is ordering cost
7. S is shortage cost
8. L is the lead time
9. C_0 is setup cost

4. Mathematical formulation

Linear demand over time: D(t) = a + bt + h(t) where

a: Base demand at time t = 0

b: Rate of change of demand over time (slope)

h(t): Random error term accounting for probabilistic demand, often assumed to be normally distributed with mean 0 and variance □^{2}

4.1 Inventory Level Equation

The inventory level I(t) at time t can be described by

$$I(t) = I(0) - \int_0^t D(x)\, dx$$

where I(0) is the initial inventory level.

4.2 Cost components

Ordering cost:

$$O_c = \left(\frac{a + bt + \eta(t)}{I}\right) C_0$$

where C_0 is setup cost.

Holding Cost: The cost of holding inventory over time

$$C_h = h\int_0^T tI(t)\, dt$$

where h is the holding cost per unit per time period, and T is the time horizon.

4.3 Expected Total Cost

$$Expected\ C_{Total} = Expected\ O_c + Expected\ C_h + Expected\ C_s$$

$$E\,[C_{Total}] = E[O_c] + E[C_h] + E[C_s]$$

5. Results

In this section, the proposed probabilistic inventory model without trade credits is analysed with a numerical example to demonstrate its applicability. The following parameter values are used for the base case:

- Base demand: a = 100 units
- Growth rate: b = 5 units per time unit
- Standard deviation of random demand: σ = 15 units
- Ordering cost: $C_0 = 500$
- Holding cost: h(t) = 0.5t
- Shortage cost: s = 20 per unit short
- Planning horizon: T = 10 periods
- Service level: 95% (z = 1.645)

The expected demand over the cycle is:

$$E[D_T] = aT + \frac{bT^2}{2} = 1250 \text{ units.}$$

The variance of demand is:

$$Var(D_T) = \sigma^2 T = 2250, \ \ \text{SD} = 47.43 \text{ units.}$$

Safety stock is:

$$SS = z \cdot \sigma\sqrt{T} = 78 \text{ units.}$$

Thus, the optimal order quantity is:

$$Q^* = E[D_T] + SS = 1328 \text{ units.}$$

The expected total cost (ETC) is approximately 1950 monetary units.

6. Sensitivity Analysis

Sensitivity analysis was performed to investigate the effect of varying key parameters on the expected total cost. One parameter was varied at a time, keeping all others fixed at their base values.

6.1 Effect of Demand Variability (σ)

When σ increased from 15 to 25, safety stock rose from 78 to 130 units, and ETC increased by nearly 20%. Figure 1 shows that total cost rises sharply as demand uncertainty grows.

6.2 Effect of Holding Cost (h)

As h increased from 0.5 to 1.0, carrying costs doubled, reducing the optimal order size to approximately 1250 units. Figure 2 illustrates that higher holding costs discourage large inventories.

6.3 Effect of Service Level (z)

At 90% service level (z = 1.28), safety stock was 62 units, while at 99% service level (z = 2.33), safety stock rose to 110 units, increasing ETC by about 12%. This demonstrates that higher service levels reduce shortage but increase total cost (Figure 3).

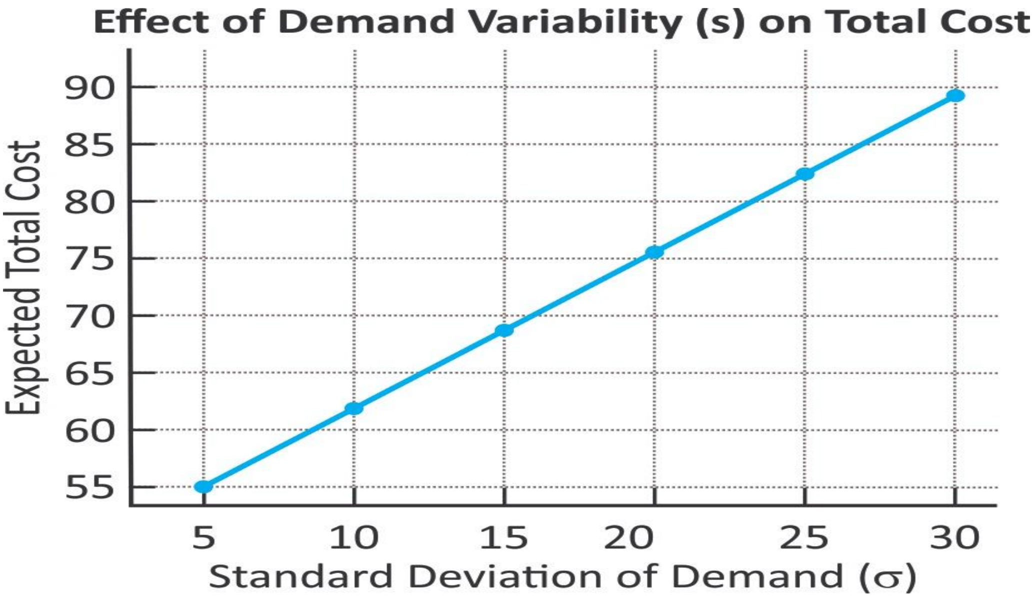

Figure 1: Effect of demand variability (σ) on expected total cost.

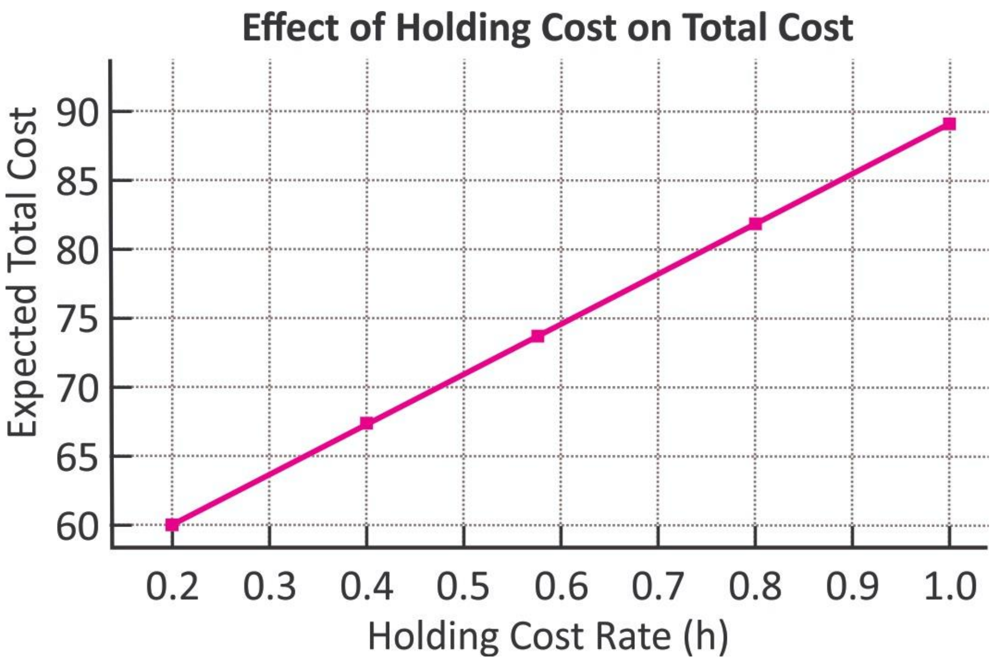

Figure 2: Effect of holding cost on expected total cost.

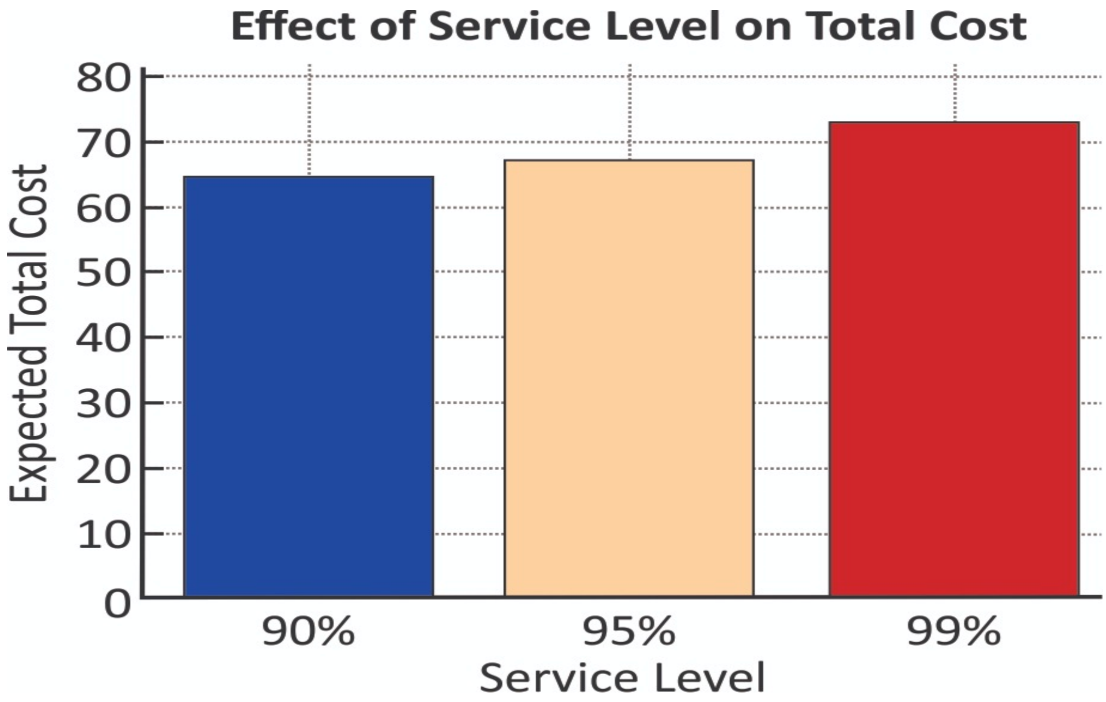

Figure 3: Effect of service level on expected total cost.

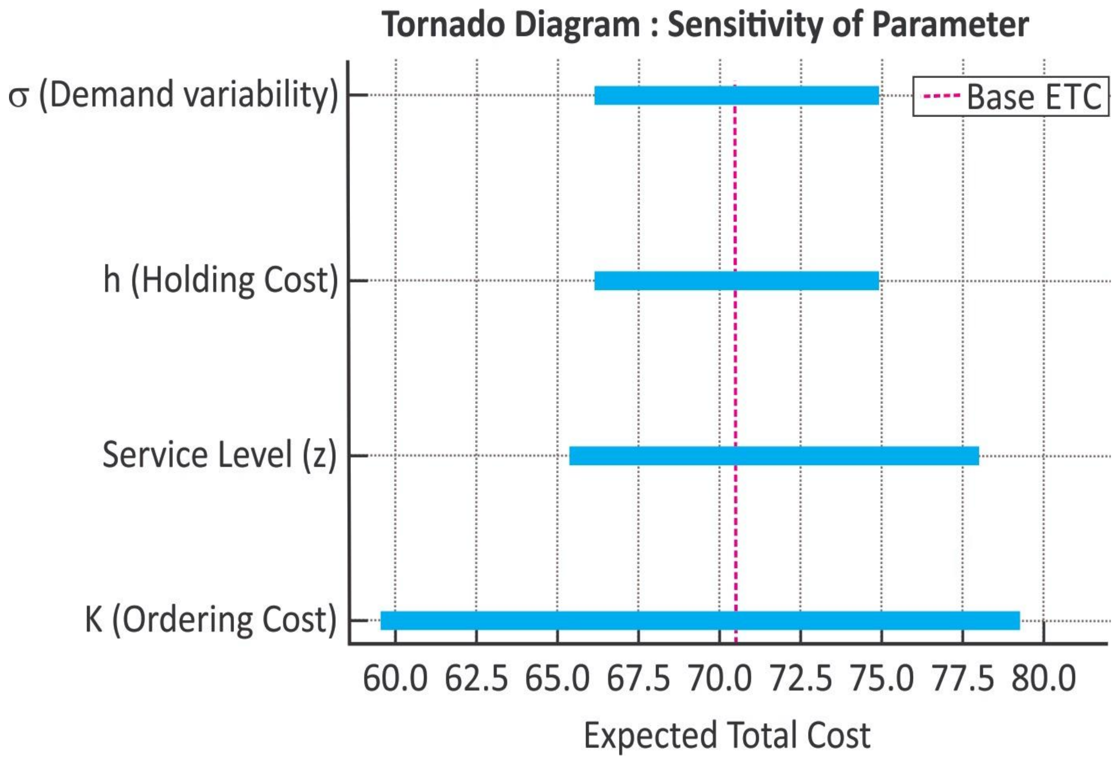

Figure 4: Tornado diagram showing sensitivity of parameters on expected total cost.

6.4. Tornado Diagram

A tornado diagram was prepared to compare the relative influence of all key parameters. As shown in Figure 4, demand variability (□) had the greatest effect on total cost, followed by holding cost (h), service level (z), and ordering cost (K). This highlights that uncertainty in demand is the most critical factor influencing inventory costs.

7. Conclusion

In this paper, we have developed a probabilistic inventory model without trade credits in which demand follows a linear trend with an added random component. The model incorporates time-dependent holding costs and allows for shortages, making it suitable for real-world situations where demand uncertainty cannot be ignored and payment for goods must be made immediately.

A numerical example was presented to illustrate the applicability of the model, and the optimal order quantity was derived as the sum of expected demand and safety stock. The results show that uncertainty in demand significantly influences the total cost, and ignoring this variability may lead to suboptimal decisions.Sensitivity analysis revealed that demand variability (□) has the most significant impact on the expected total cost, followed by holding cost (h), service level (z), and ordering cost (K). In particular, higher variability in demand substantially increases safety stock requirements, while higher holding costs discourage maintaining large inventories. Similarly, higher service levels reduce the risk of shortages but increase the total cost.The findings emphasize the importance of considering stochastic demand in inventory decision-making, especially for businesses that operate without trade credit facilities. The model provides a decision-making framework that can be applied by retailers, wholesalers, and manufacturers facing linear growth in demand combined with uncertainty.

Future research can extend this work by incorporating multiple items, inflationary effects, deteriorating items, or trade credit policies to make the model more general and applicable to complex supply chain settings.

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